Helical chiral polyhedra

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- vertices (points)

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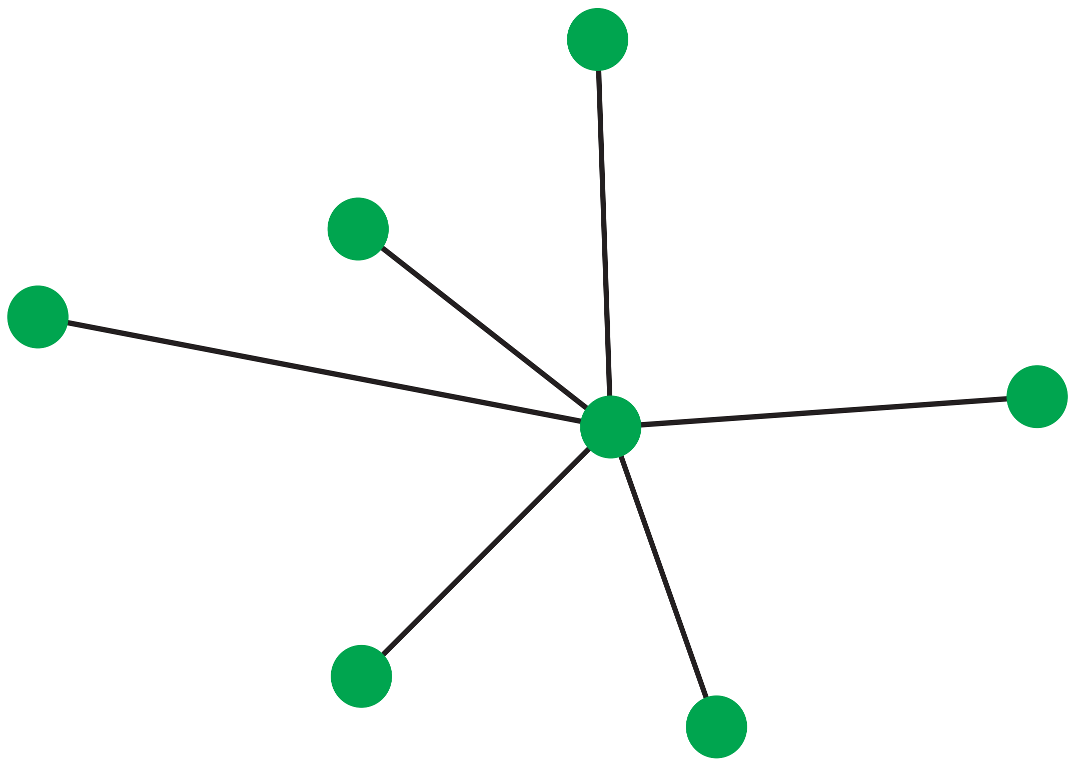
- The graph is connected
- the vertex-figures are cycles

Polyhedra



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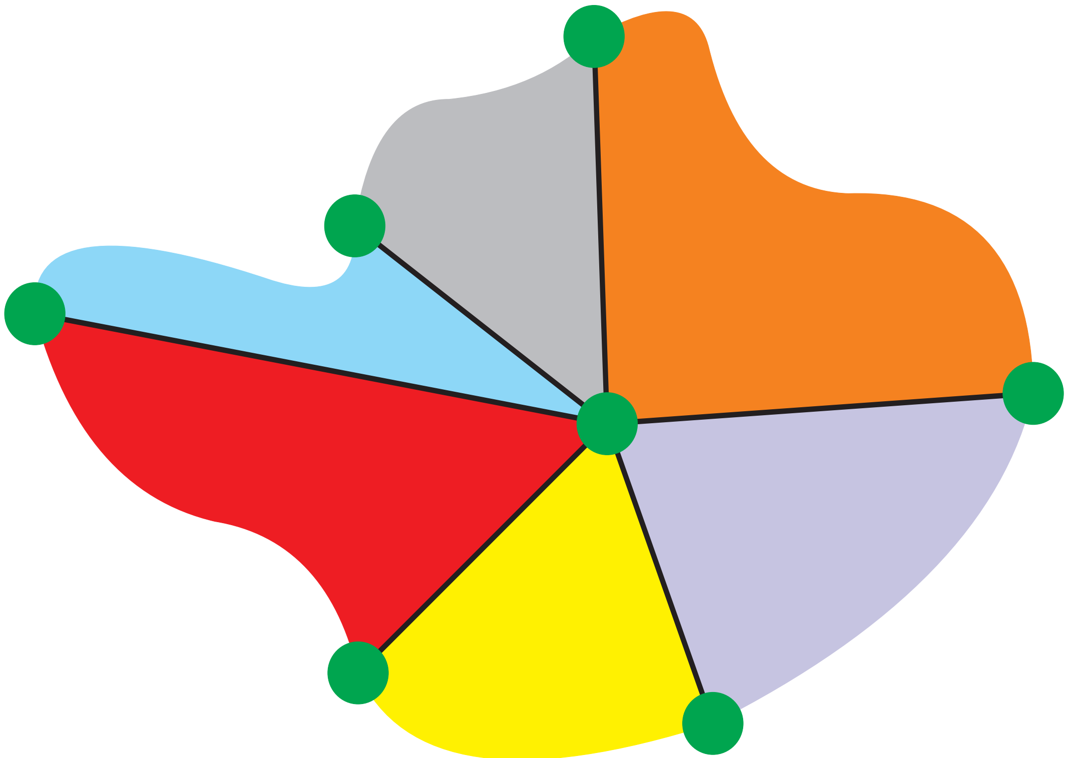

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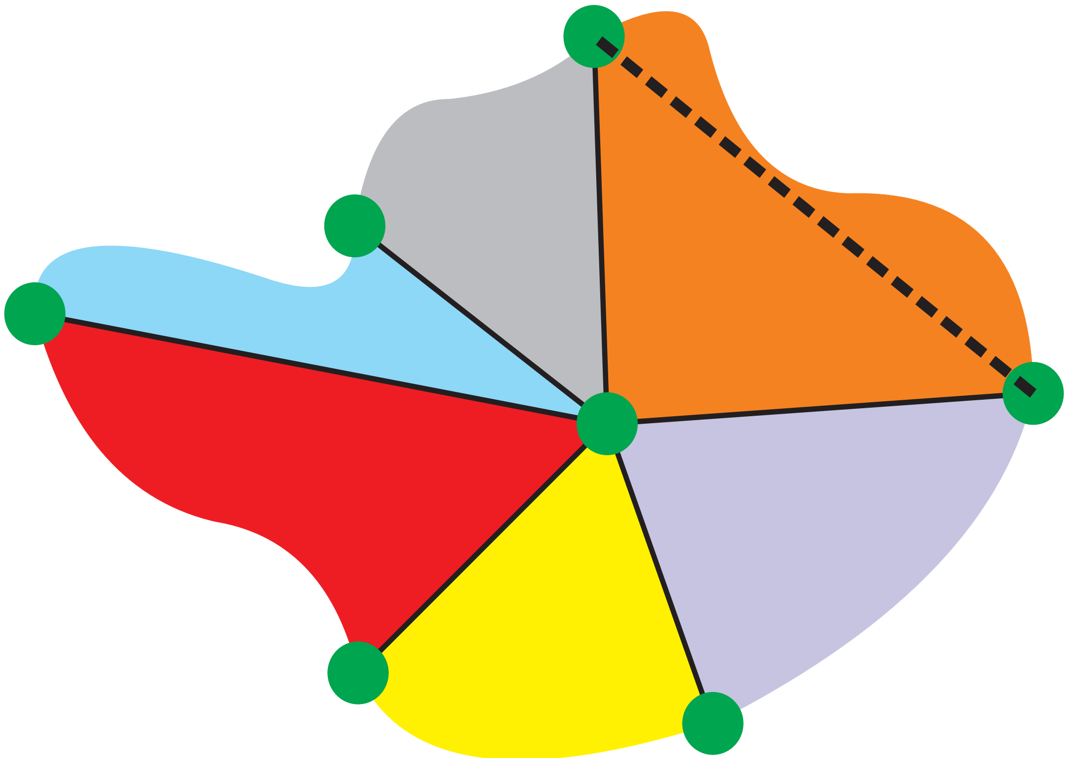

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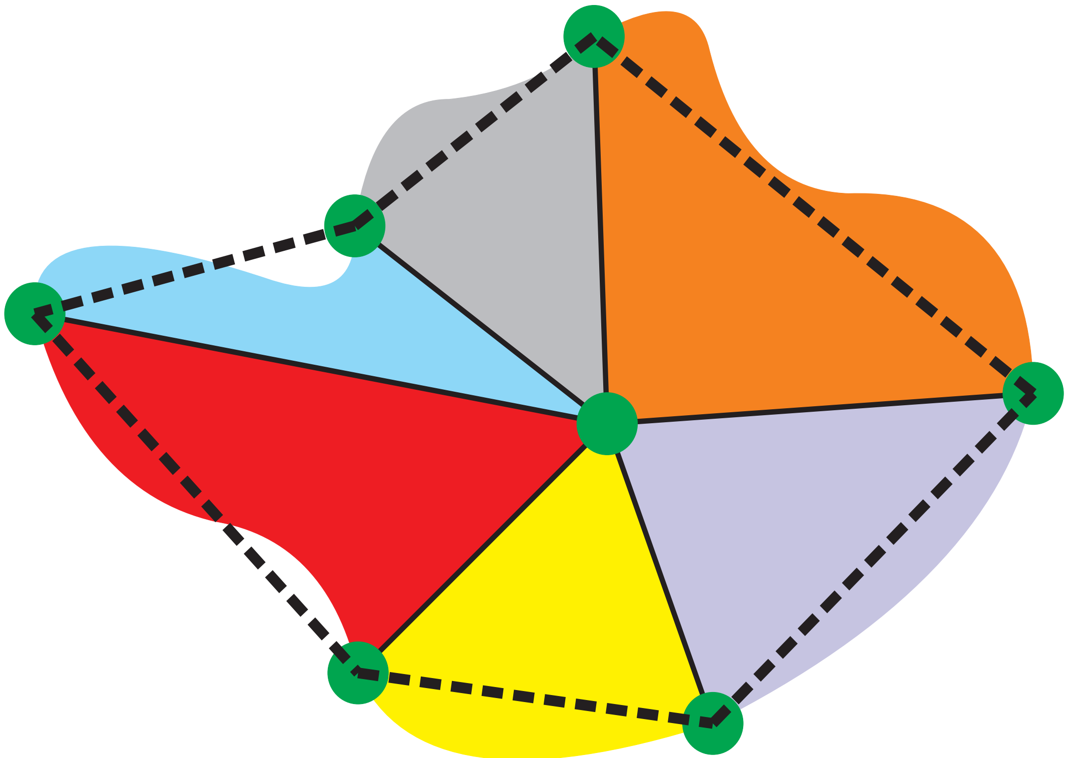

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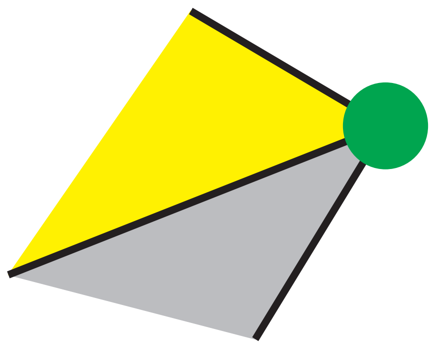

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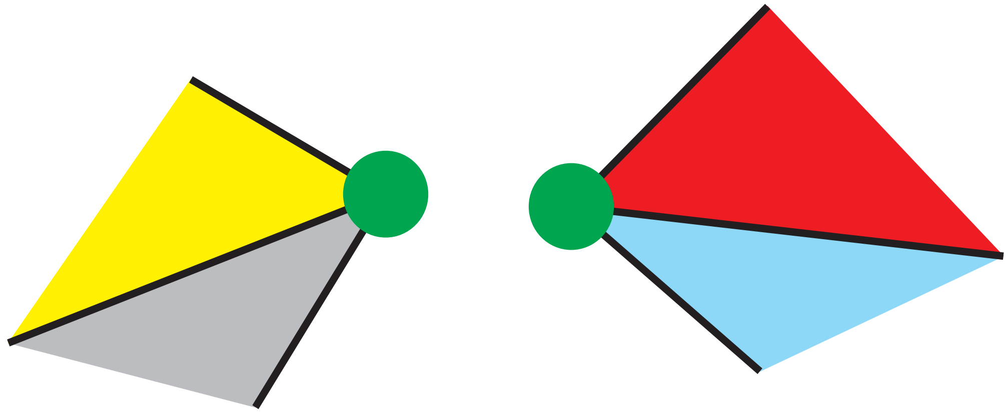

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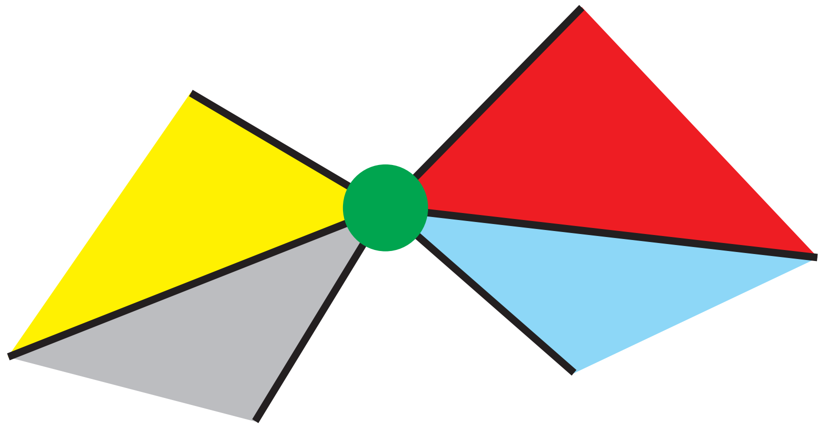

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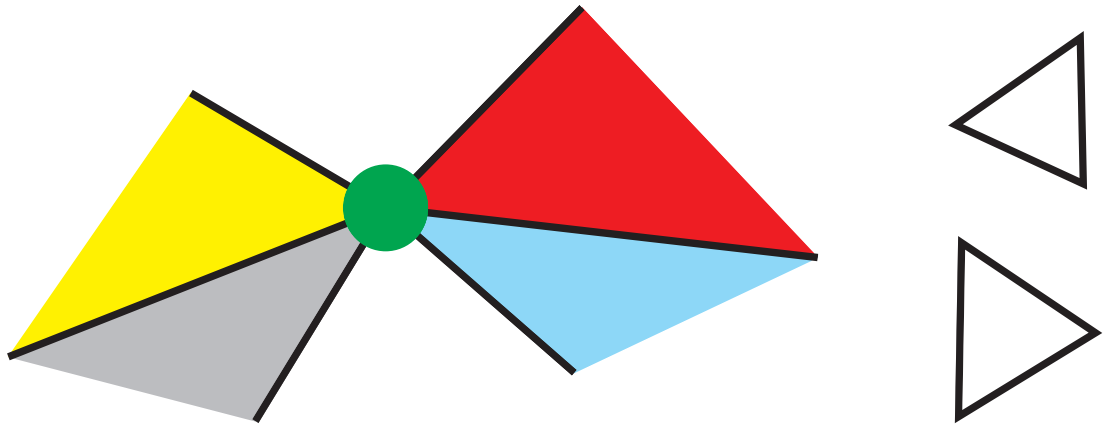

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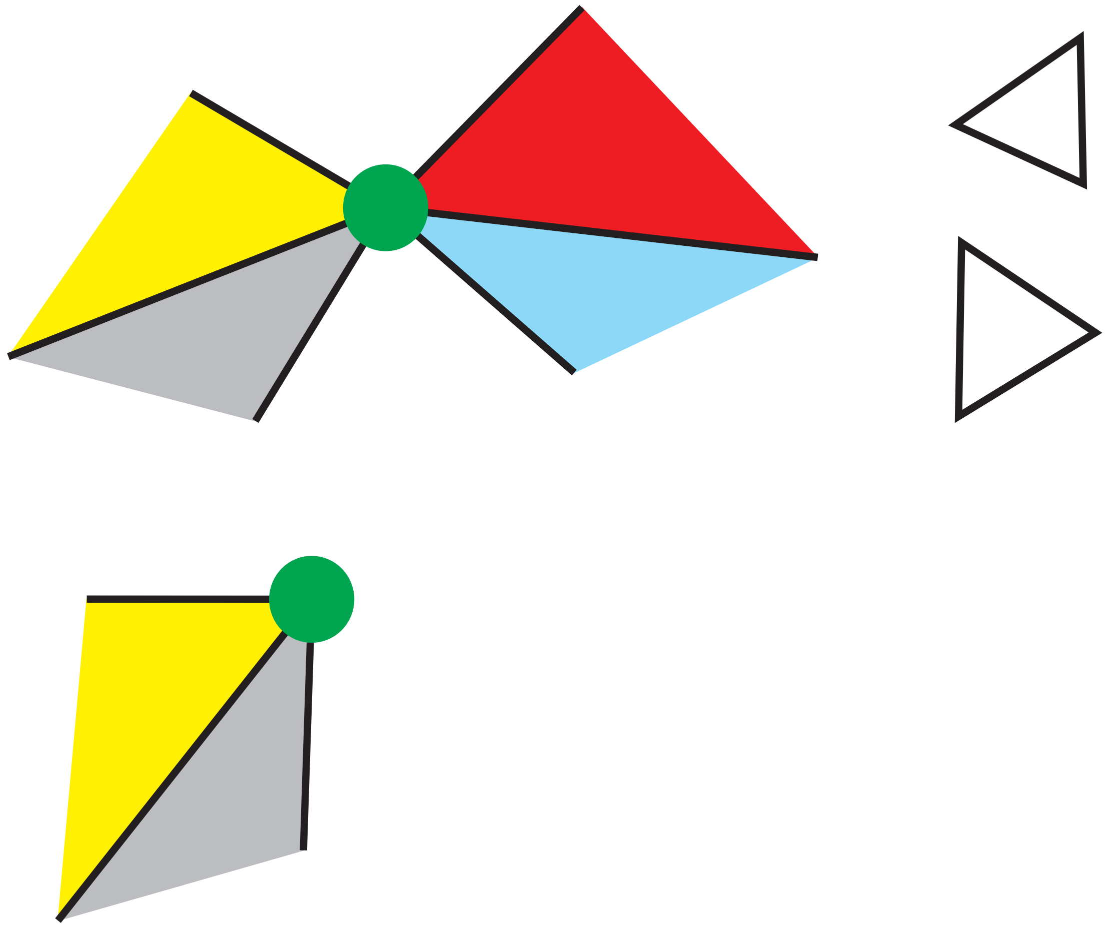

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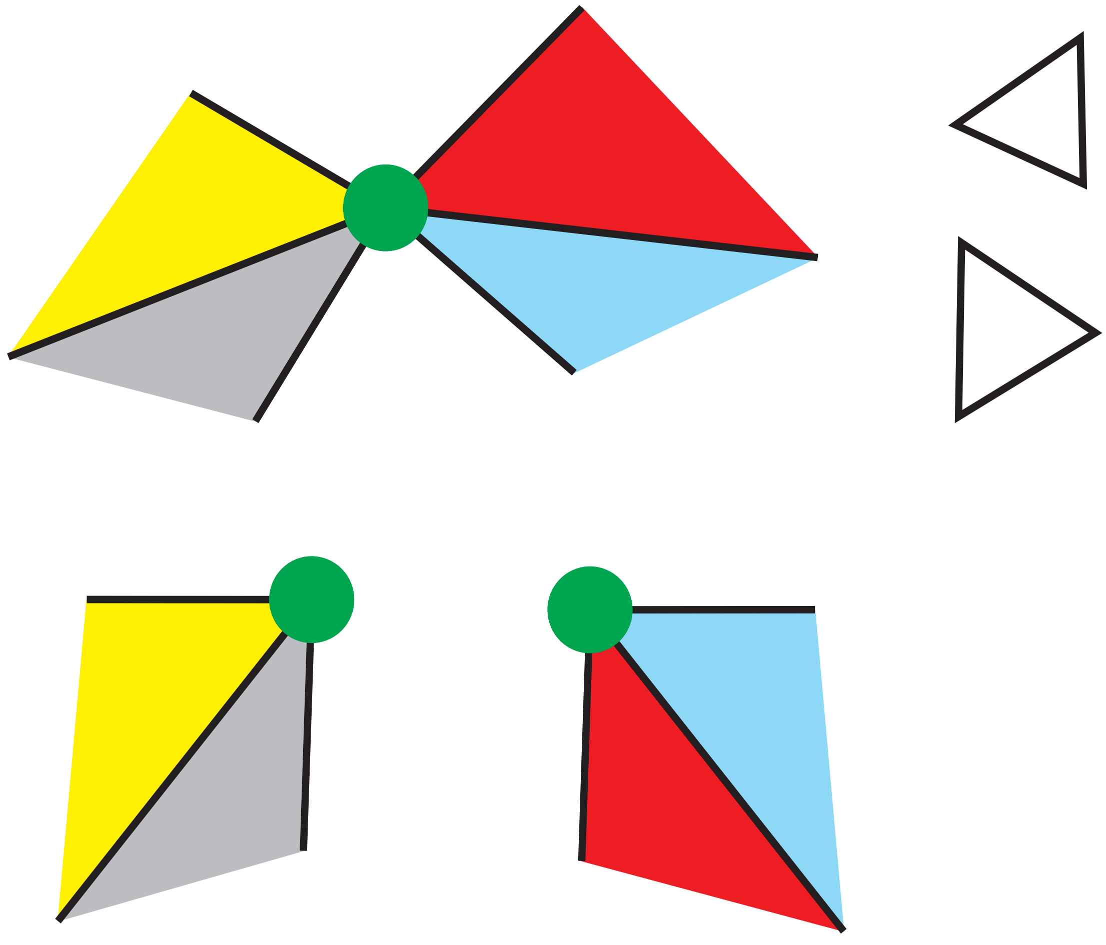

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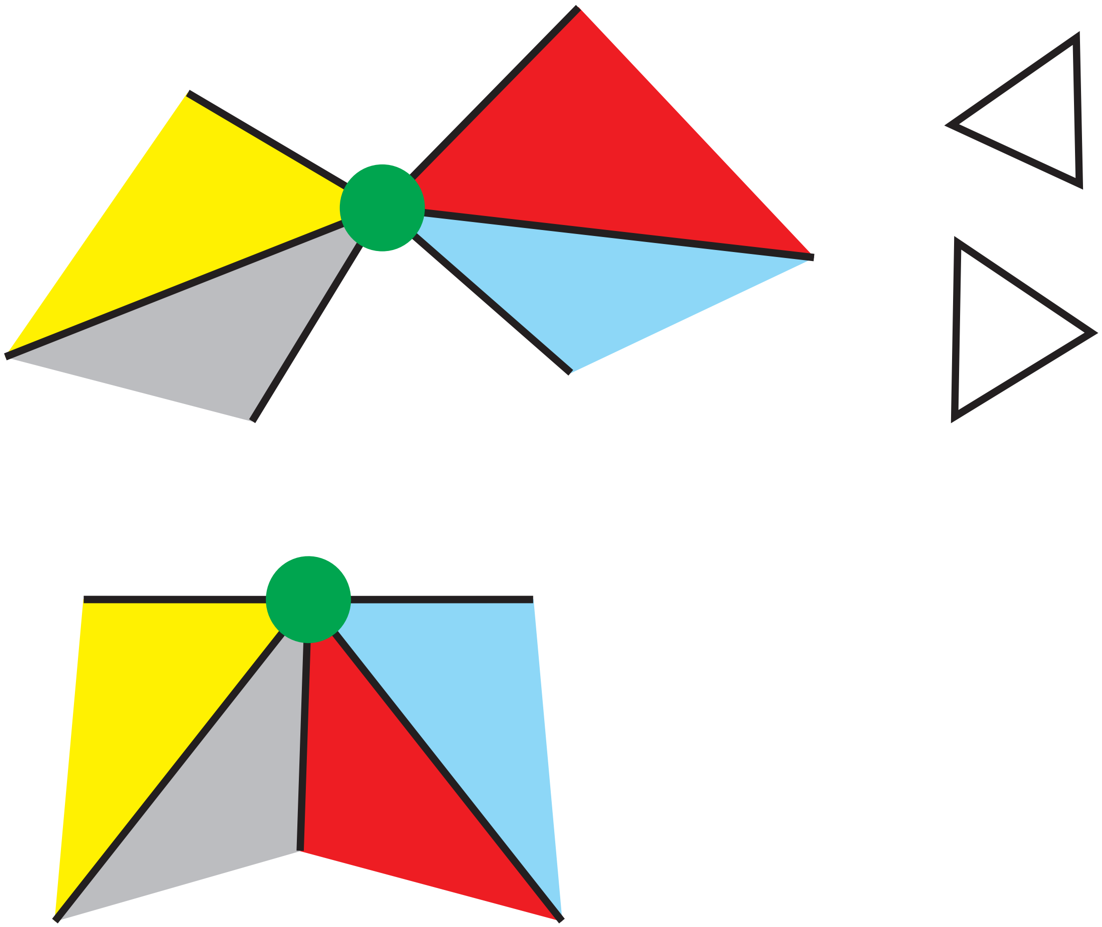

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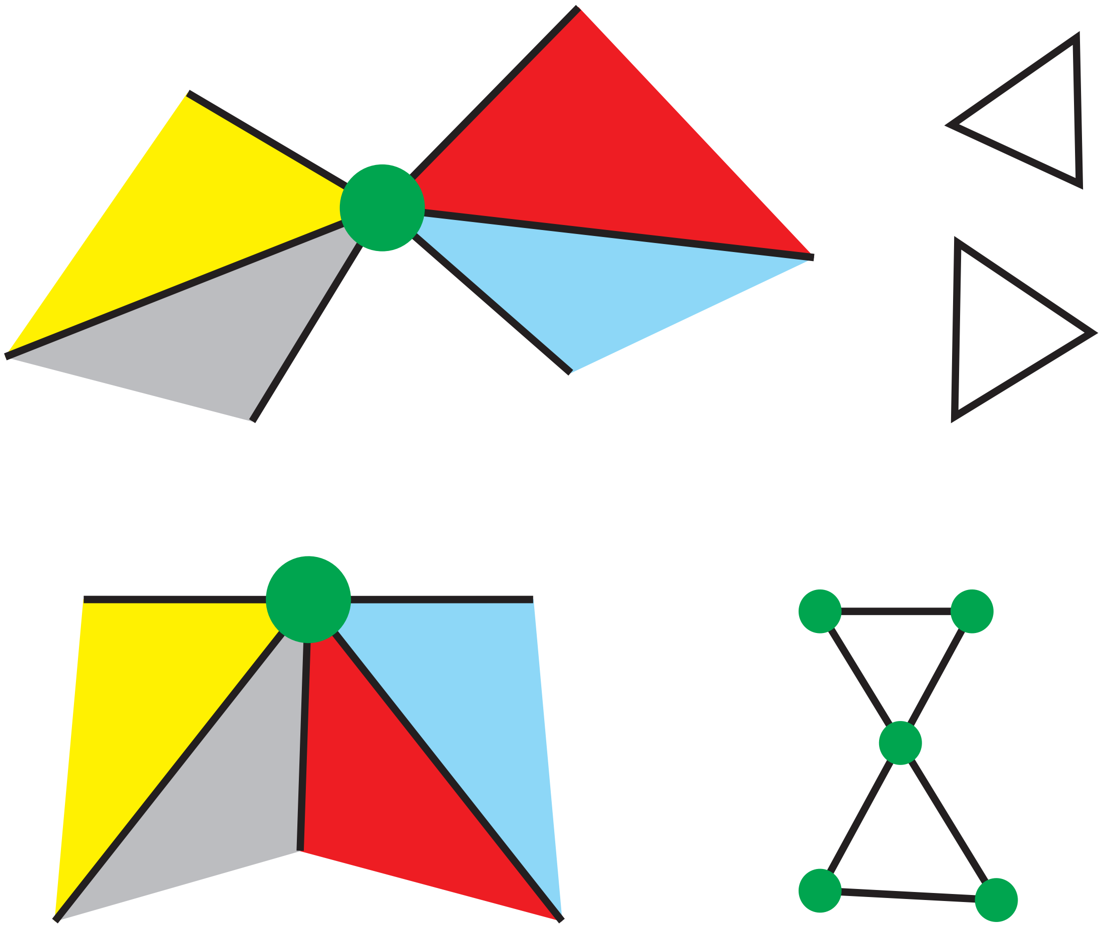

Examples

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- Convex polyhedra

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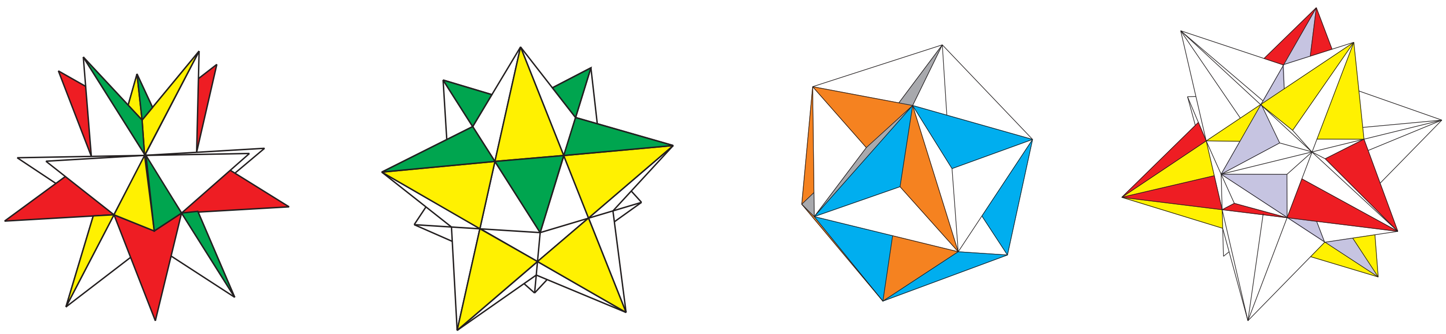

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▶ Faces may or may not be planar!

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▶ Faces may or may not be planar!
▶ Faces may or may not be finite!

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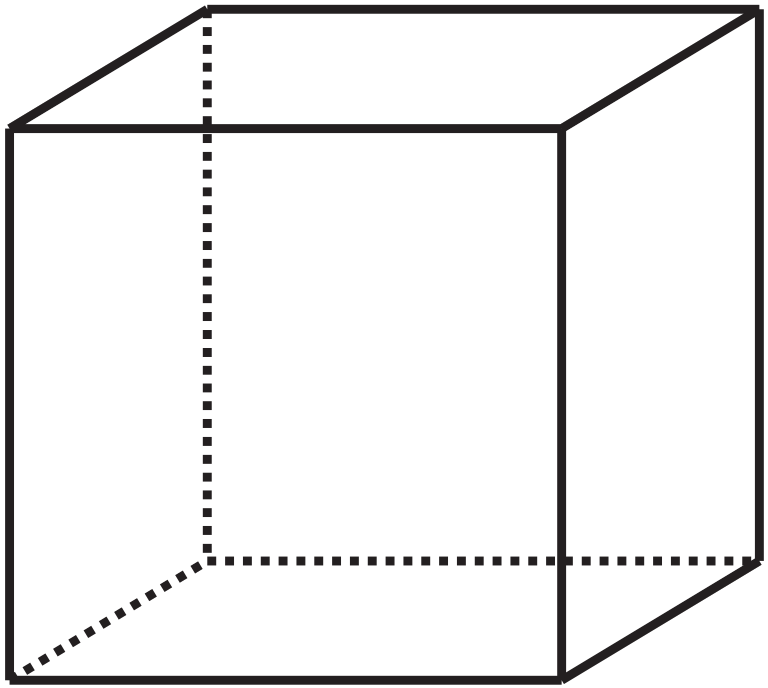

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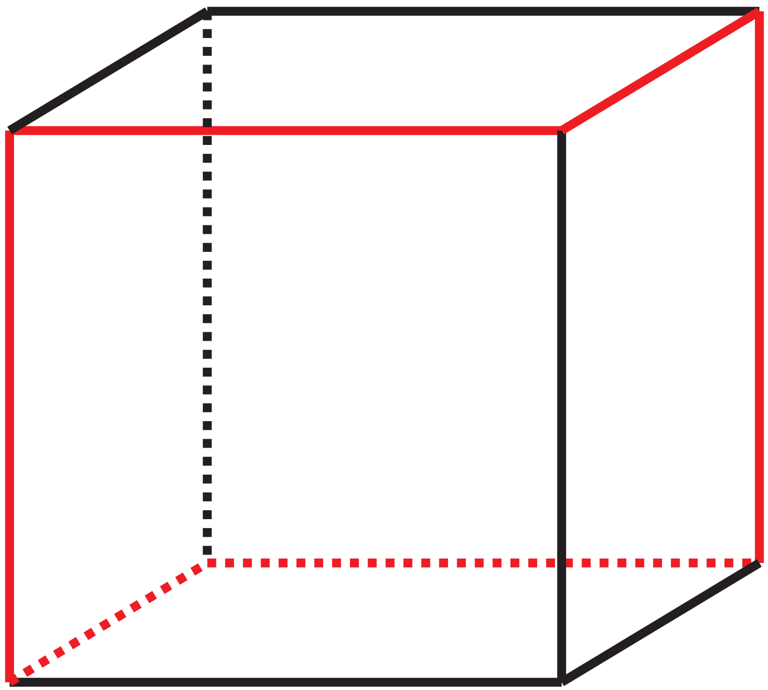

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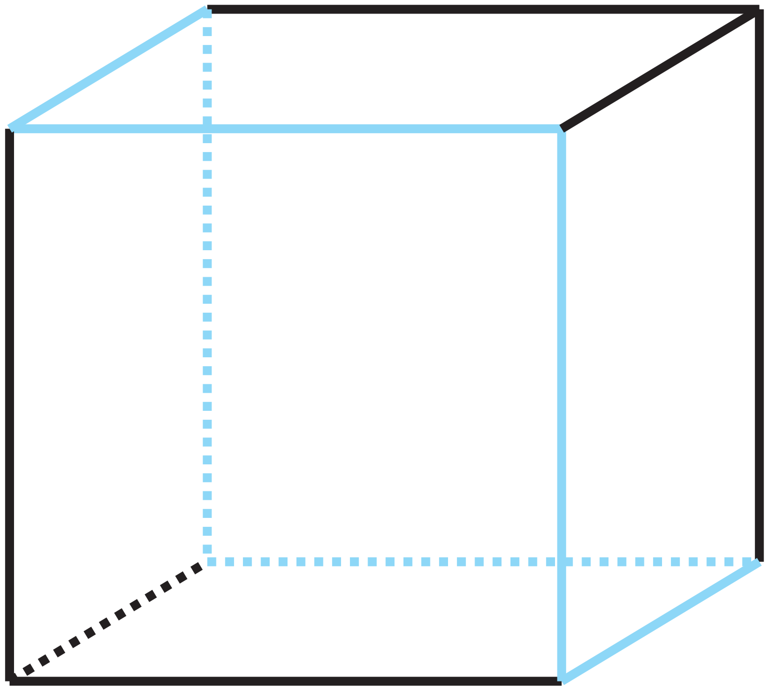

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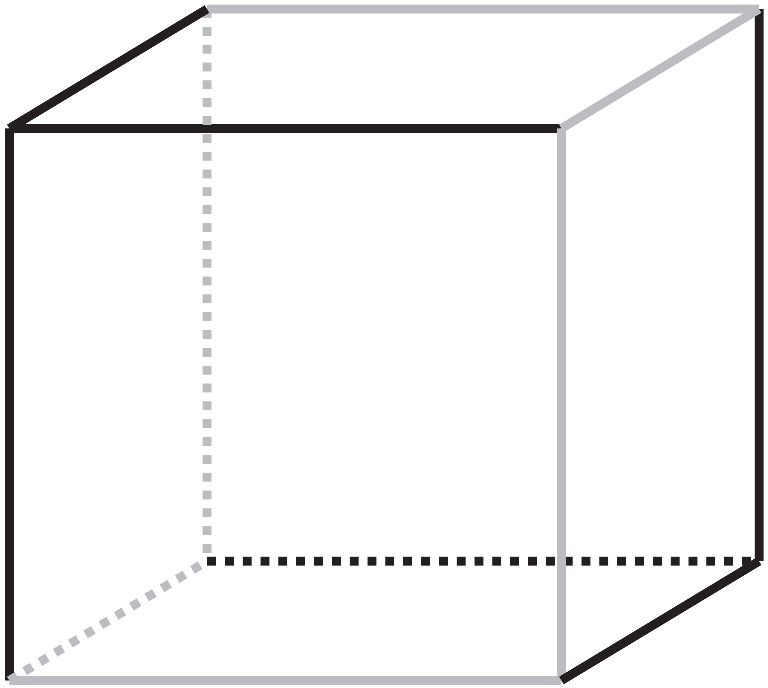

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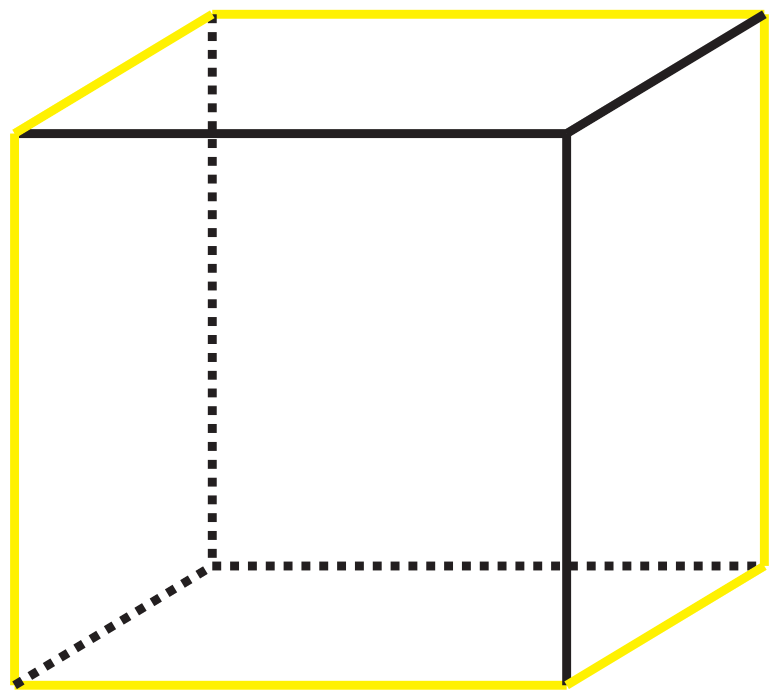

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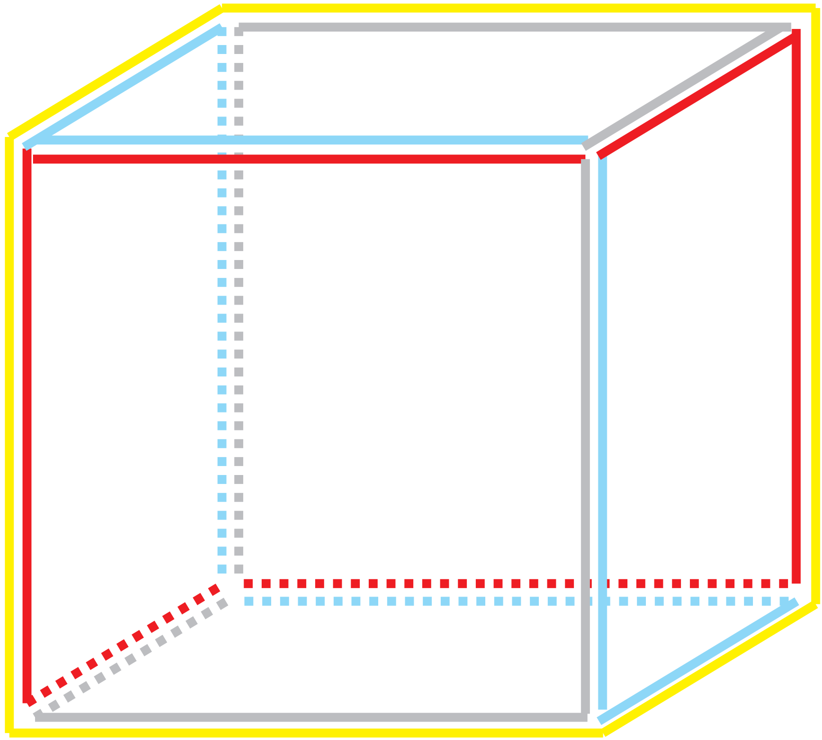

Regular polyhedra

- **flag** $\longrightarrow$ triple of incident vertex, edge and face

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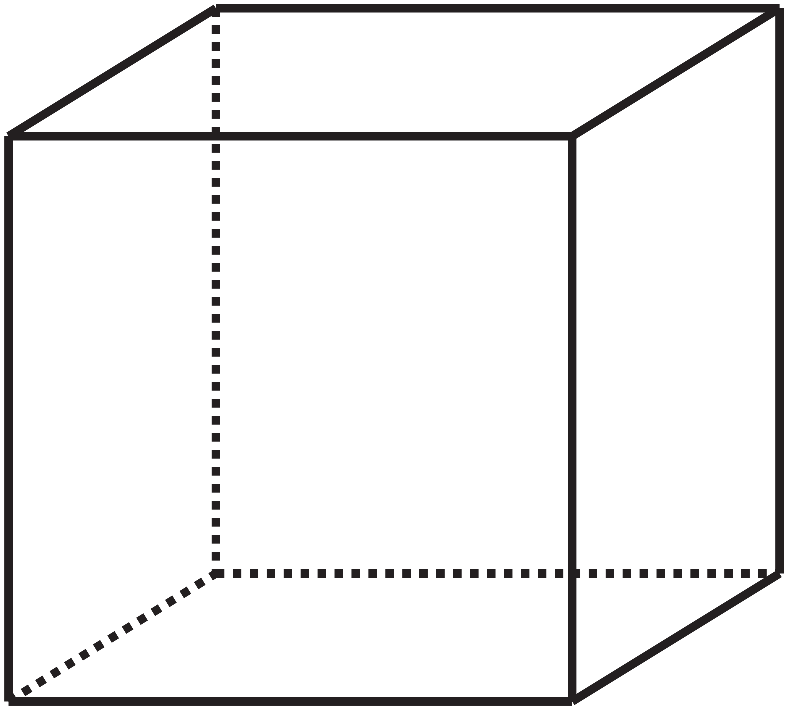

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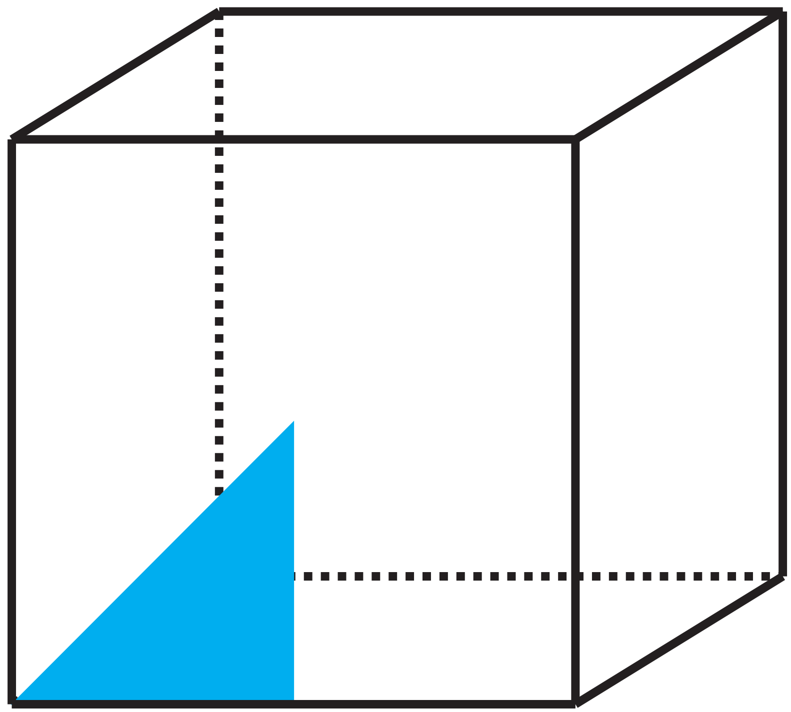

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- All polygons are combinatorially regular
- All rectangular prisms are combinatorially regular

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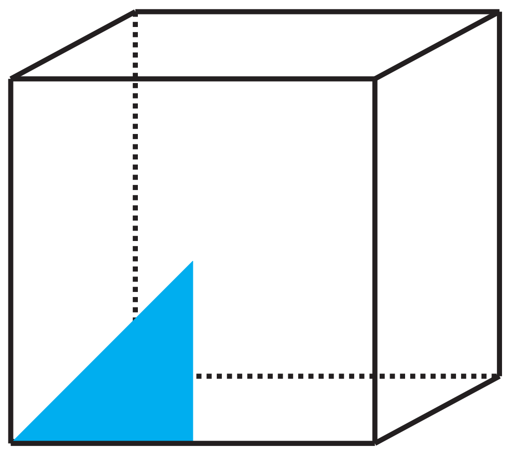

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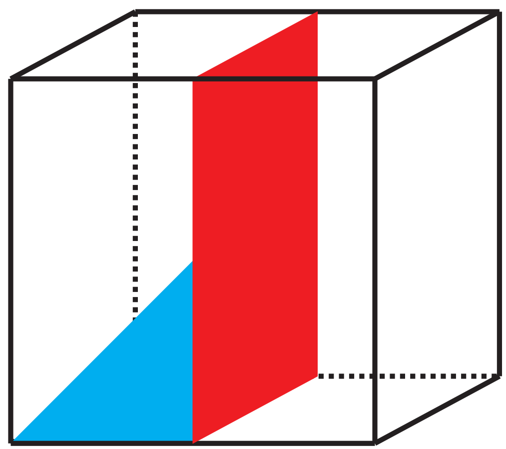

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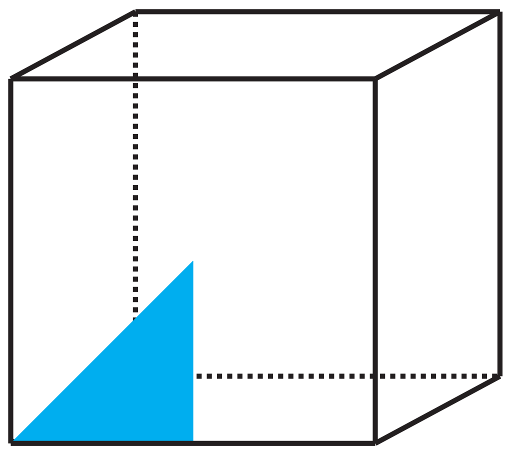
ρ_1 changes the edge while fixing the vertex and face



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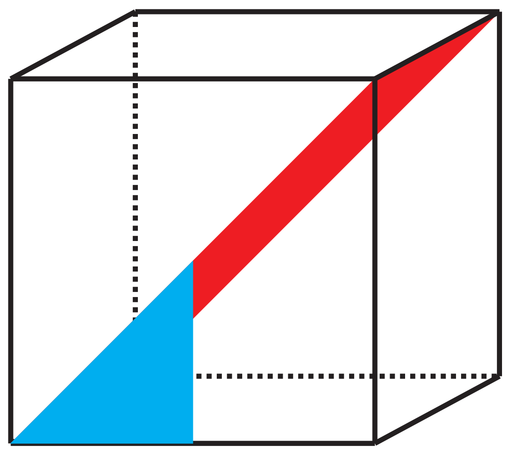

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$Sym(\mathcal{P}) = \langle \rho_0, \rho_1, \rho_2 \rangle$

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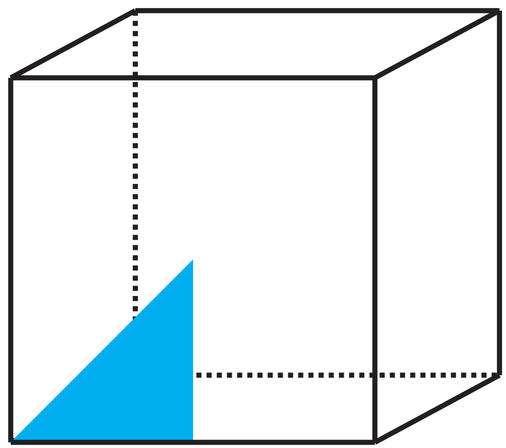

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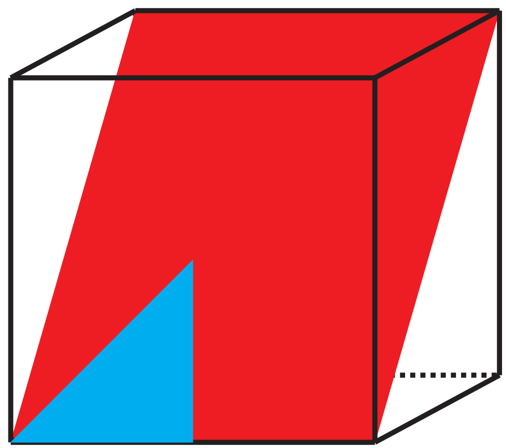

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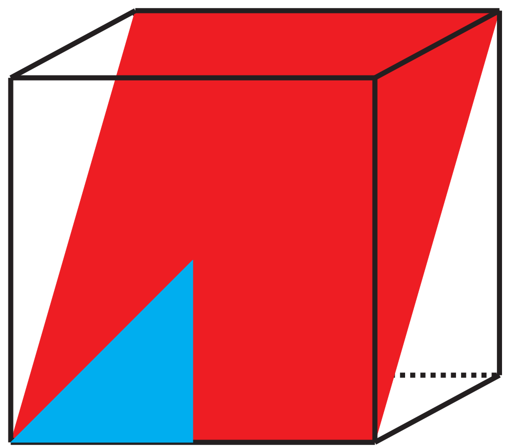
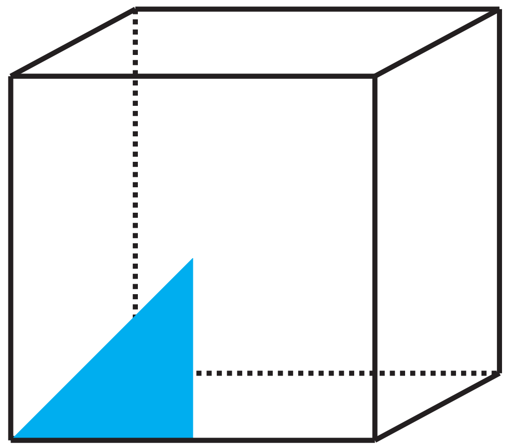


They are not always reflections!

Regular polyhedra

$\sigma_1 := \rho_0\rho_1$ rotates along the face



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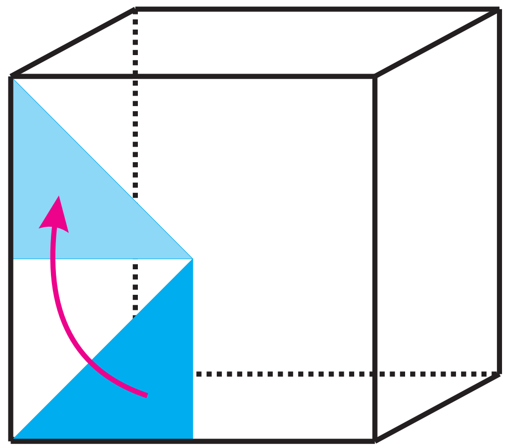

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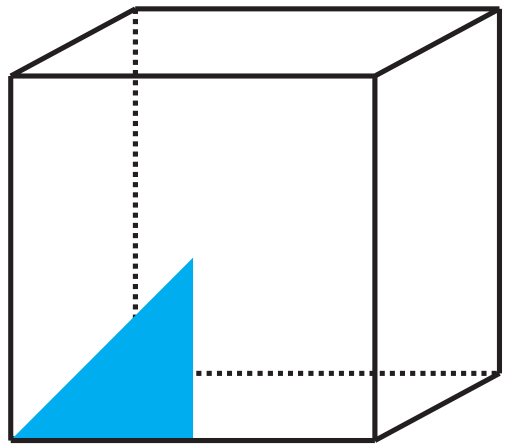

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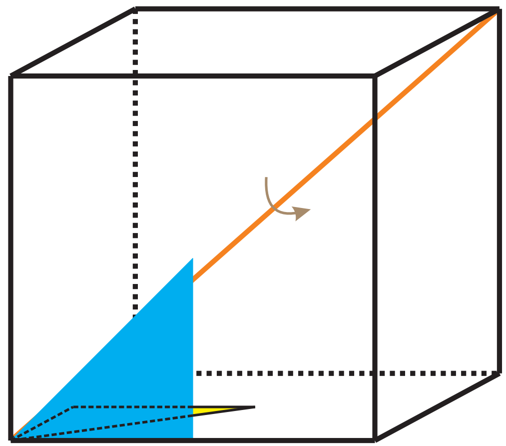

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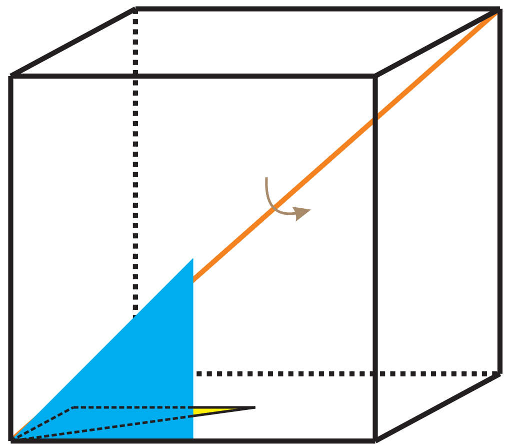

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Regular polyhedra in $\mathbb{E}^3$

- Planar faces

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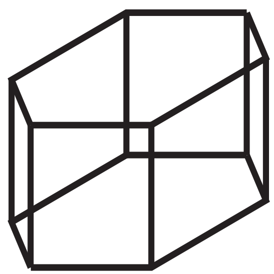

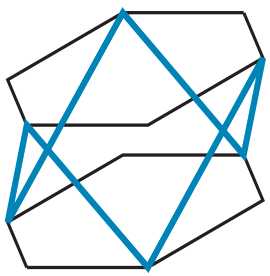

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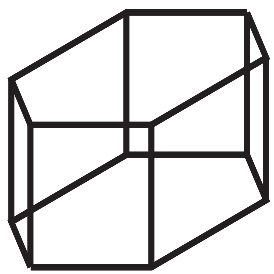

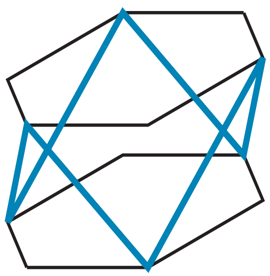

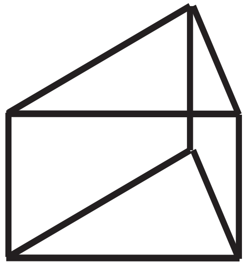

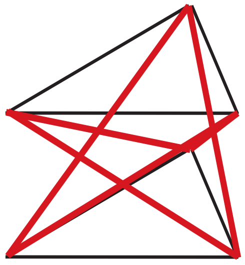

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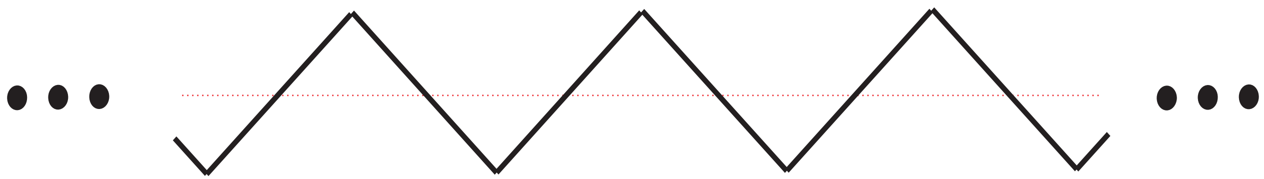

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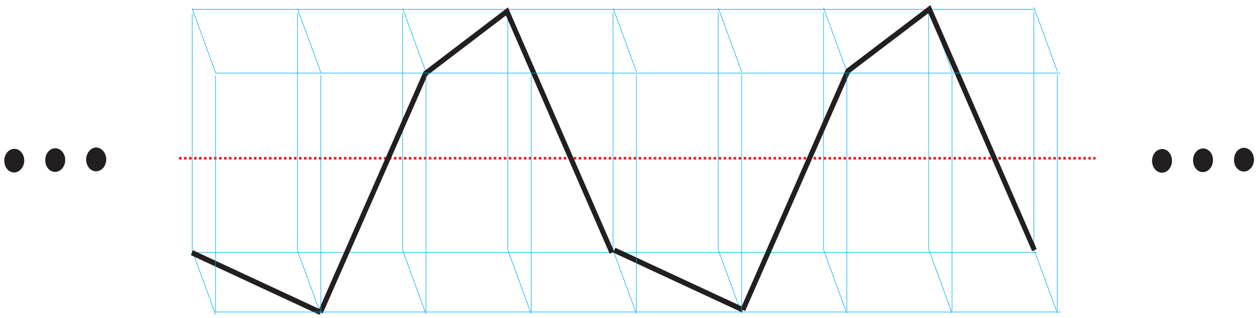

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Regular polyhedra in $\mathbb{E}^3$ (Grünbaum, Dress)

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- 6 infinite with finite skew faces

Regular polyhedra in $\mathbb{E}^3$

Regular polyhedra in $\mathbb{E}^3$ (Grünbaum, Dress)

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- 9 with zigzag faces

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Regular polyhedra in $\mathbb{E}^3$ (Grünbaum, Dress)

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Regular polyhedra in $\mathbb{E}^3$ (Grünbaum, Dress)

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- 6 infinite with finite planar faces
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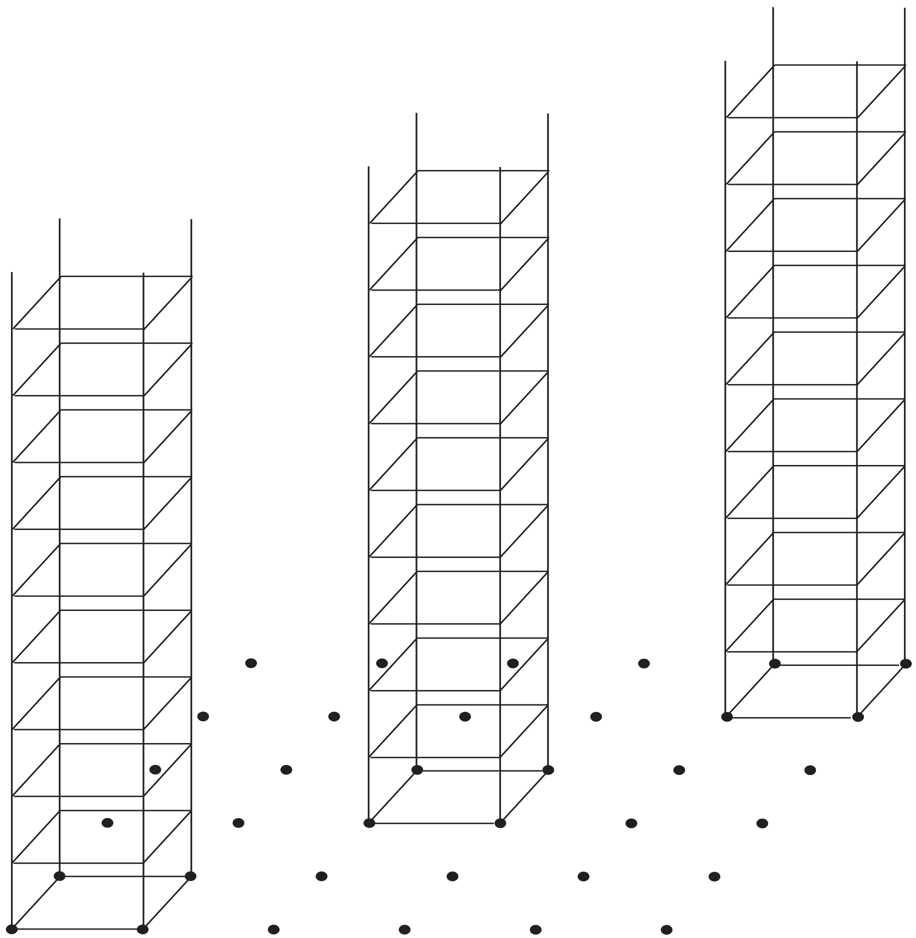
- 9 with zigzag faces
- 9 with helical faces
  - 6 with skew vertex-figures
  - 3 with planar vertex-figures

$\{\infty, 3\}^{(b)}$



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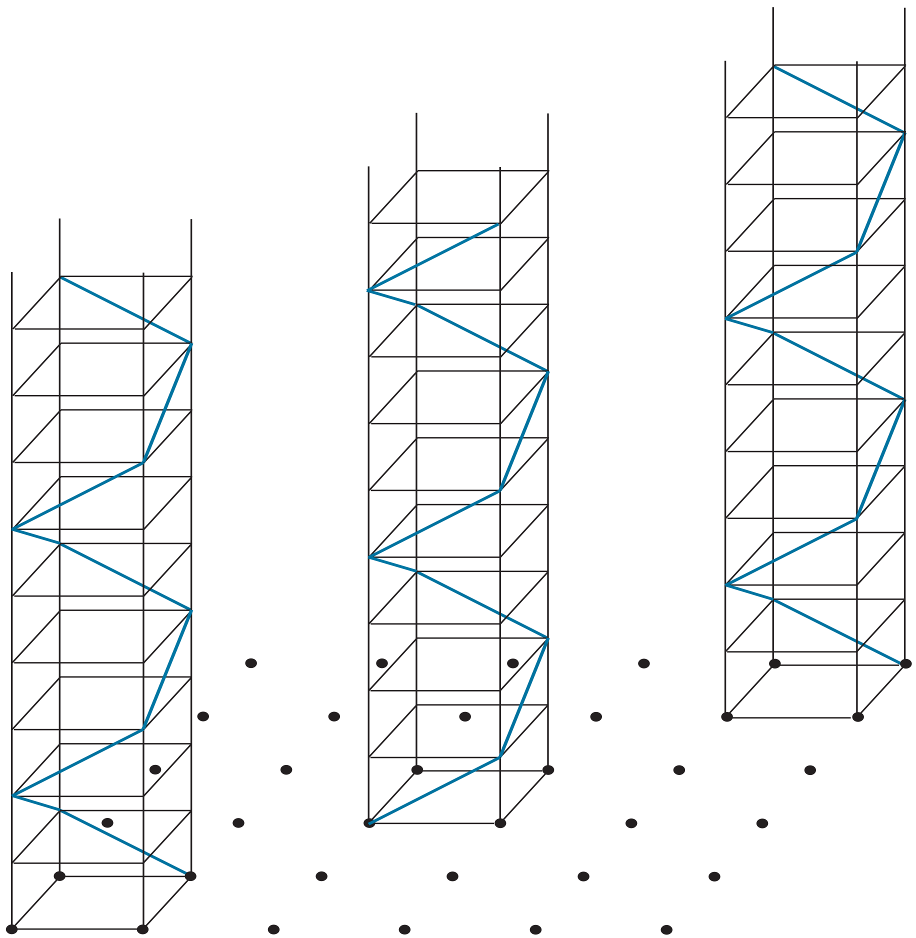

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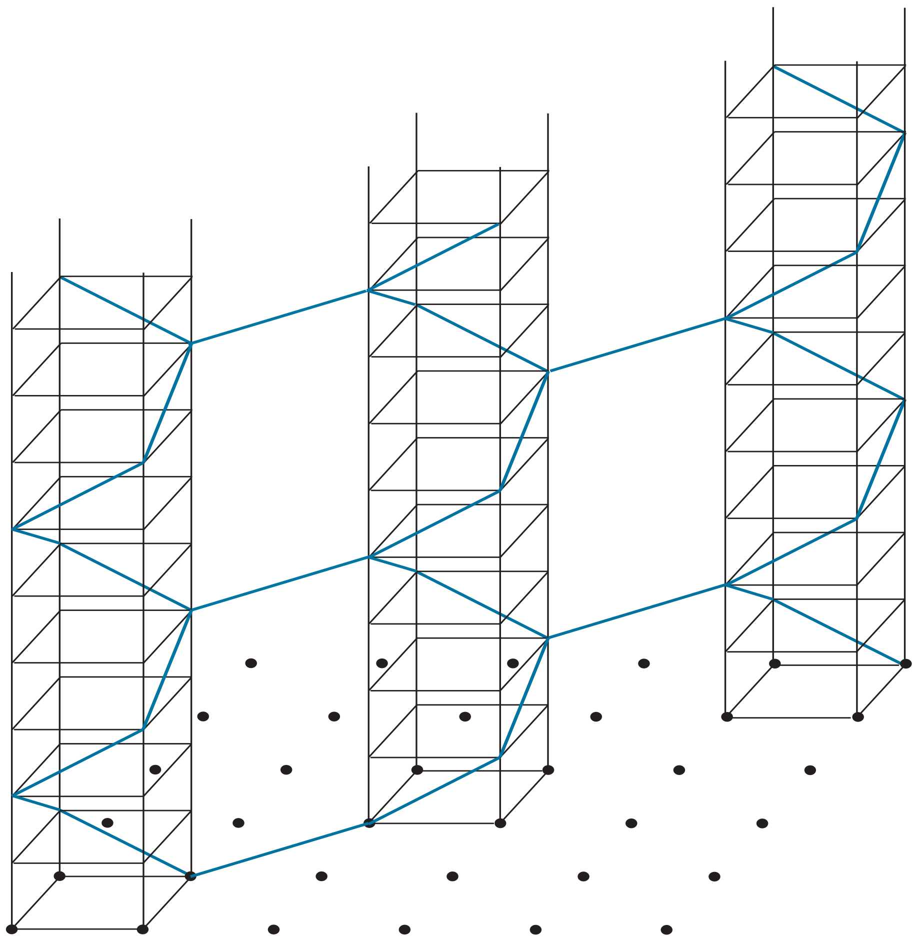

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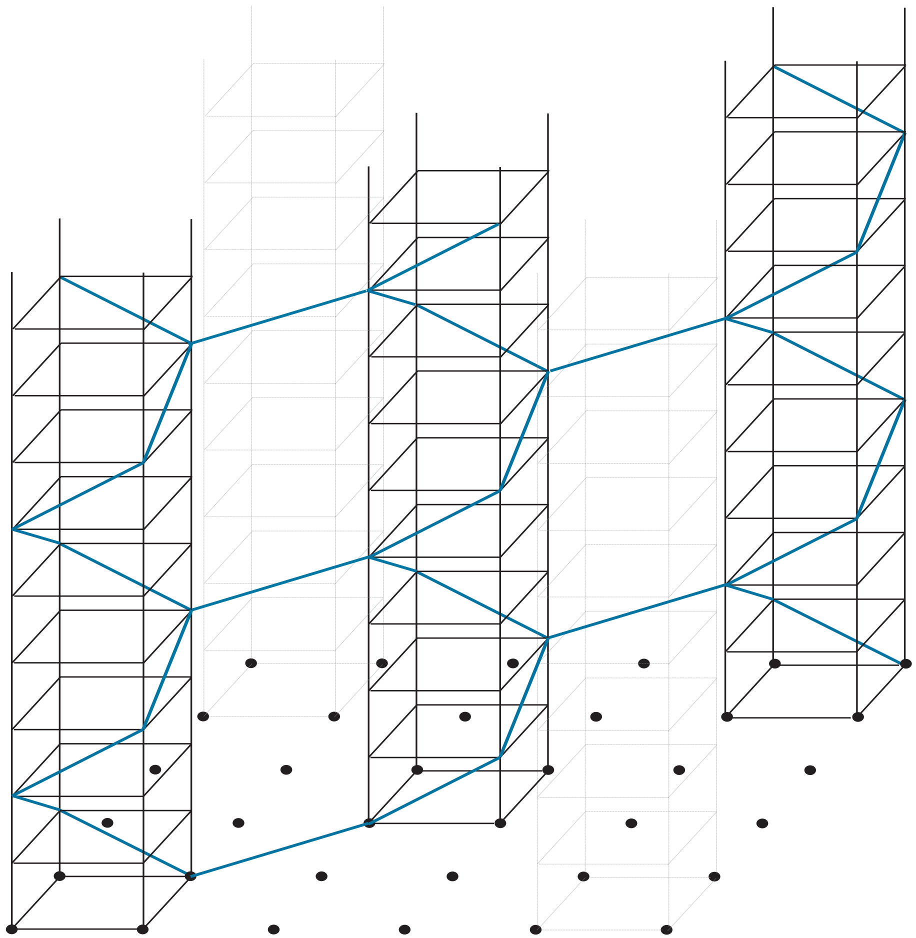

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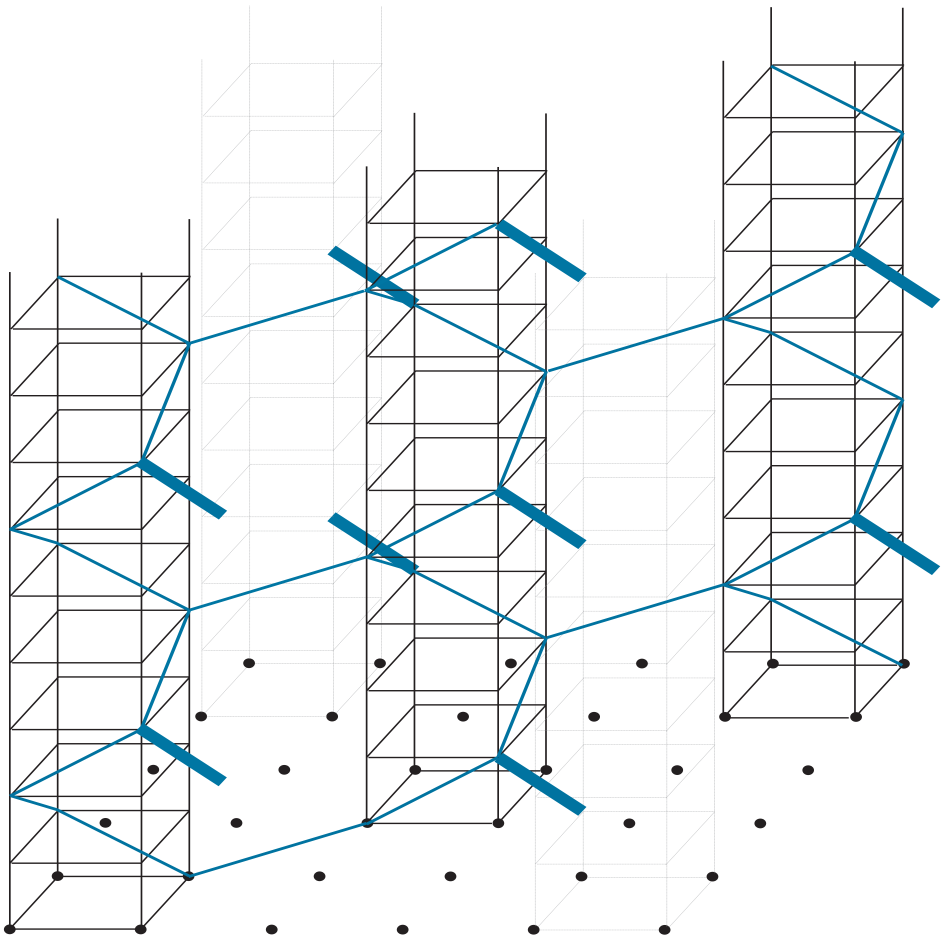

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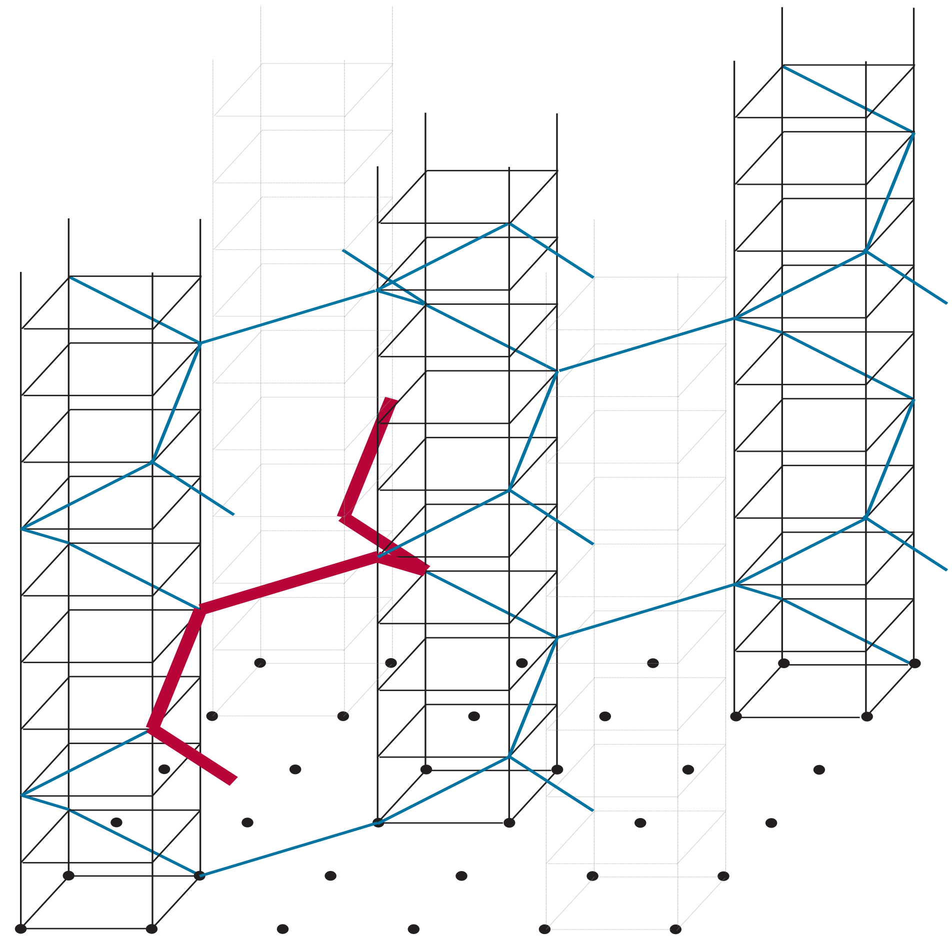

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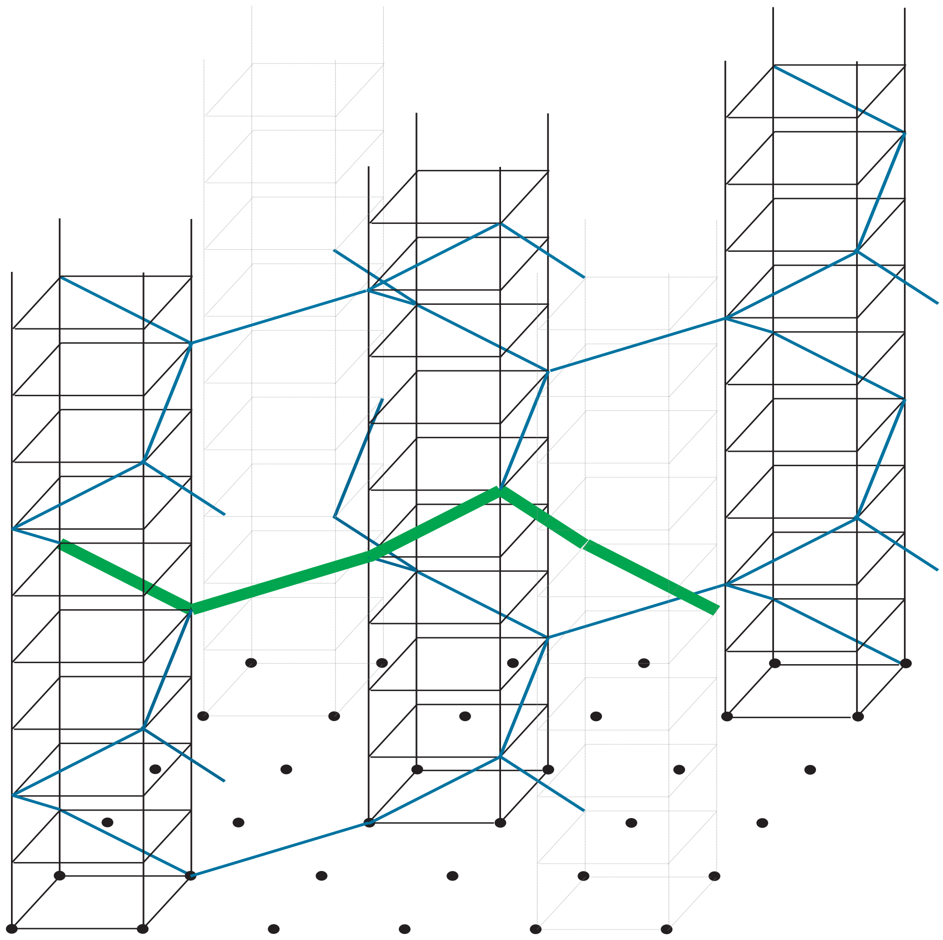

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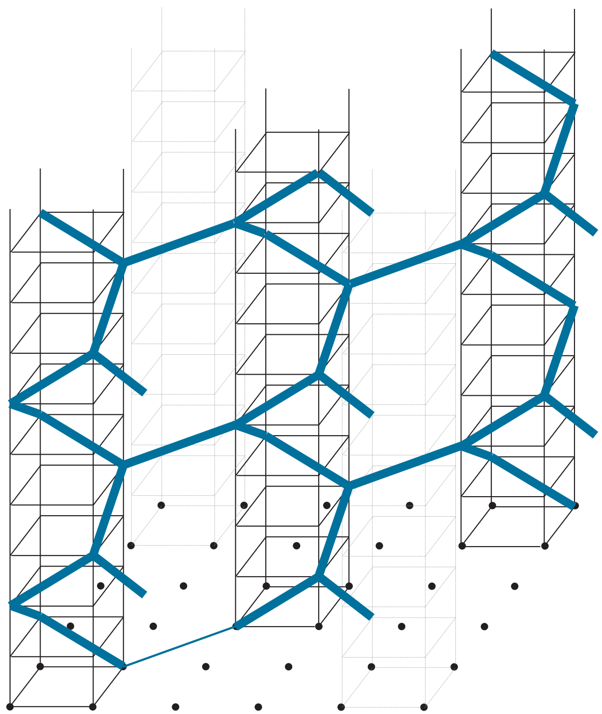

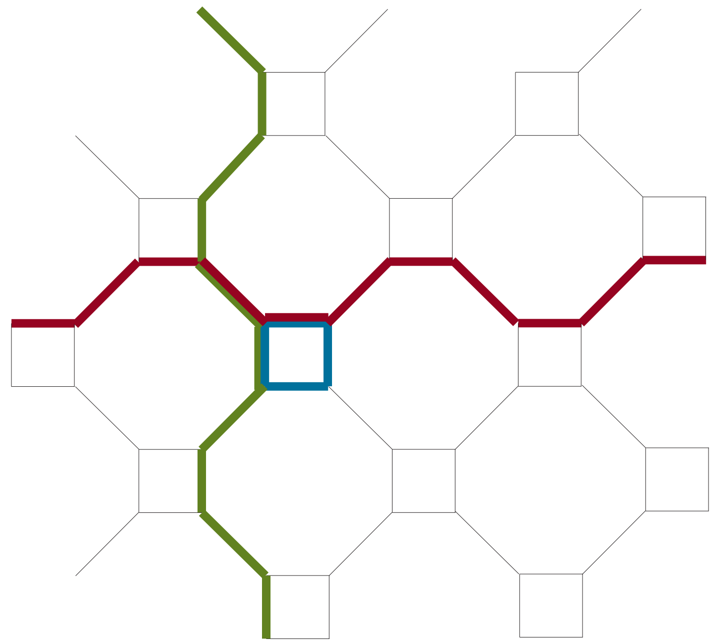

The projective space $\mathbb{P}^3$

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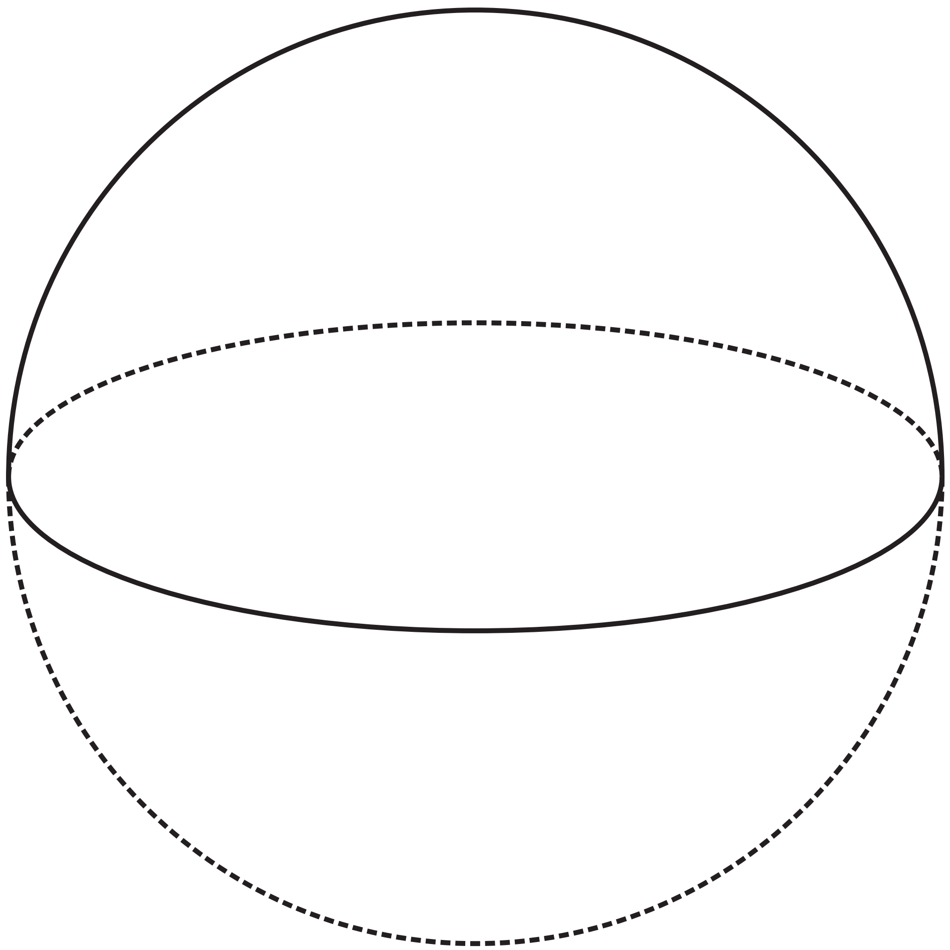

The projective space $\mathbb{P}^3$

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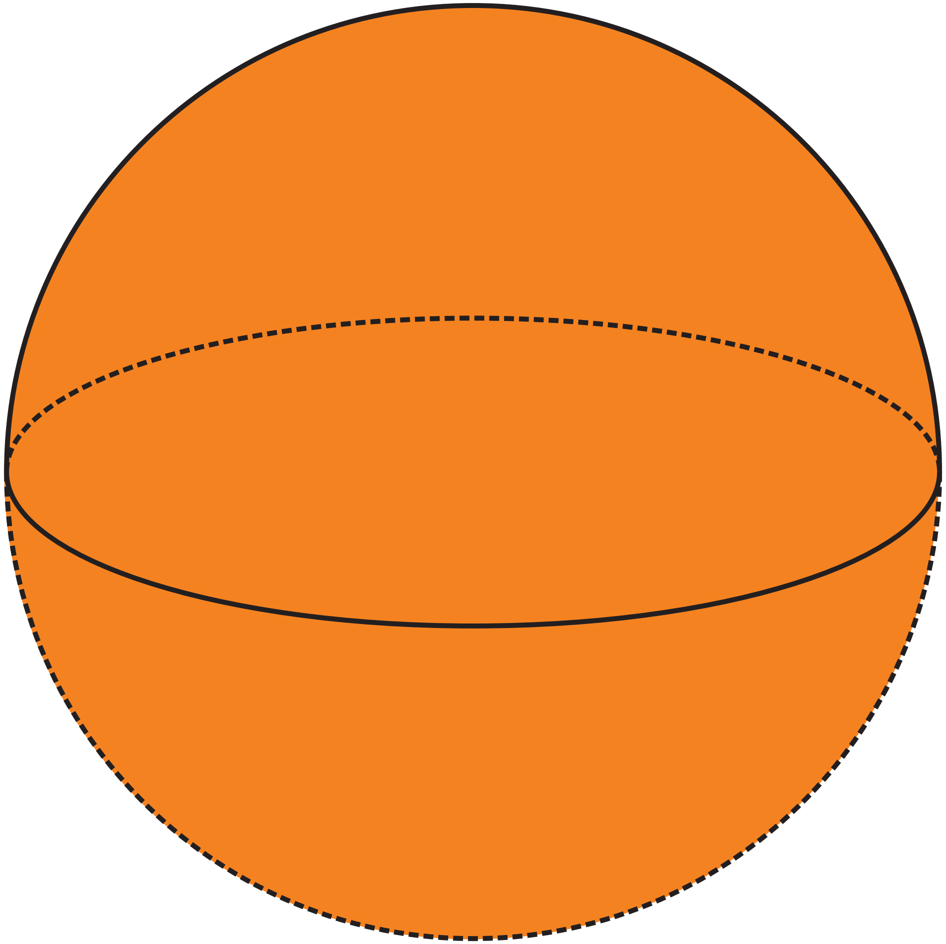
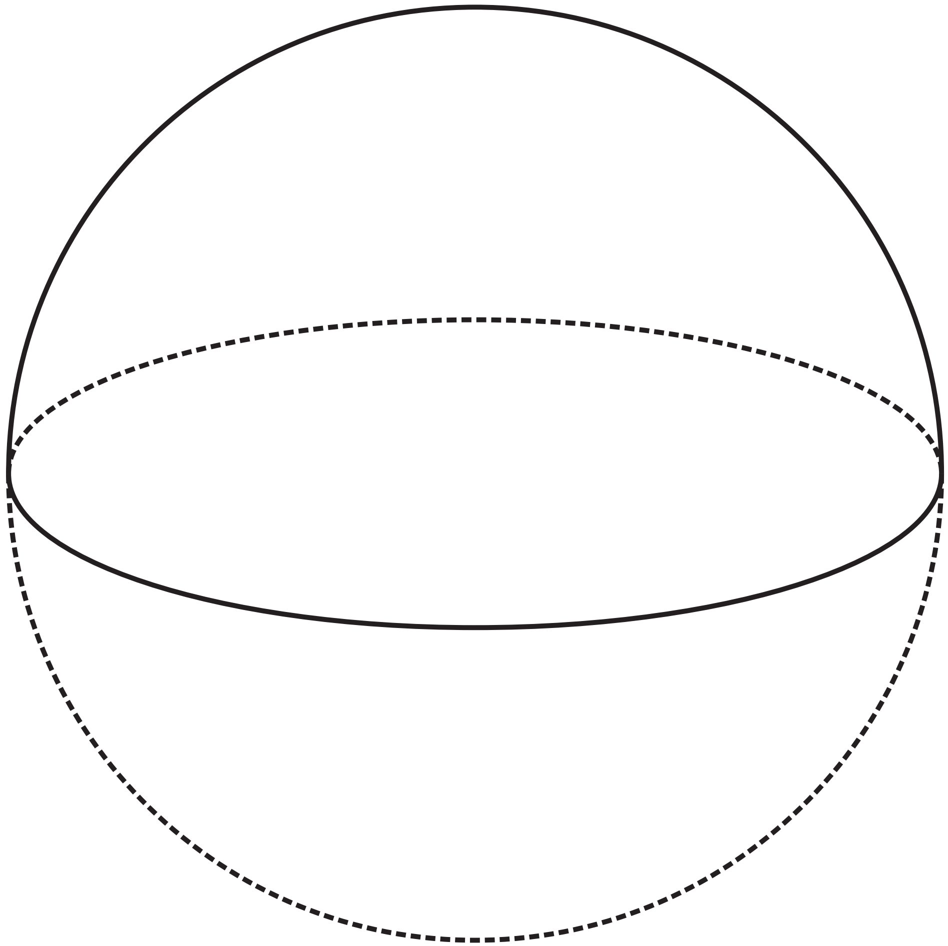


Lines



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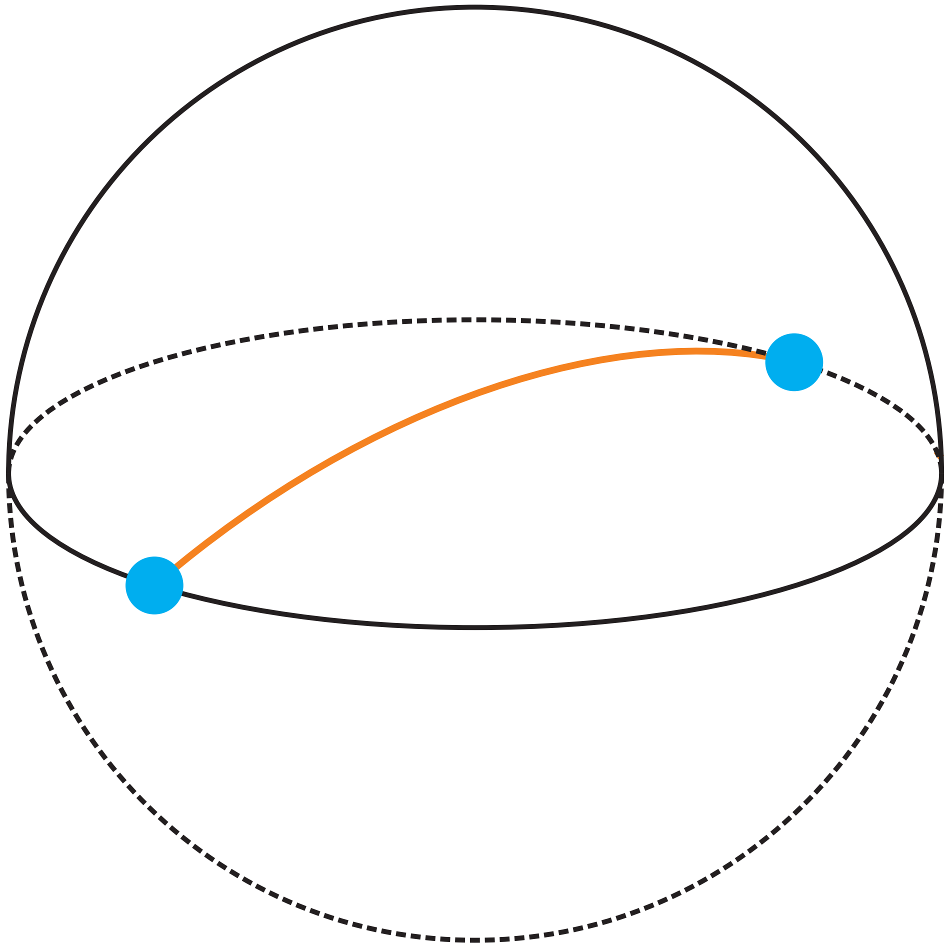

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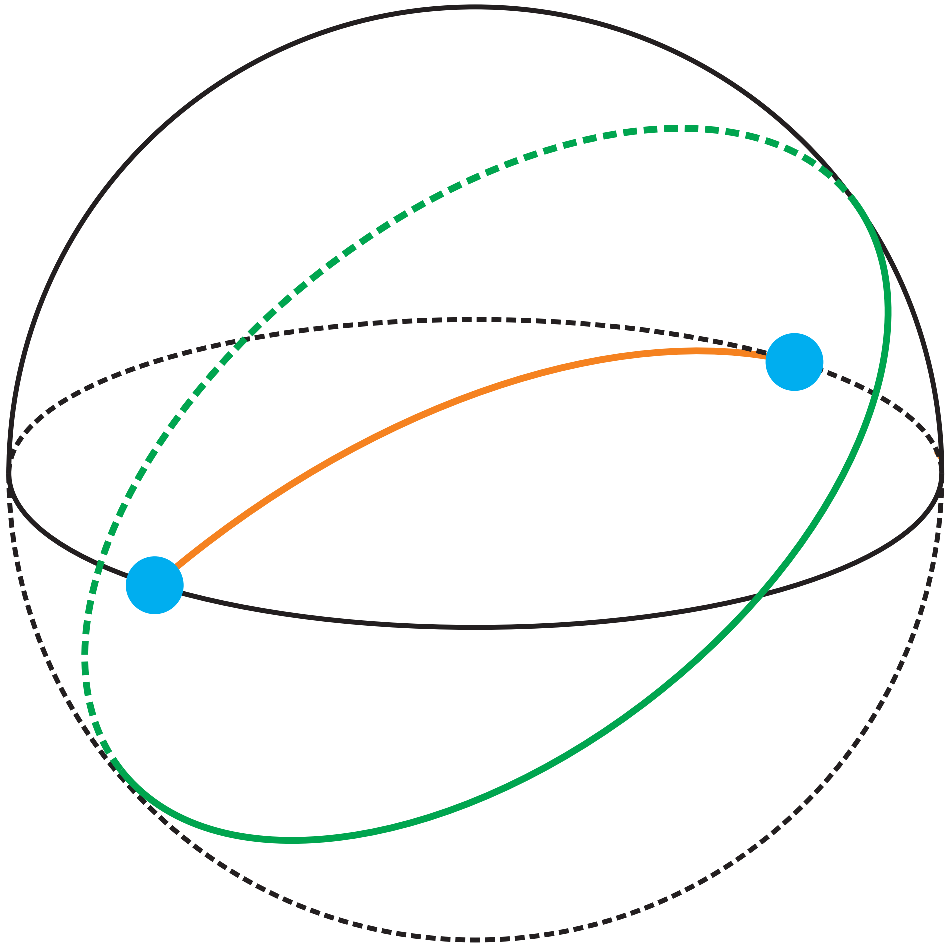
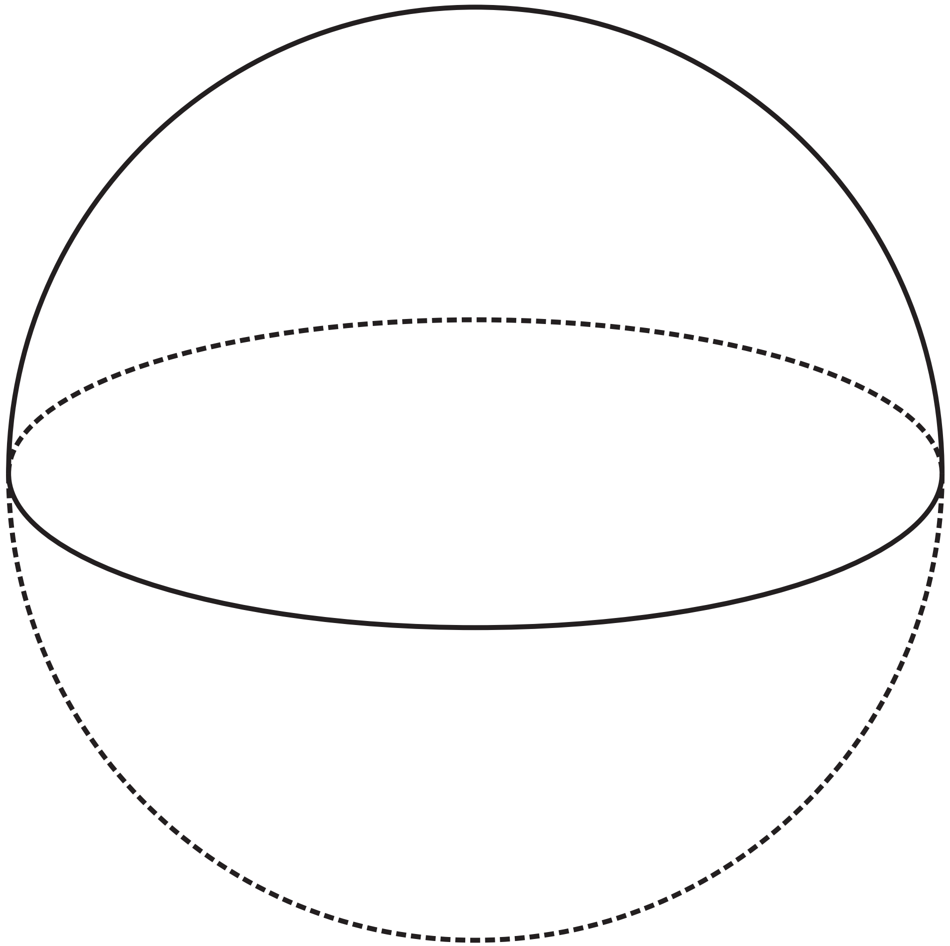


Planes



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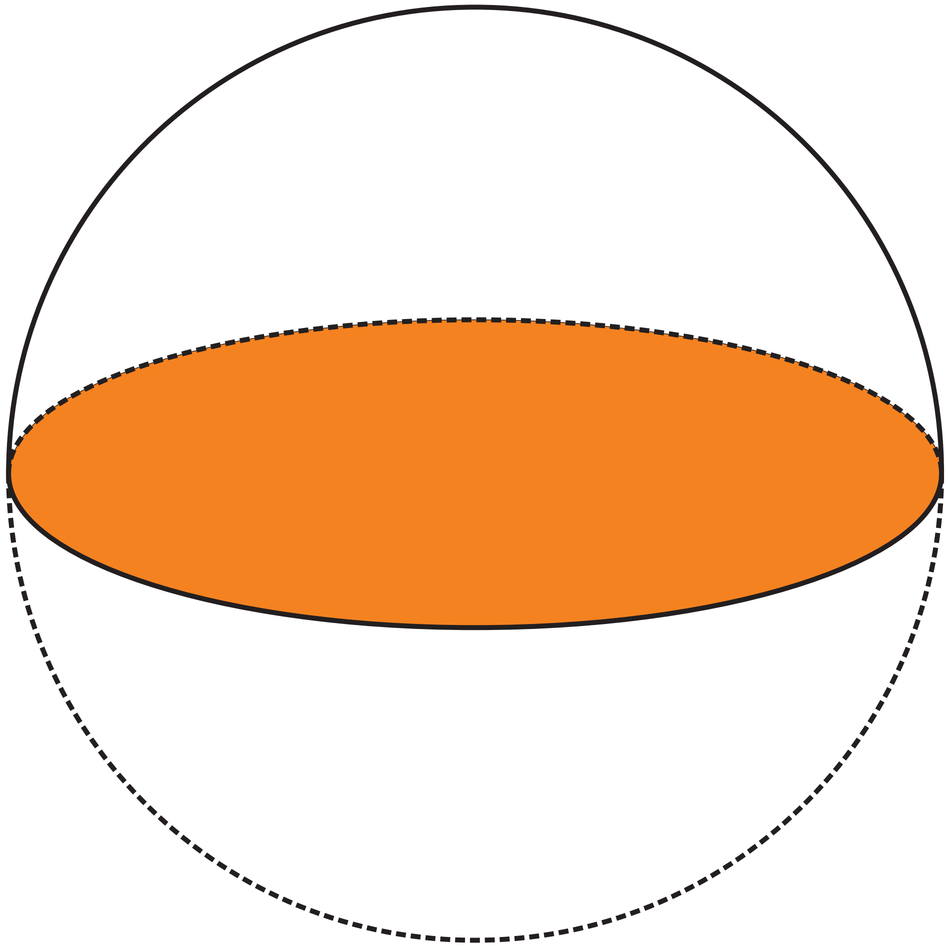

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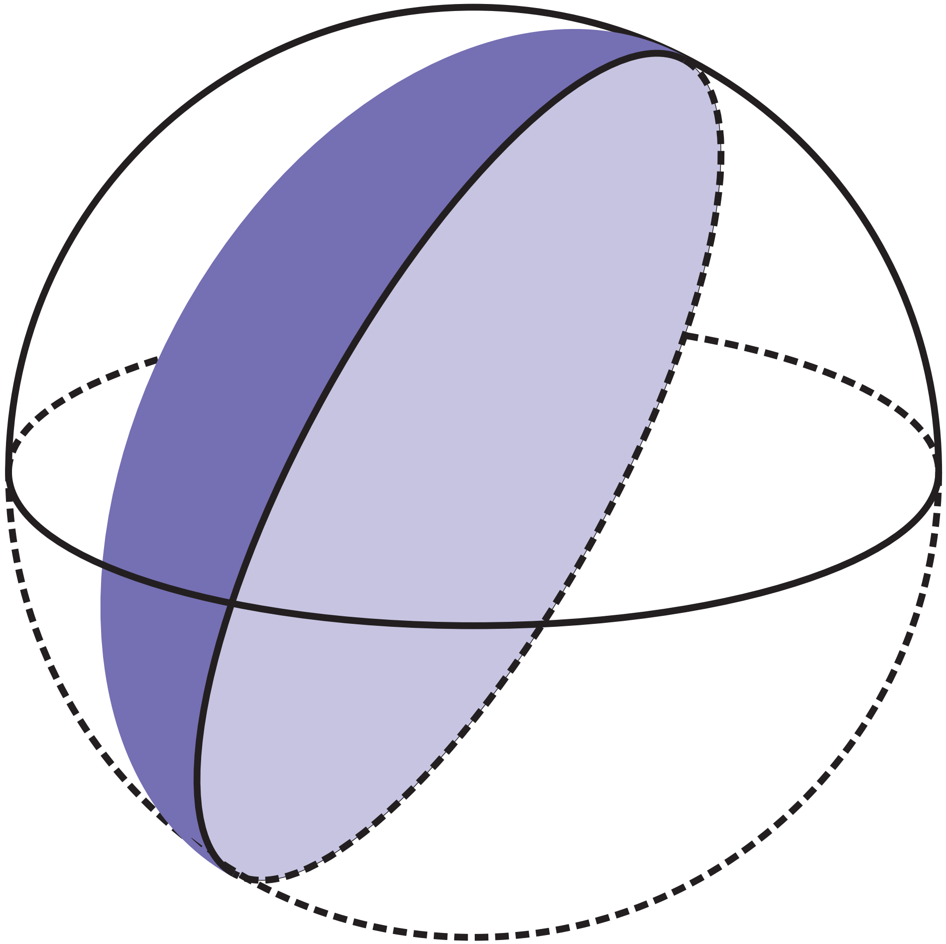

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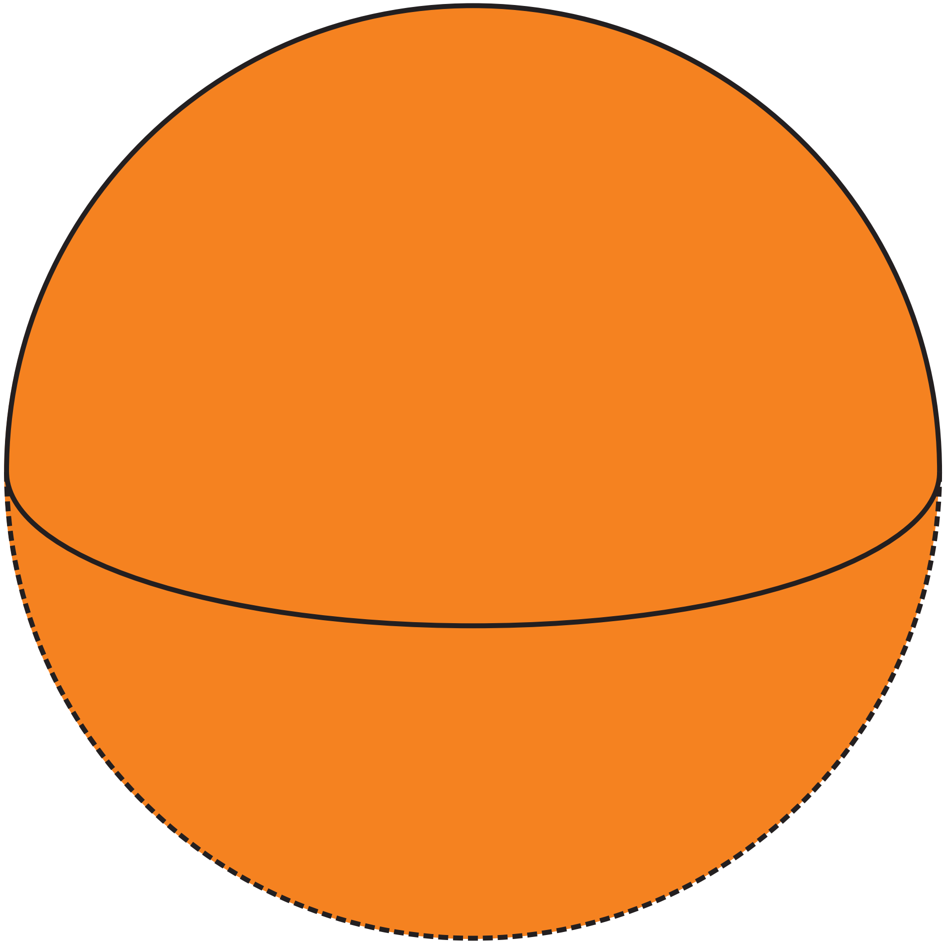

Metric

- Distance $\longrightarrow$ arc length on $\mathbb{S}^3$

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- Angles $\longrightarrow$ angle between tangents

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Regular polyhedra in $\mathbb{P}^3$

- Planar faces

Regular polyhedra in $\mathbb{P}^3$

- Planar faces (convex or star-shape)

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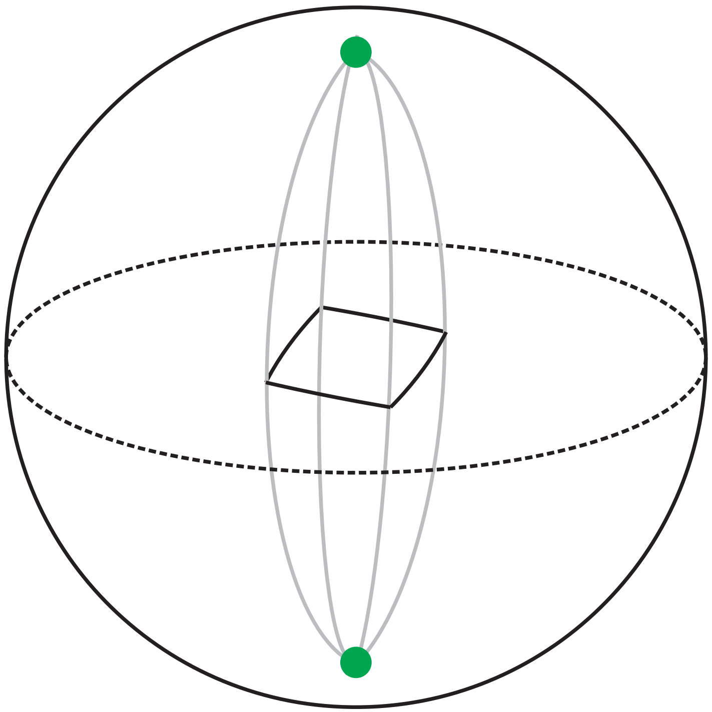

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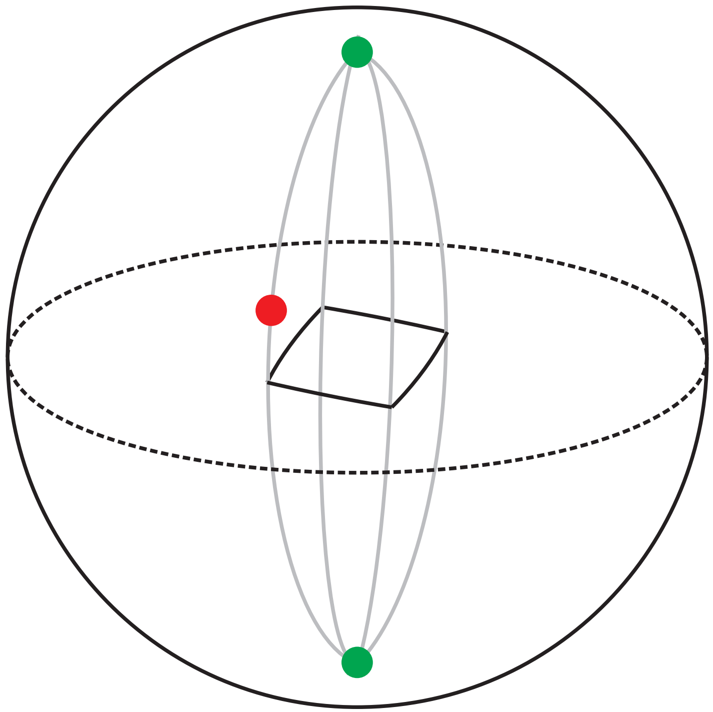

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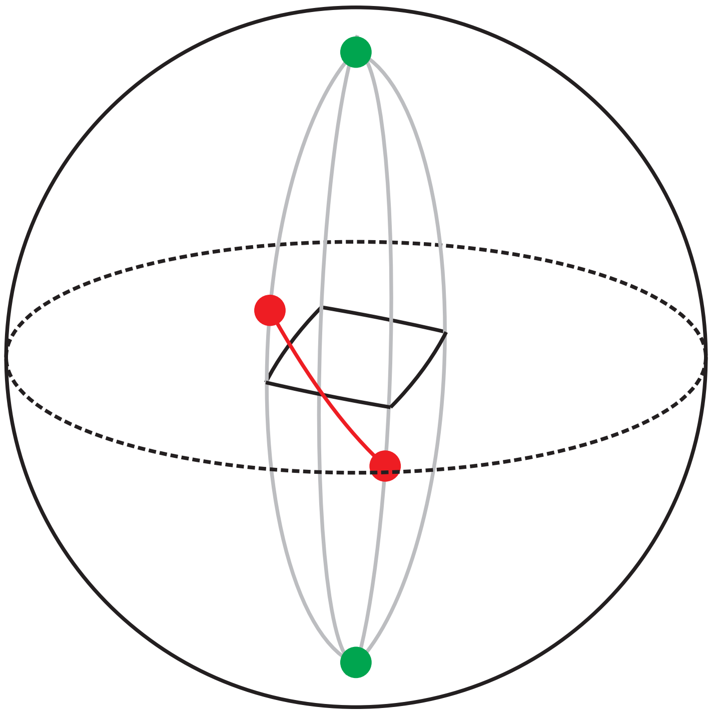

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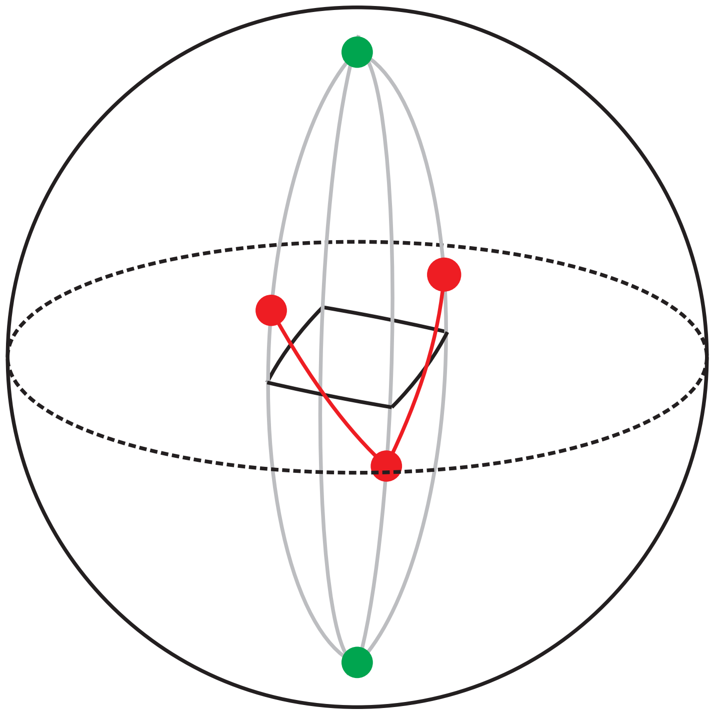

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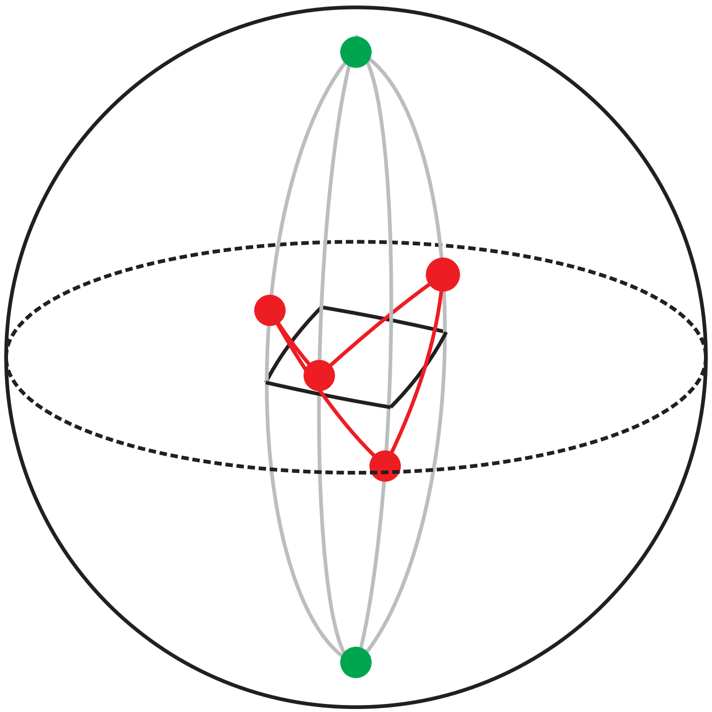

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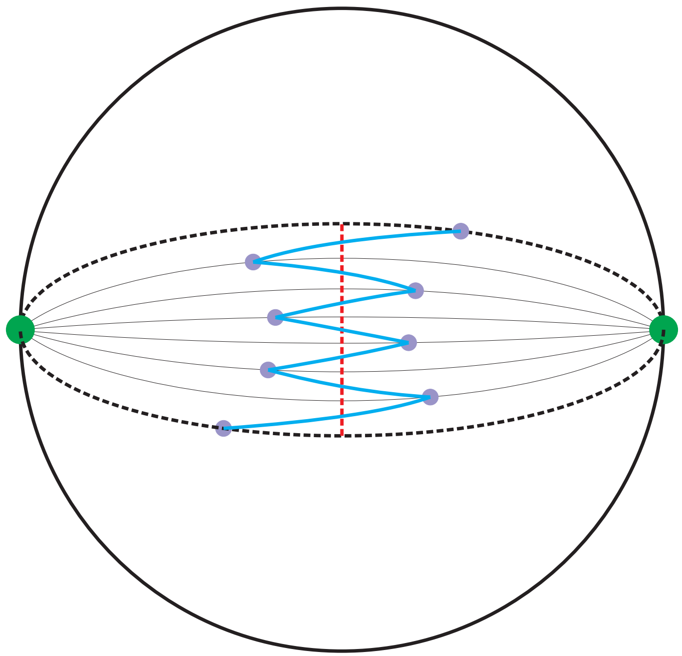

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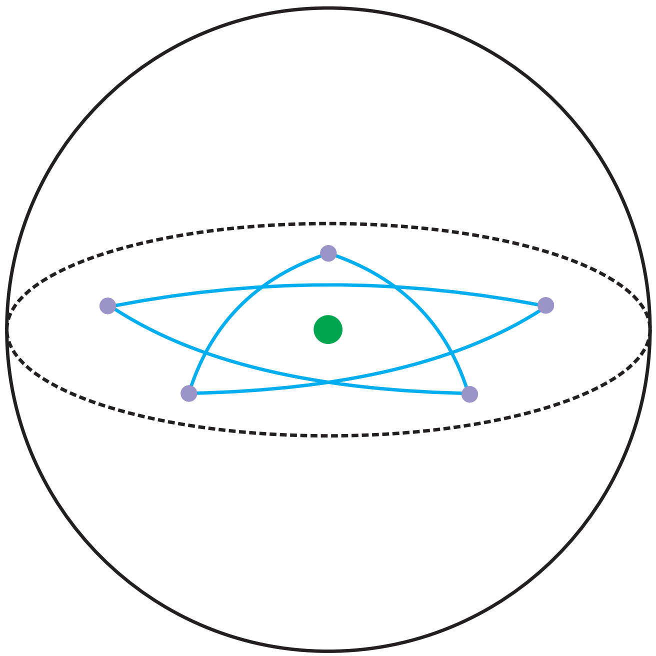

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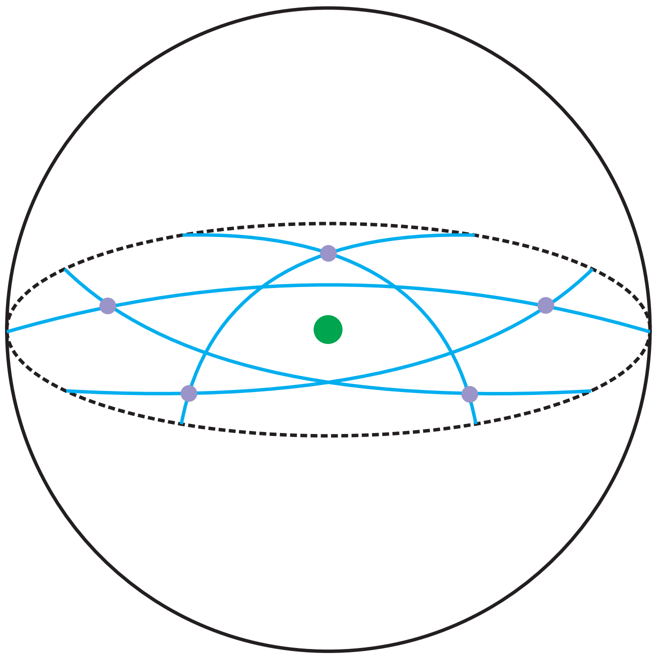

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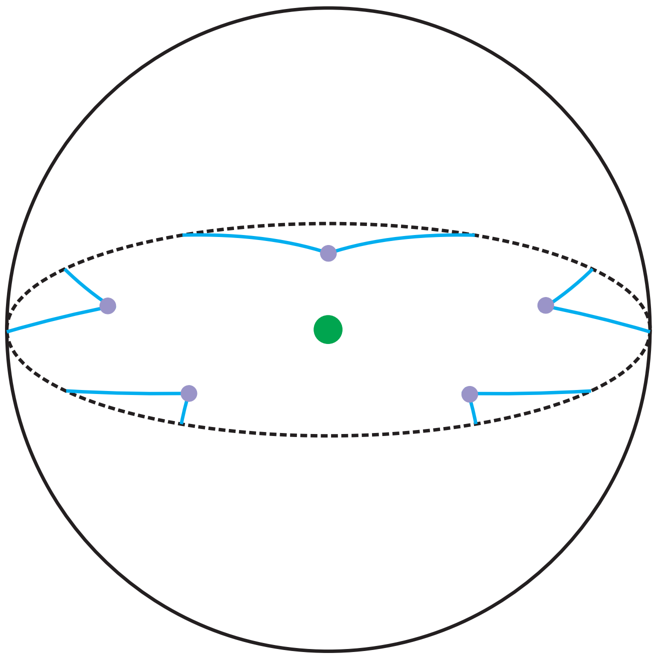

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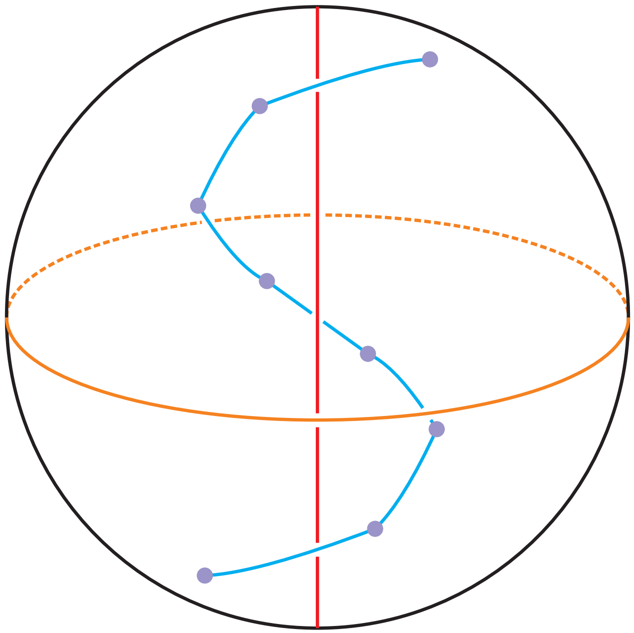

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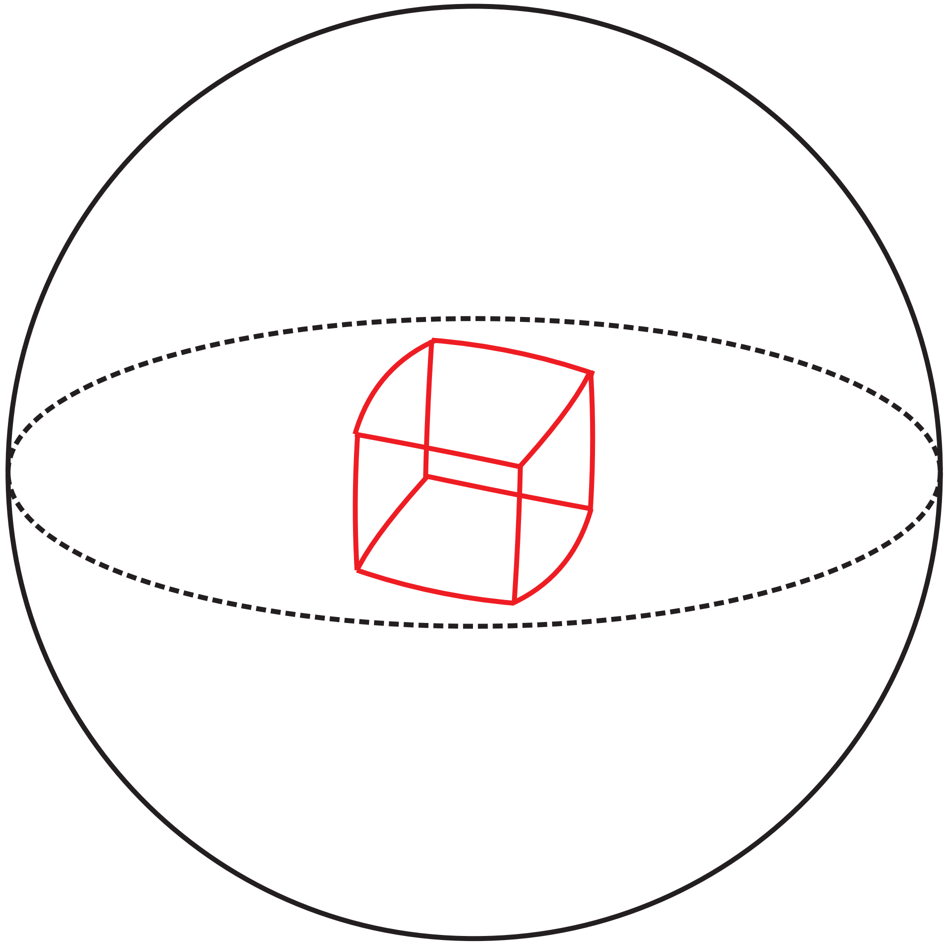

Regular polyhedra in $\mathbb{P}^3$

$\{2p, p\}$

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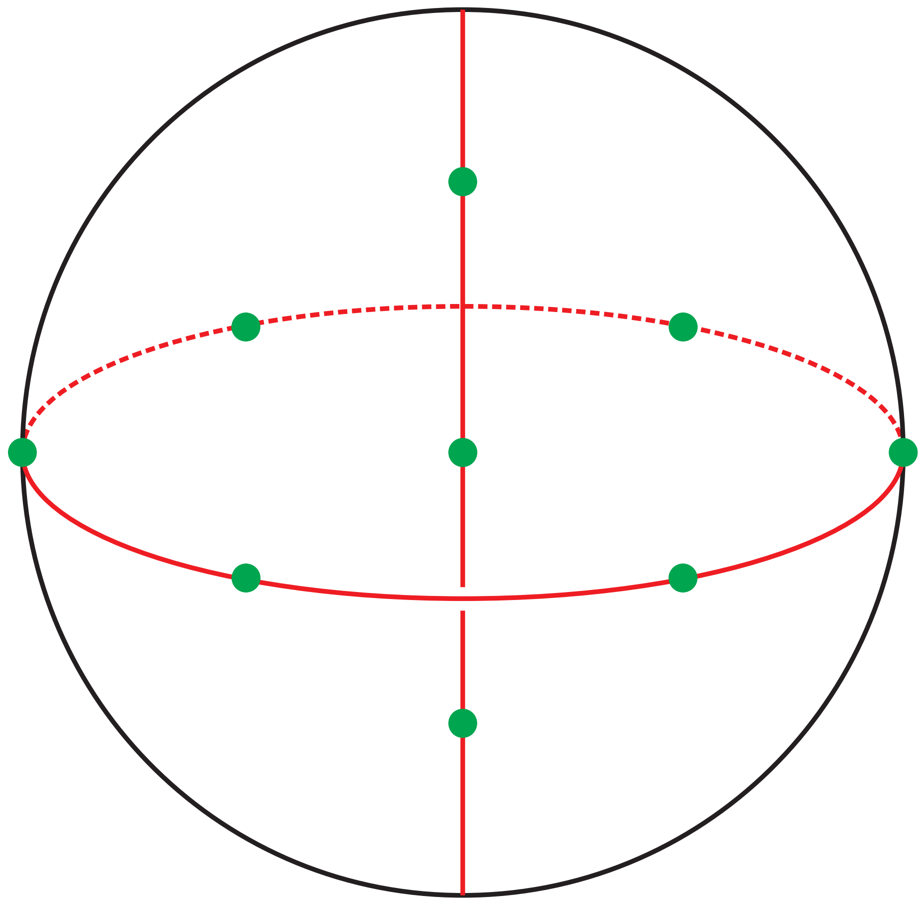

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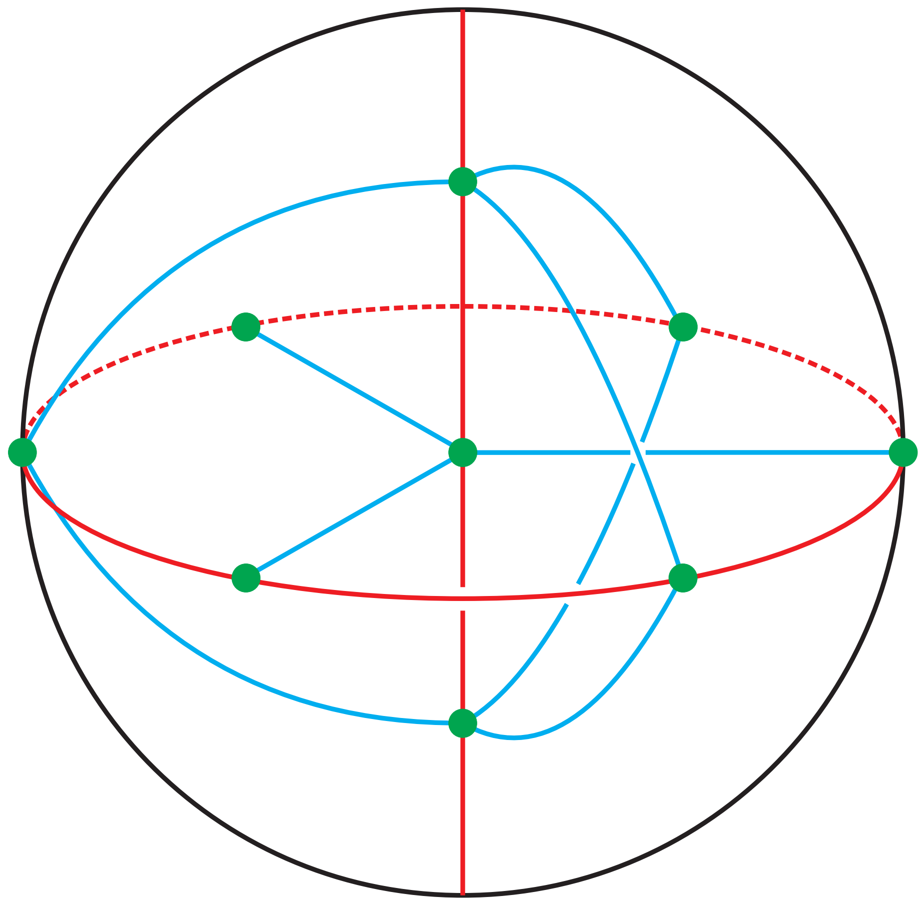

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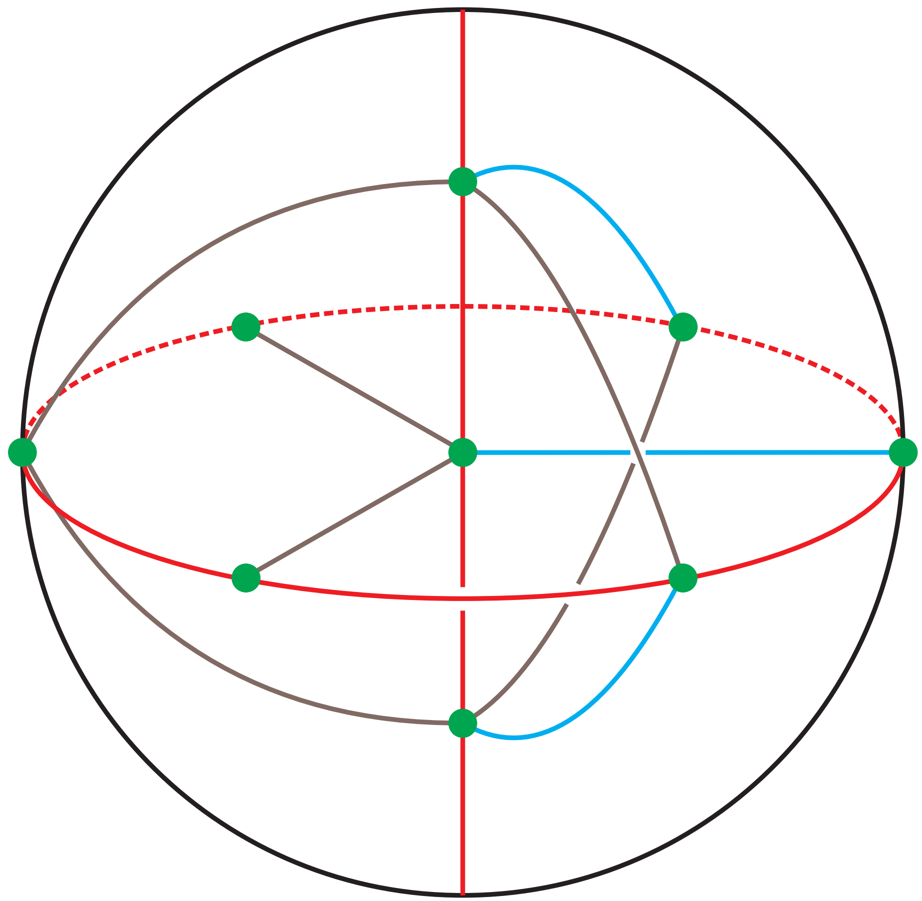

Chiral polyhedra

- **Chiral polyhedron** $\longrightarrow$ symmetry group induces two orbits on flags with adjacent flags on different orbits

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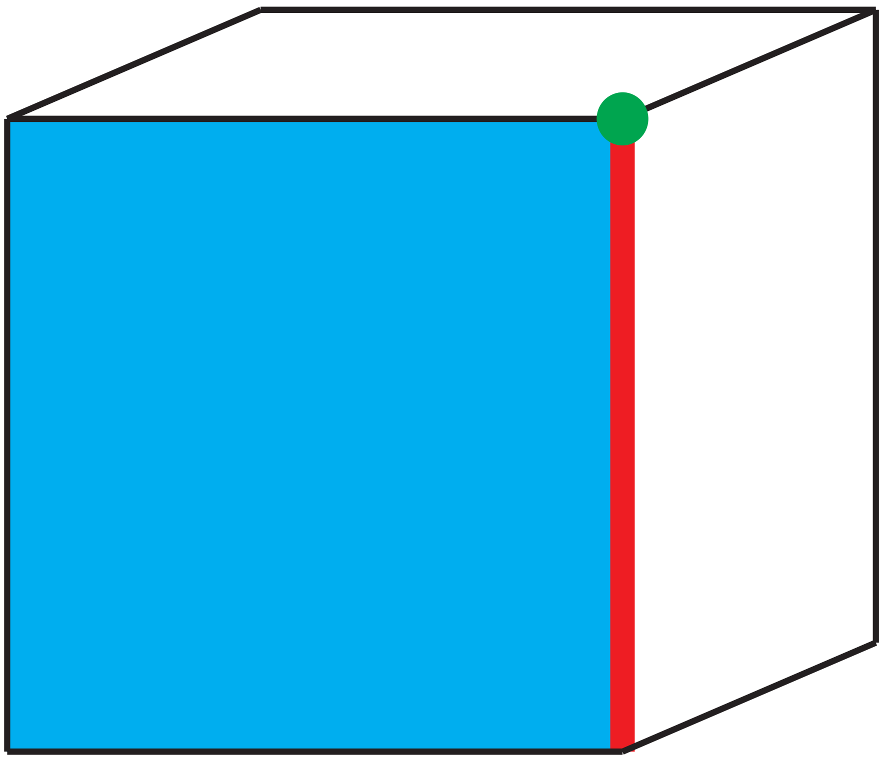

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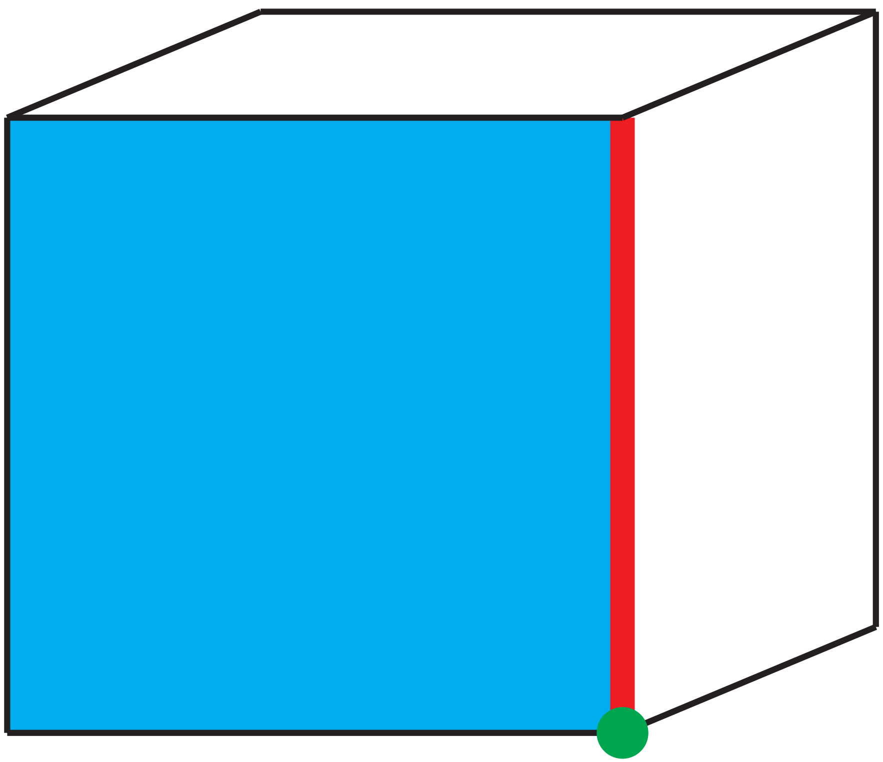

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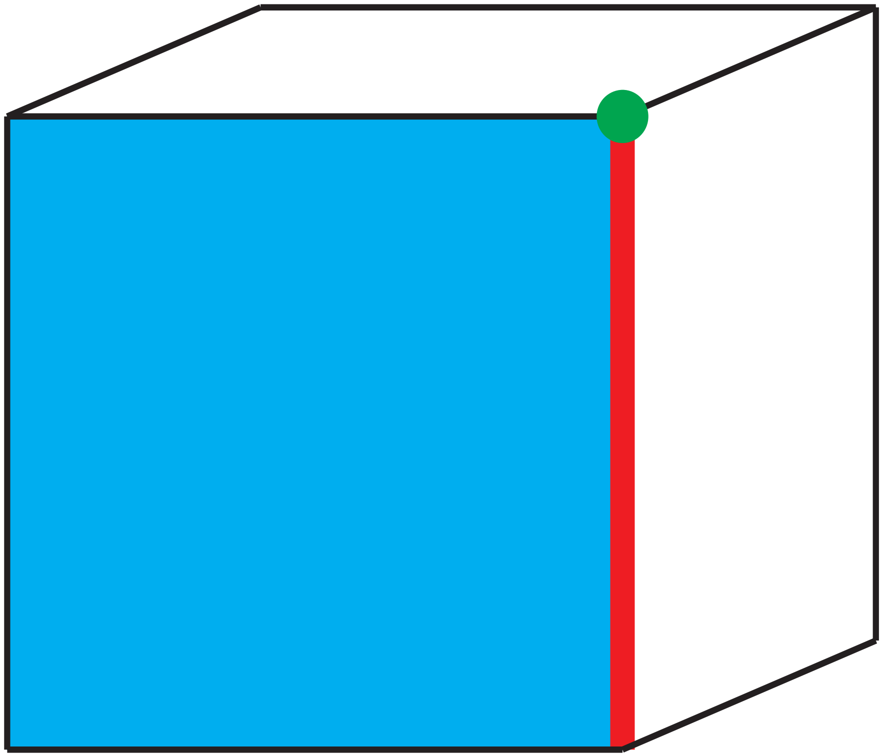

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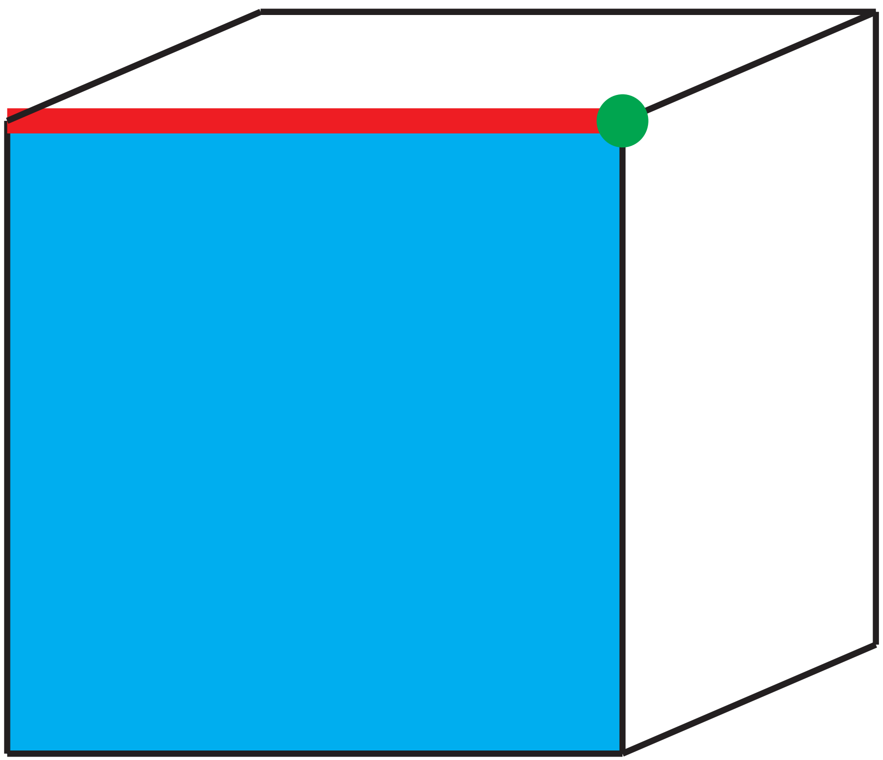

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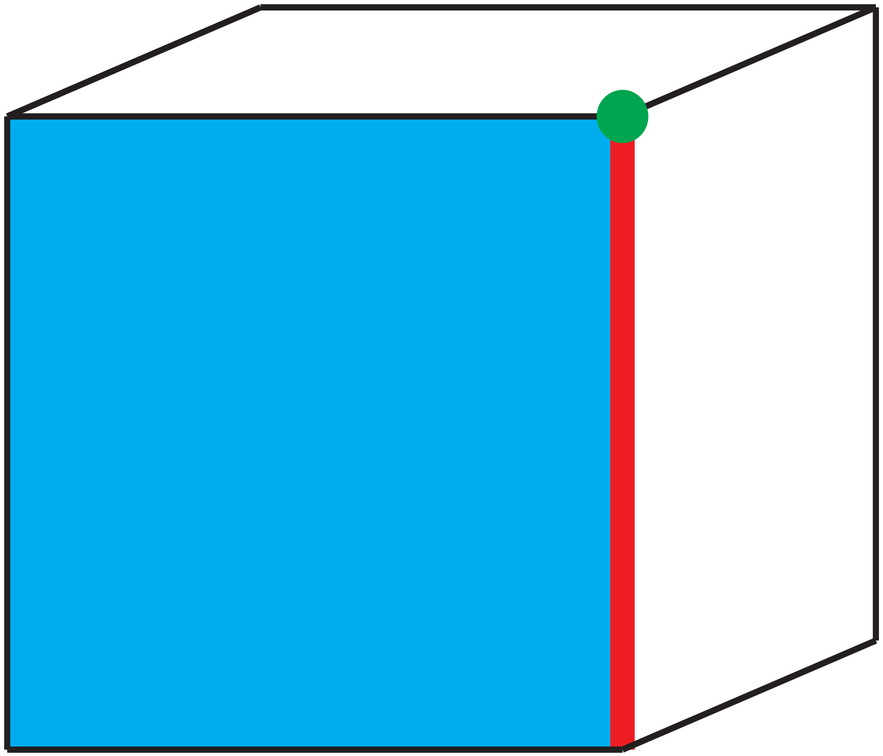

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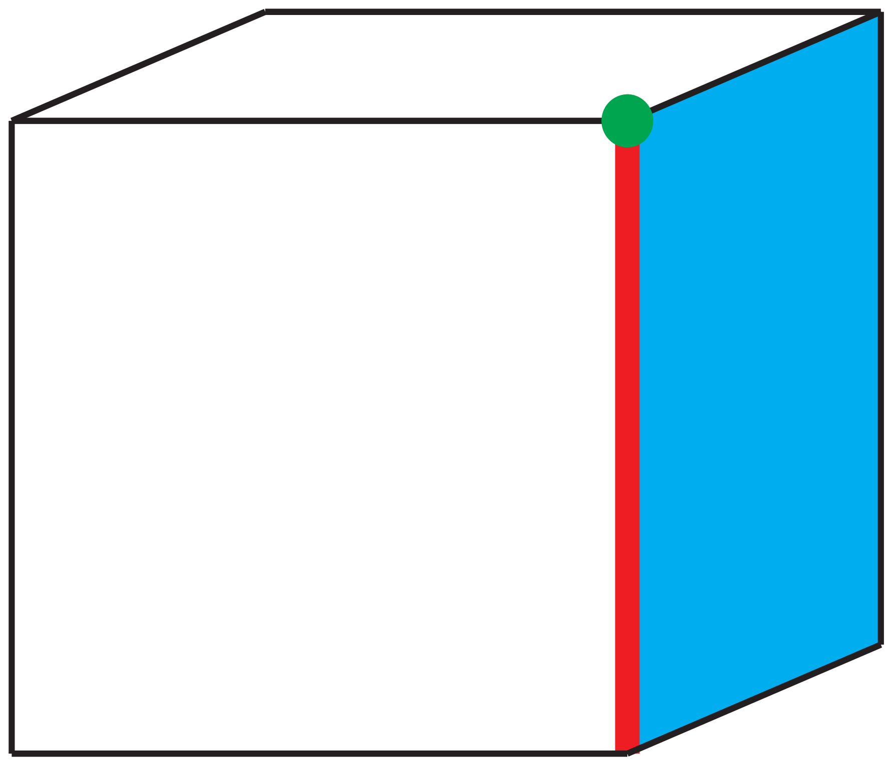

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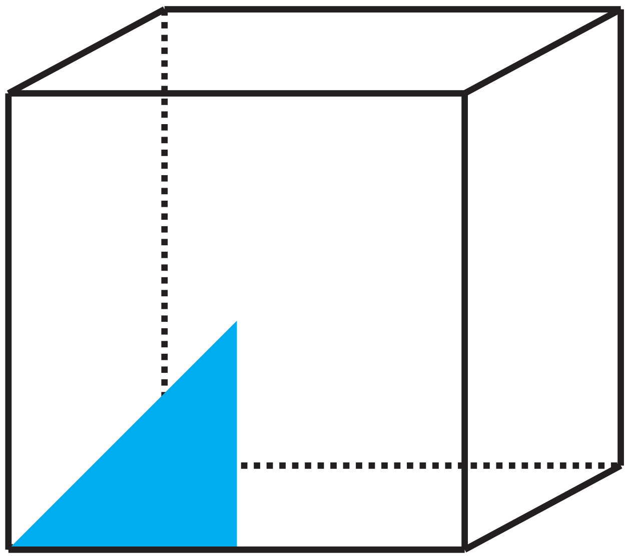

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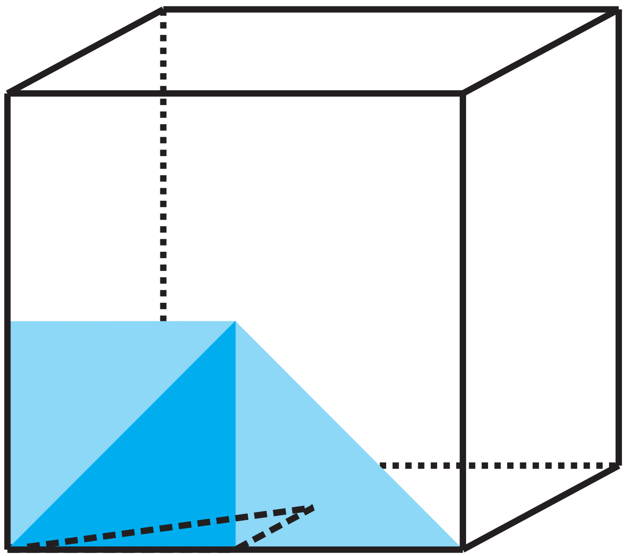
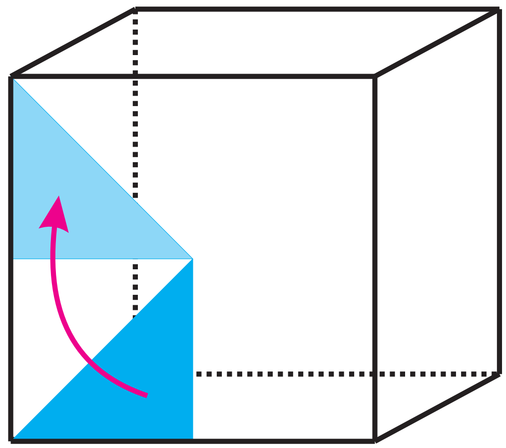


Chiral polyhedra

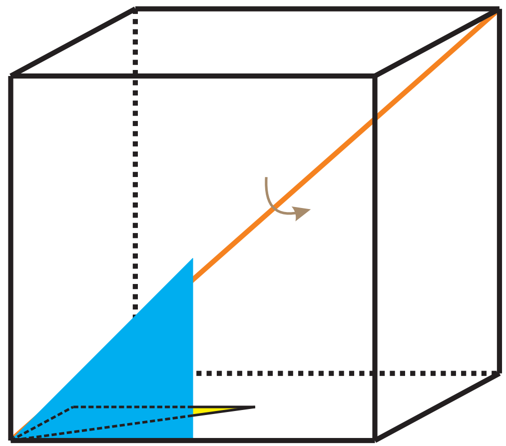
σ_1 rotates along the face



Chiral polyhedra

σ_1 rotates along the face

σ_2 rotates along the vertex



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$Sym(\mathcal{P})$ contains no element inverting σ_1 and σ_2

Chiral helical polyhedra

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- Polyhedra in 3 of the families have skew faces
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Chiral helical polyhedra

Problem 1 (2011) Find all chiral polyhedra in $\mathbb{P}^3$

Chiral helical polyhedra

Problem 1 (2011) Find all chiral polyhedra in $\mathbb{P}^3$

Subproblem 1 (2014) Find all chiral polyhedra in $\mathbb{P}^3$ with helical faces

Chiral helical polyhedra

- Helical faces are generated by double rotations

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Theorem All chiral polyhedra with helical faces in $\mathbb{P}^3$ or $\mathbb{R}^3$

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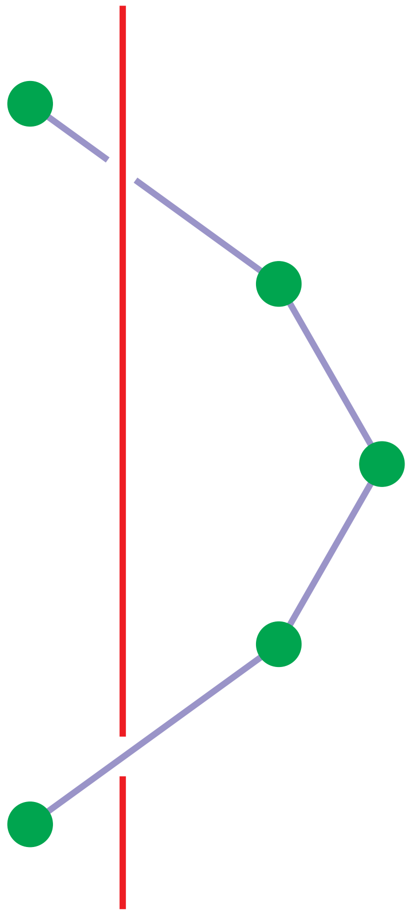
Proof

The vertex-figures are planar or skew

Chiral helical polyhedra



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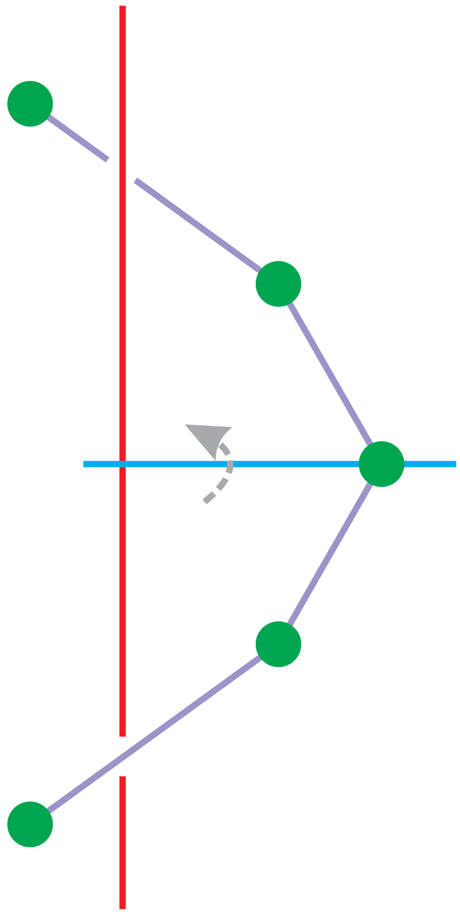

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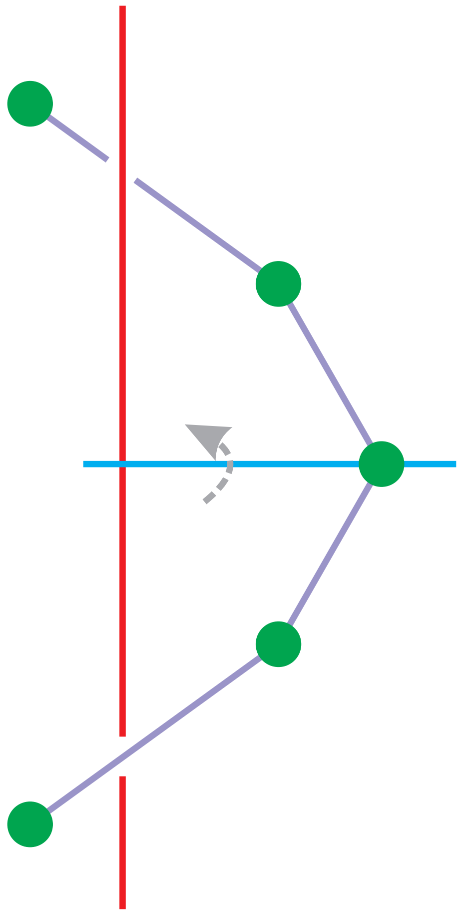

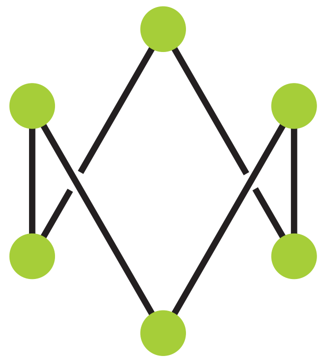

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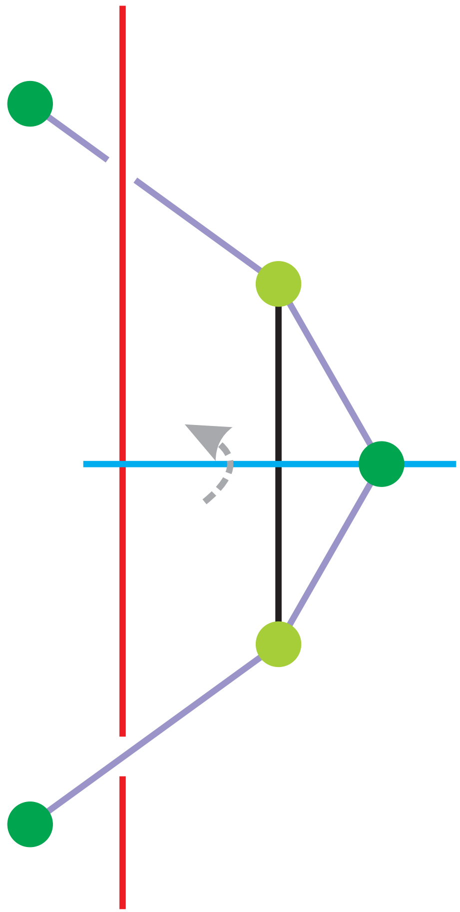

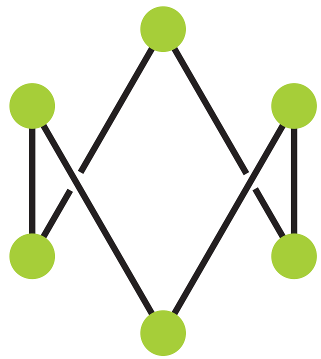

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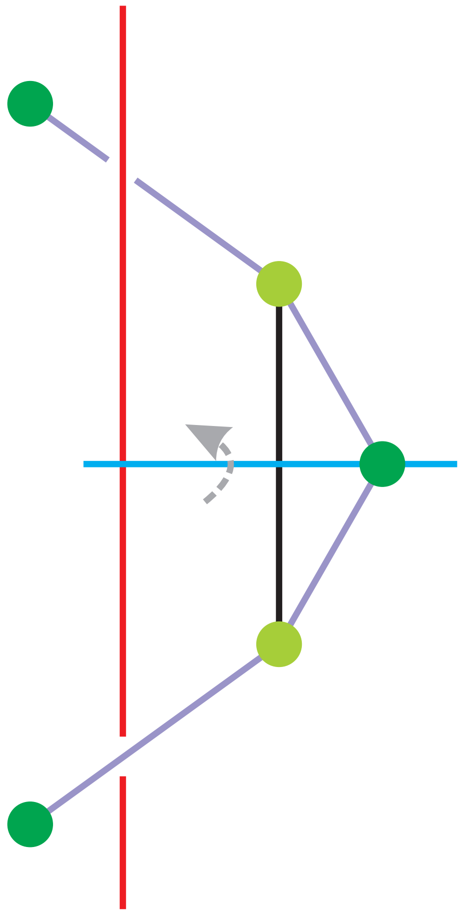

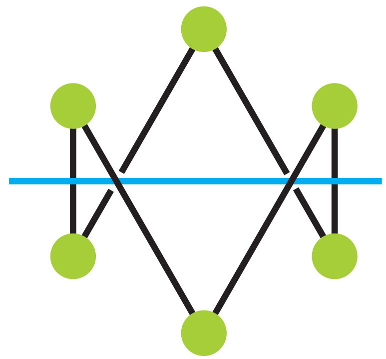

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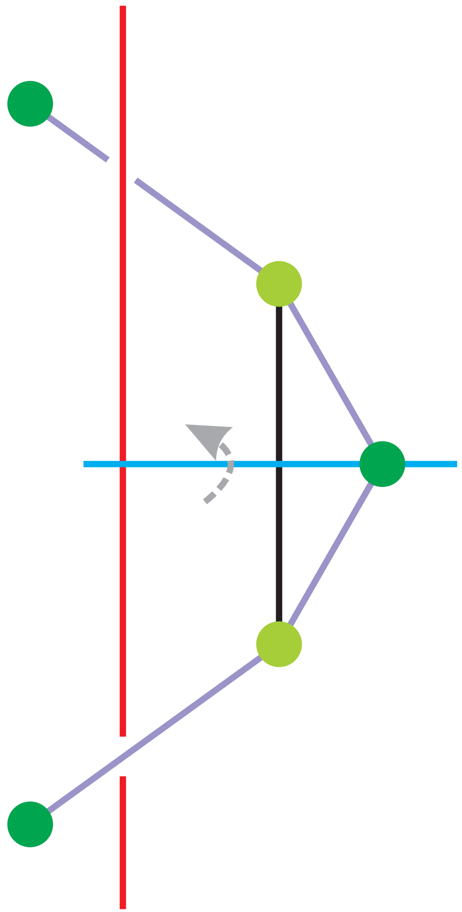
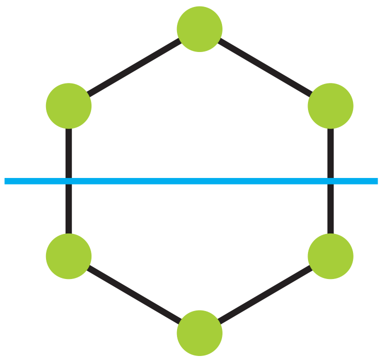
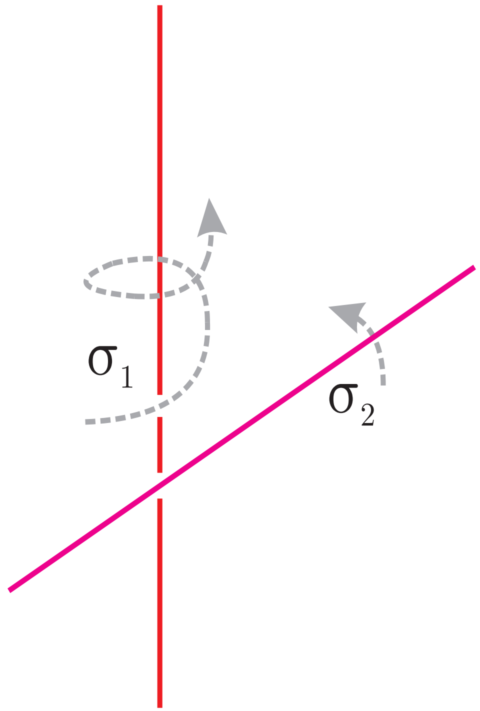




Chiral helical polyhedra

- σ_1 is a double rotation
- σ_2 is a rotation



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Chiral helical polyhedra

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- σ_2 is a rotation

- The base vertex belongs to the axis of σ_2

- Given σ_1, σ_2 and the base vertex we can reconstruct the polyhedron

- Choosing another vertex on the axis of σ_2 yields another polyhedron

Chiral helical polyhedra

- Distinct choices of base vertex on the axis of σ_2 may yield geometrically distinct polyhedra

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- These polyhedra are mildly different if the choices of base vertices are close to each other

- If for some choice of base vertex it belongs to the plane of its vertex-figure then the polyhedron is regular

Chiral helical polyhedra

- In $\mathbb{E}^3$ there is always such point!

Chiral helical polyhedra

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- That is, given a screw motion S and a line l not parallel and not intersecting the axis of S then there exists a point x on l such that $xS - x$ is perpendicular to l

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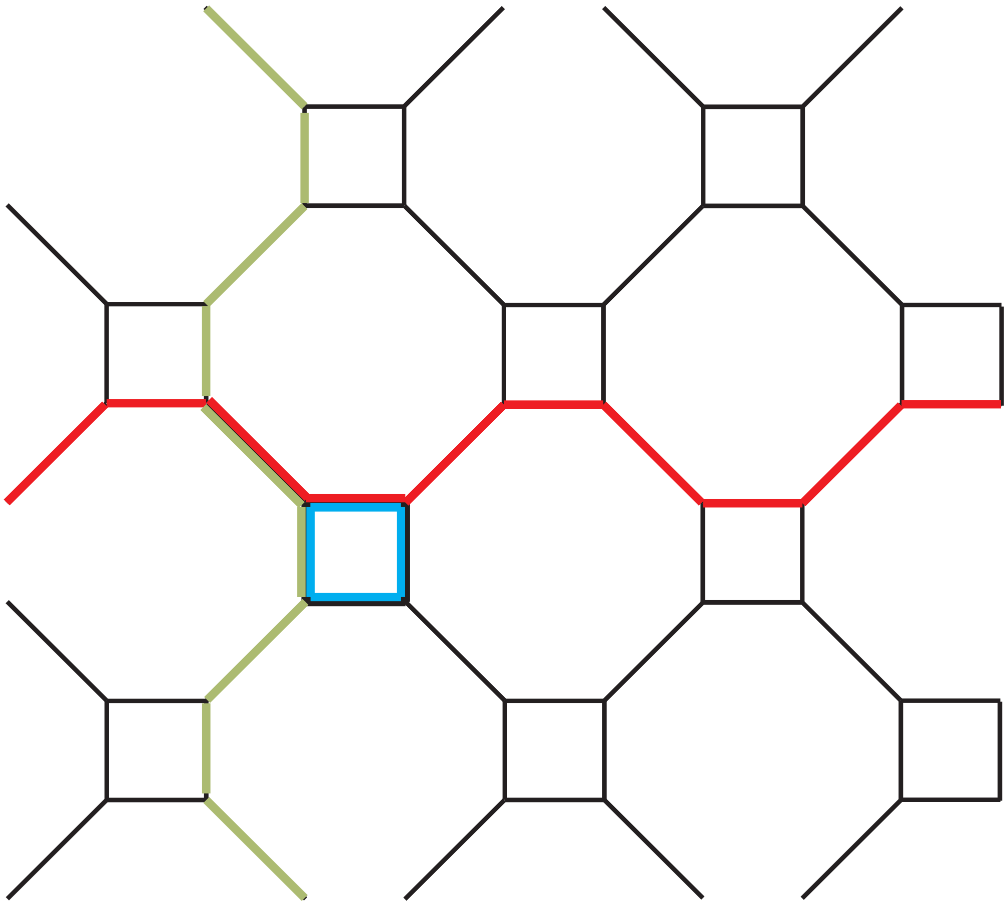
▶ In $\mathbb{E}^3$ there is always such point!

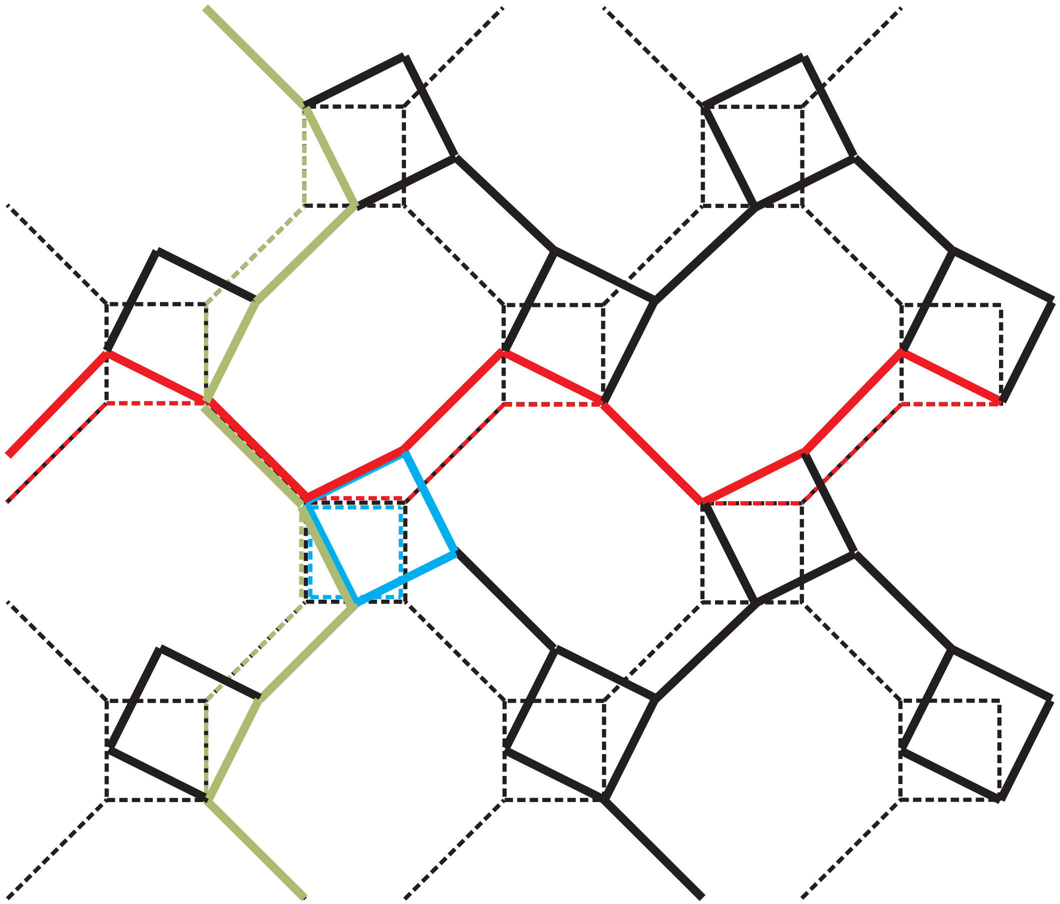
▶ That is, given a screw motion S and a line l not parallel and not intersecting the axis of S then there exists a point x on l such that $xS - x$ is perpendicular to l

Theorem (P, Weiss) All chiral polyhedra in $\mathbb{E}^3$ with helical faces can be obtained from a regular polyhedron by moving the base vertex along the axis of σ_1

Chiral helical polyhedra



Chiral helical polyhedra



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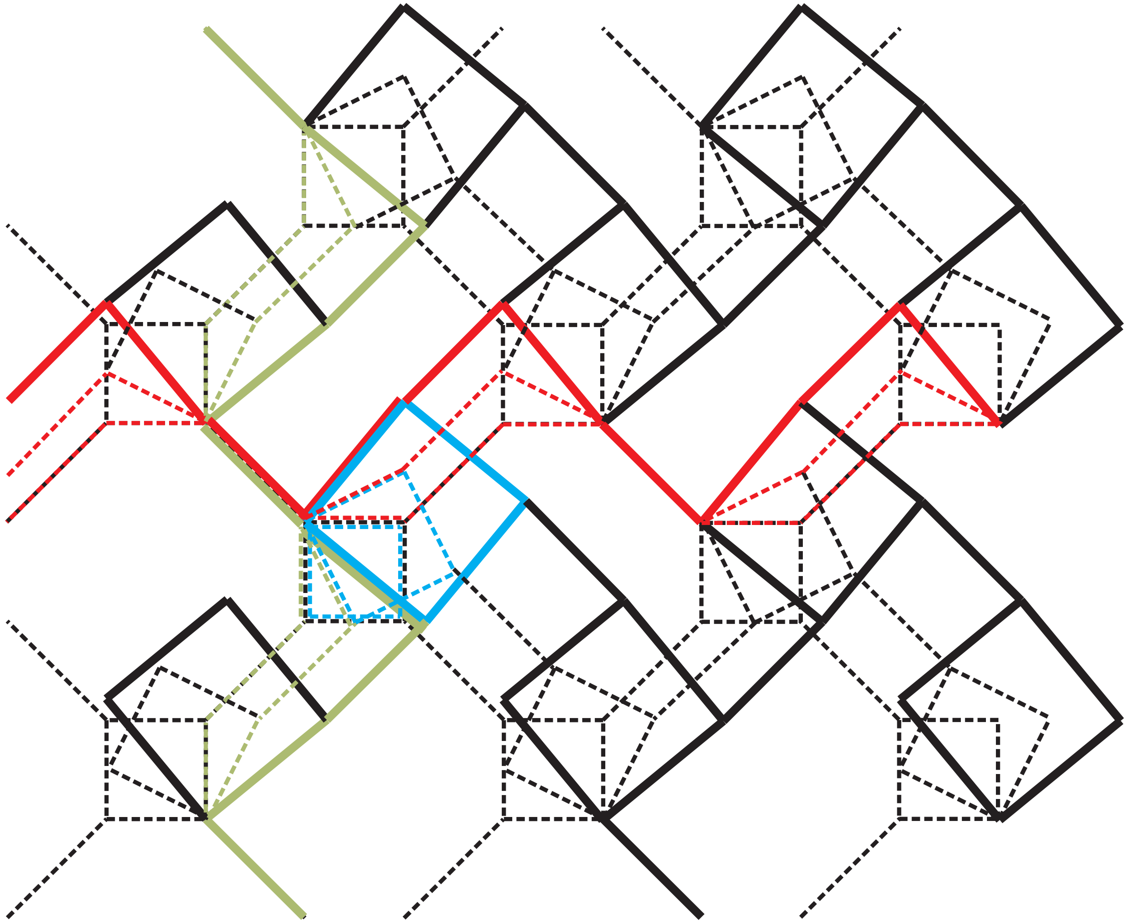

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That is, $L_1\sigma_1\sigma_2 = L_1\sigma_1^{-1}$

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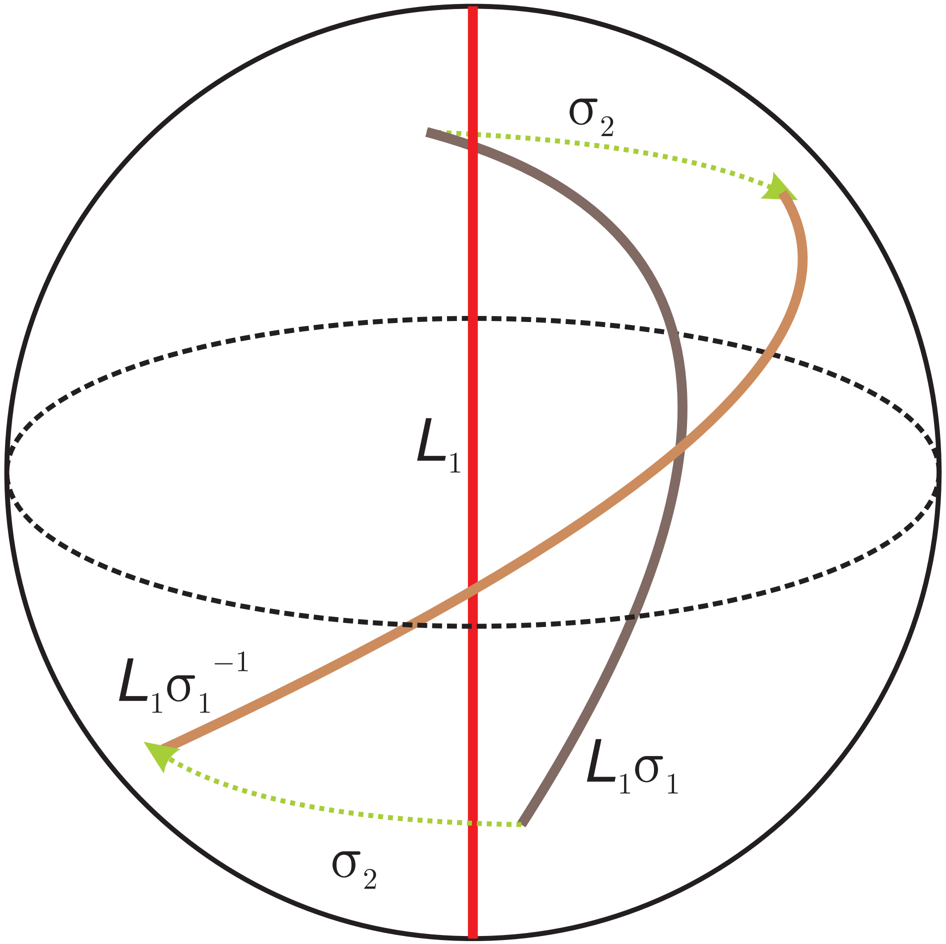

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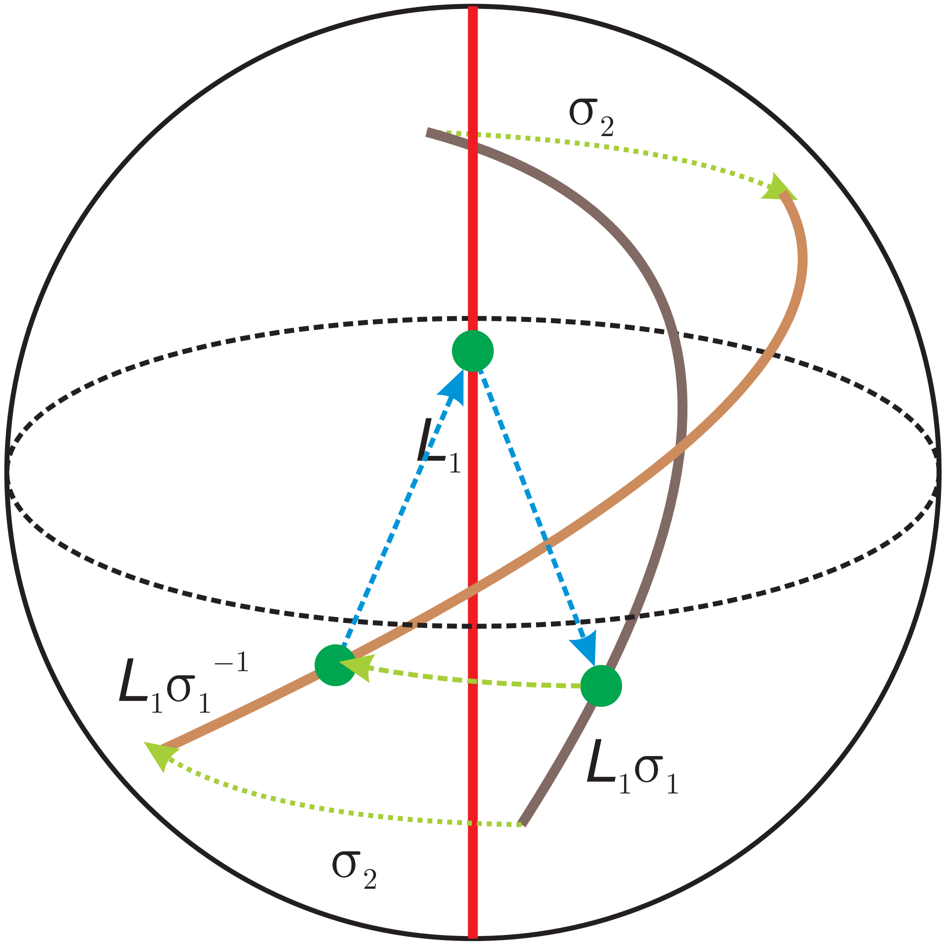

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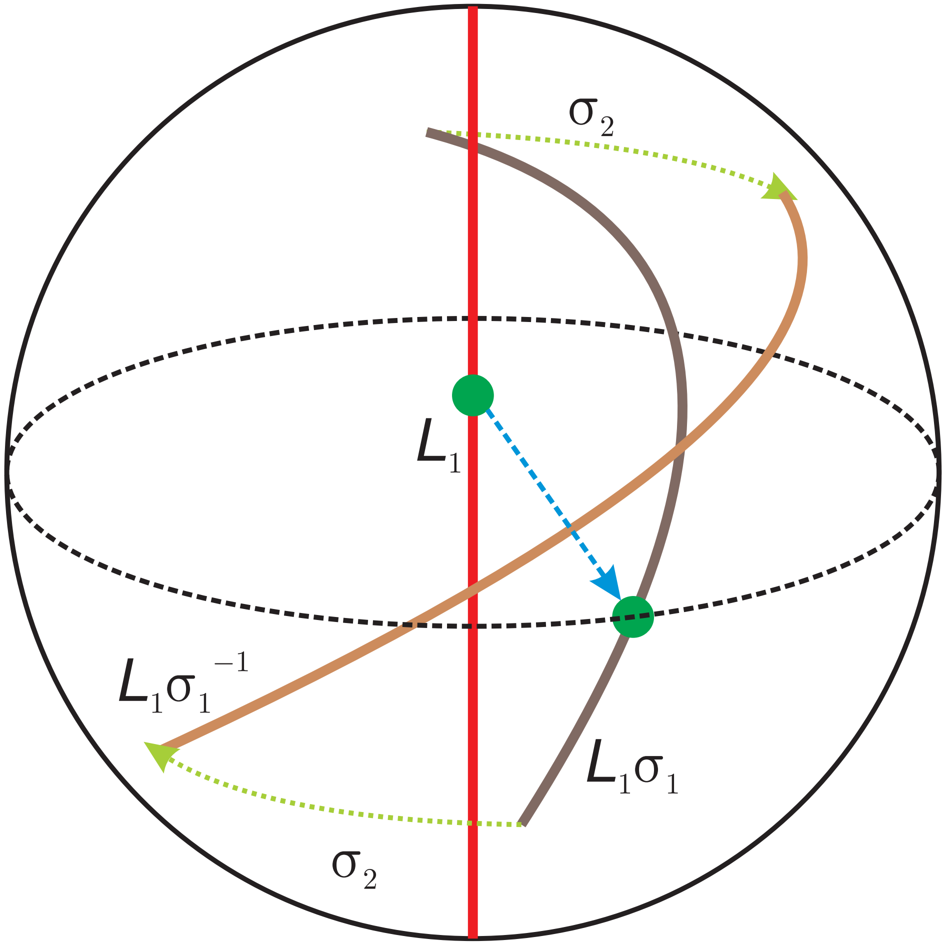

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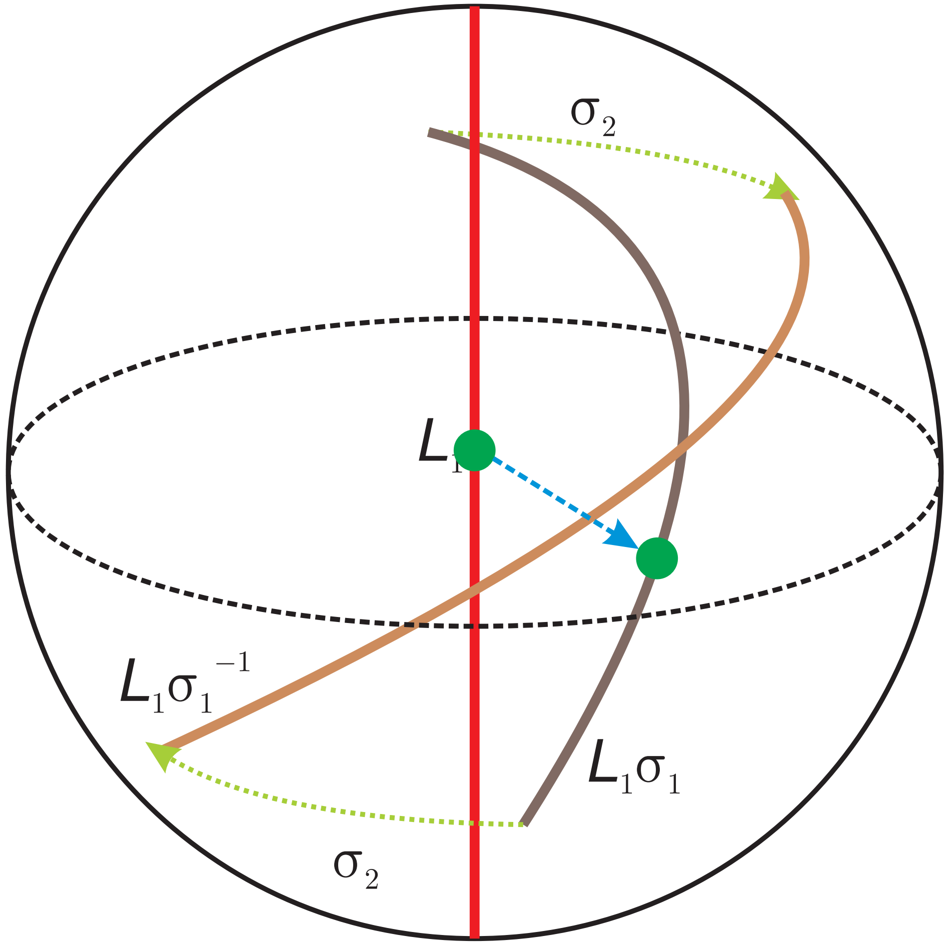

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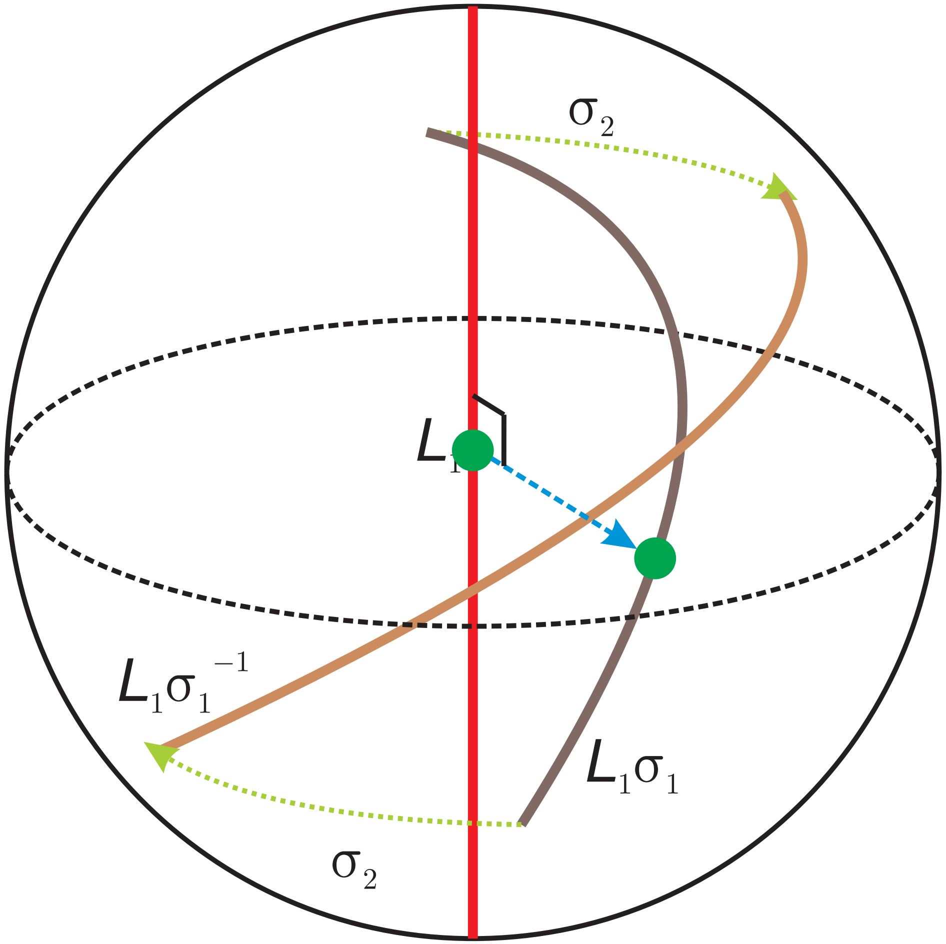

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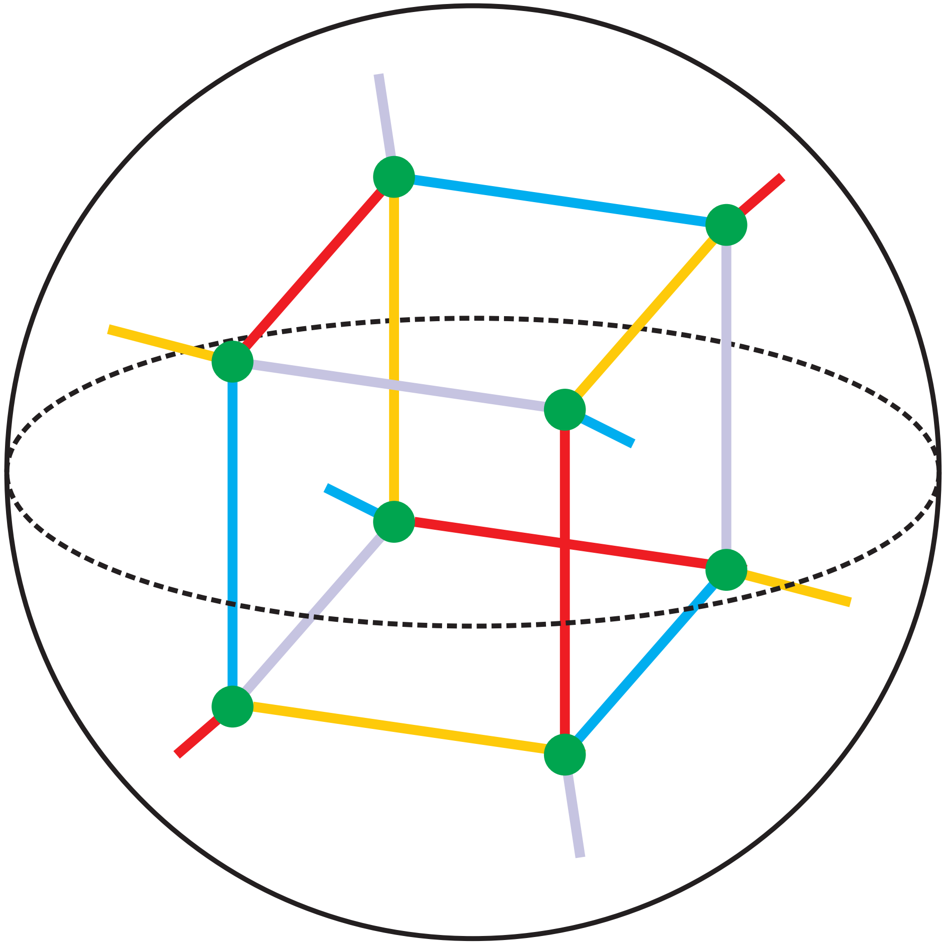

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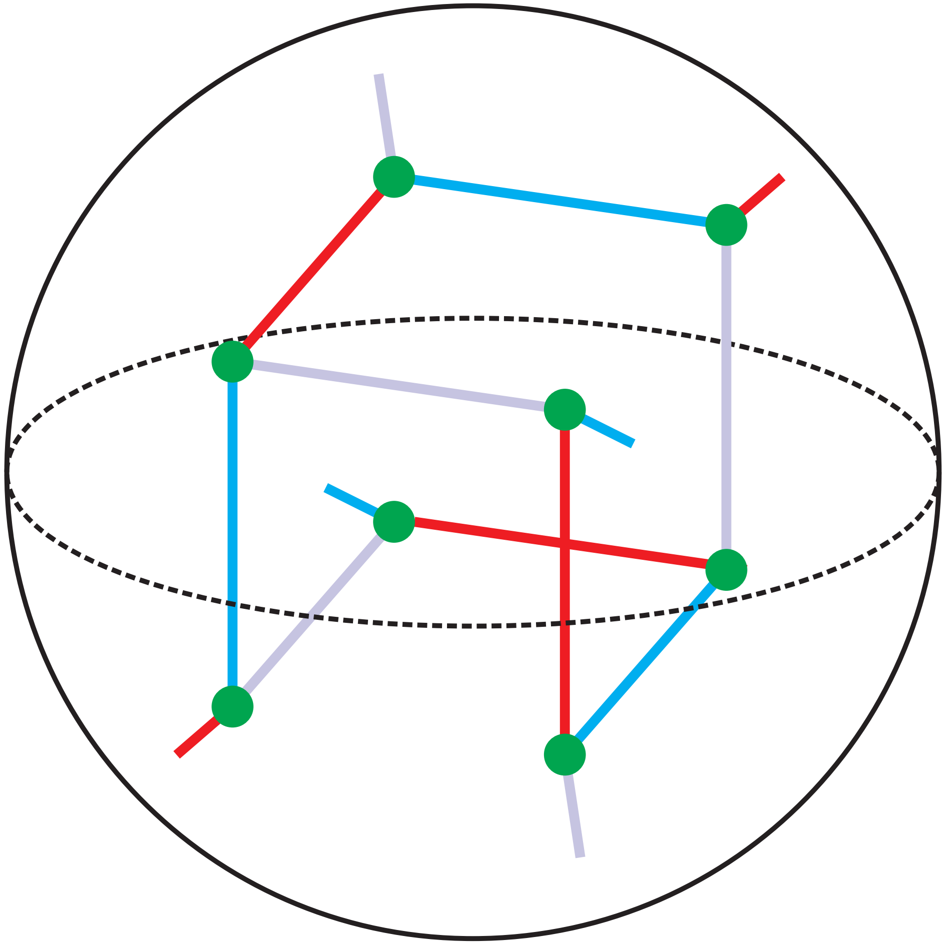

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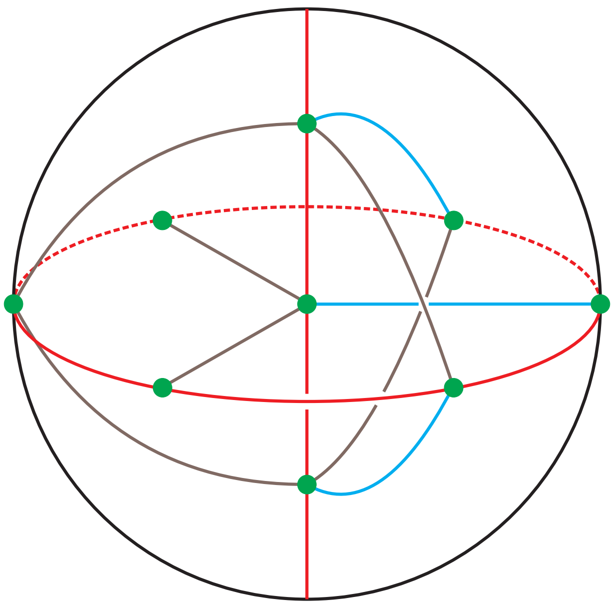

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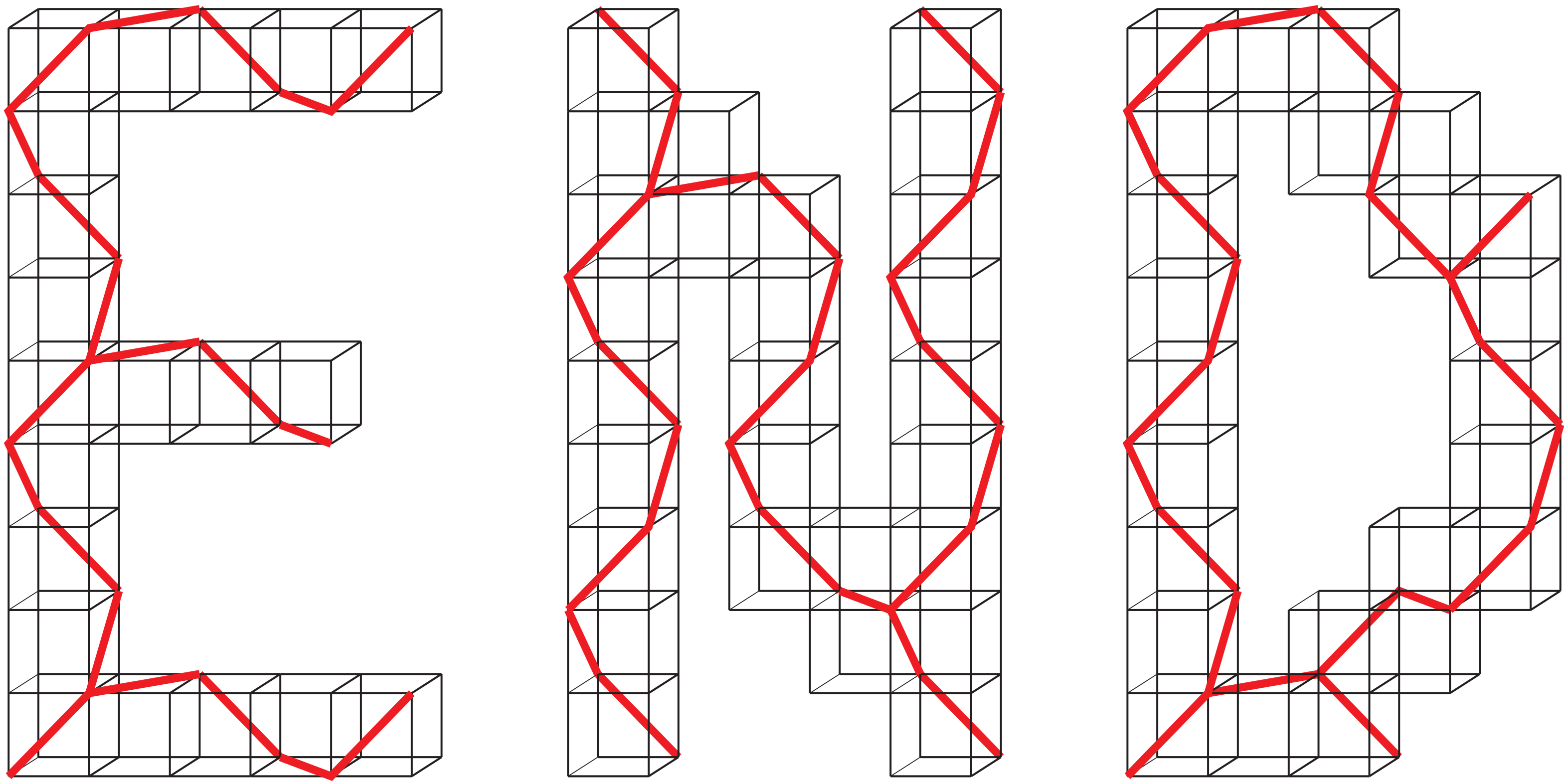
- We know helical chiral polyhedra in the hyperbolic space too!


END