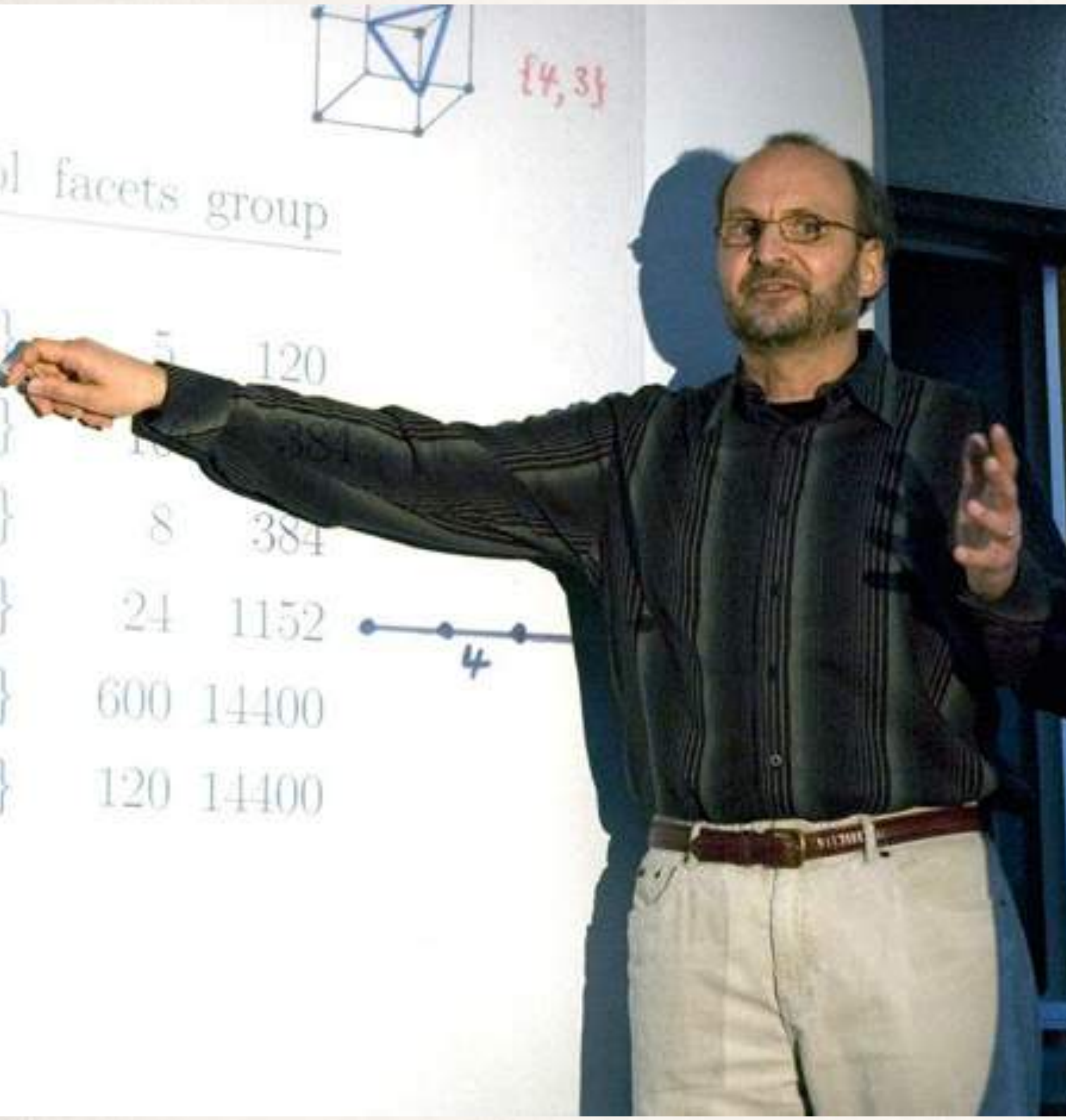
Instituto de Matemáticas UNAM
Juriquilla

Exploring the concept of perfection in 3-hypergraphs

Deborah Oliveros

University of Pannonia


{4,3}
facets group
120
8 384
24 1152
4
600 14400
120 14400

Searching for perfection in hypergraphs

Joint work with

Natalia Gacía Colin

Amanda Montejano

Luis Montejano

Some motivation

Given a Partially order set

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You may construct an oriented graph.

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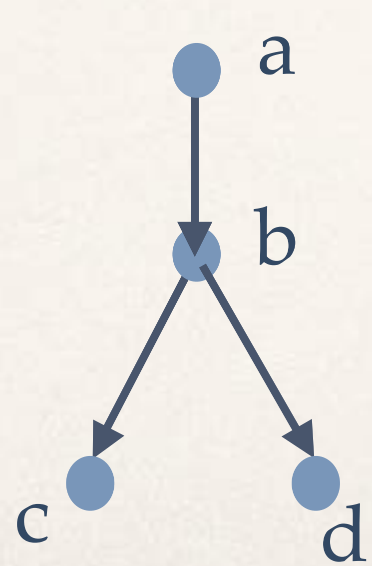
$a<b<c$, $a<b<d$



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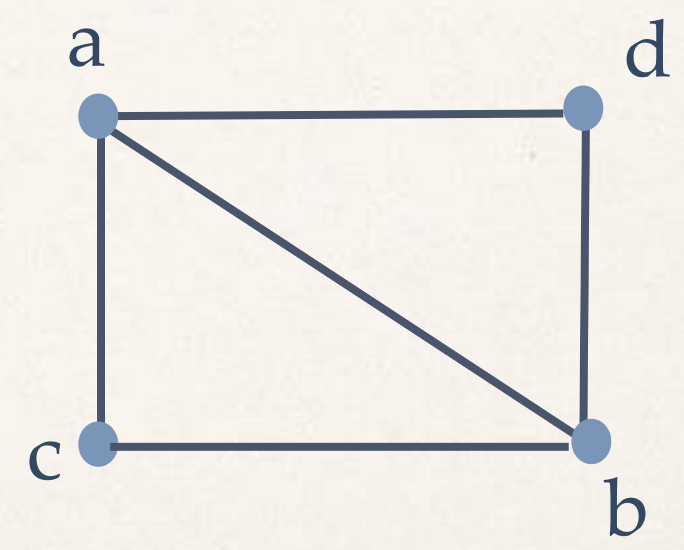
Comparability graphs

Graphs that admit a transitive orientation



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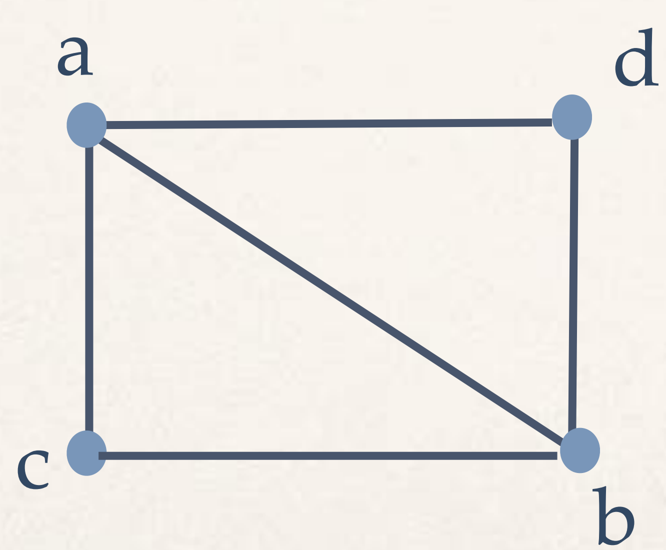

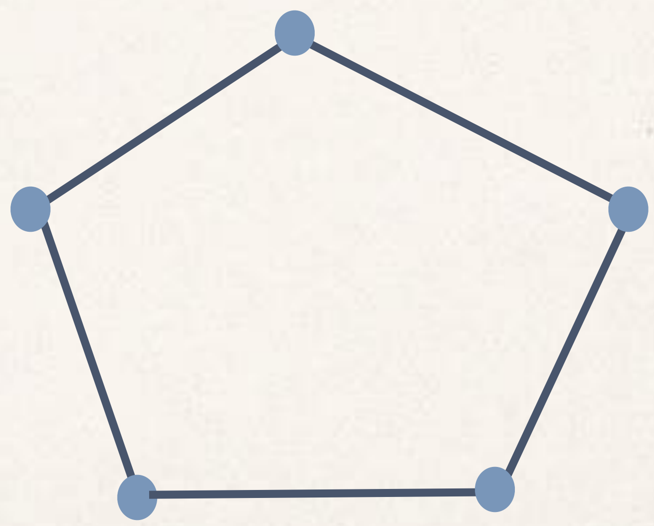

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No monochromatic edges

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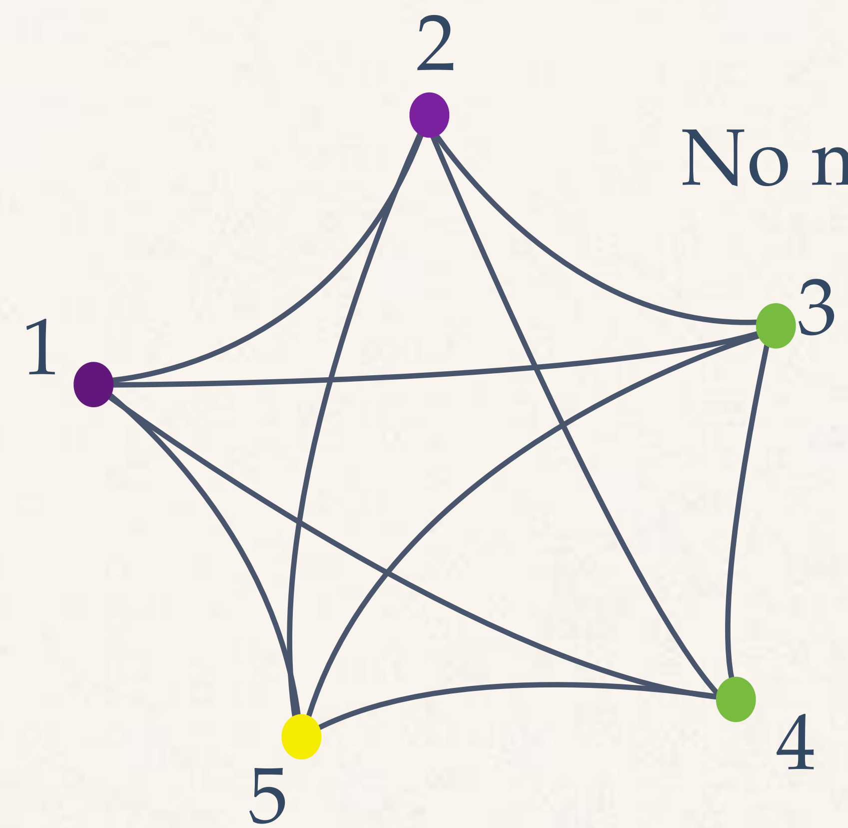

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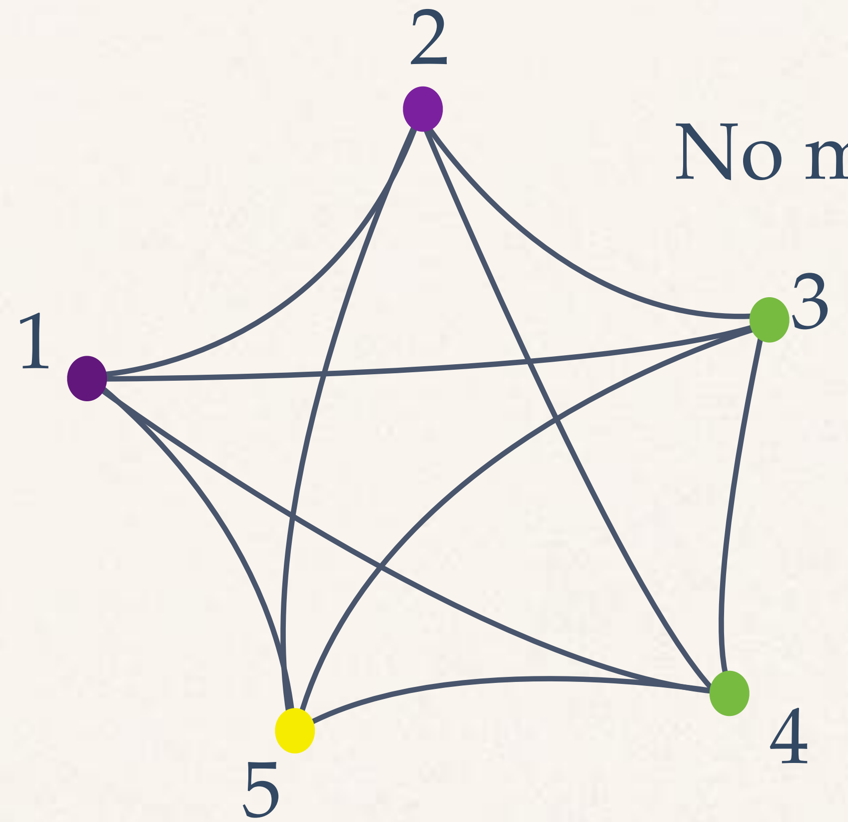

No monochromatic edges

If K_n^3 is the compleat hypergraph $\chi(K_n^3) = \lceil n/2 \rceil$

Perfection in hypergraphs?

Clearly $\lceil \omega(H)/2 \rceil \leq \chi(H)$

A 3-hypergraph H is perfect if $\chi(H) = \lceil \omega(H)/2 \rceil$.

Perfection in hypergraphs?

One nice example

Let $\mathcal{F}$ be a finite family of closed intervals in the circle $\mathbb{S}^1$,
and let $H = H(\mathcal{F})$ be the 3–hypergraph with $V(H) = \mathcal{F}$
such that a triple of intervals defines and edge of H if and only if the three intervals are pairwise disjoint.

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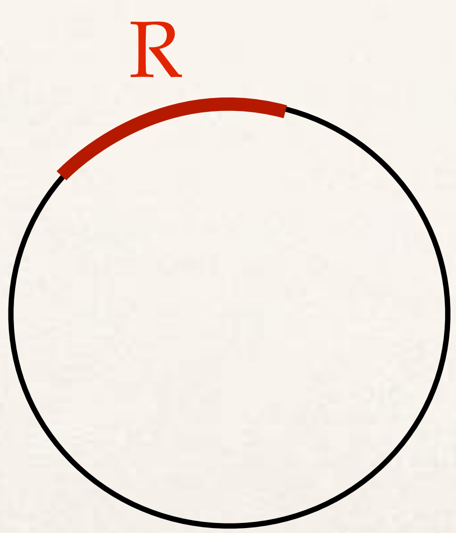

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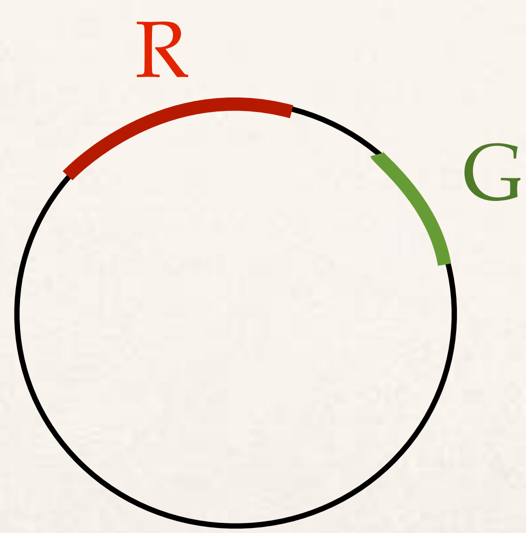

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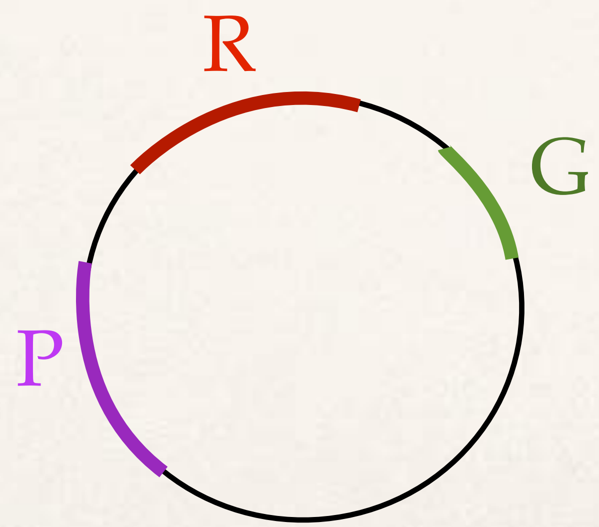

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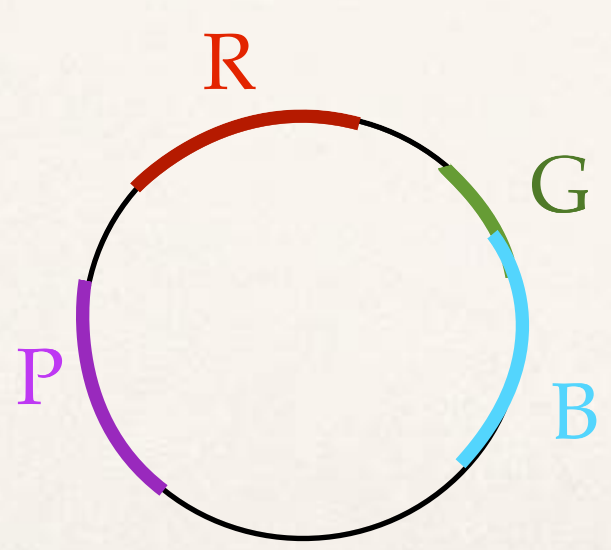

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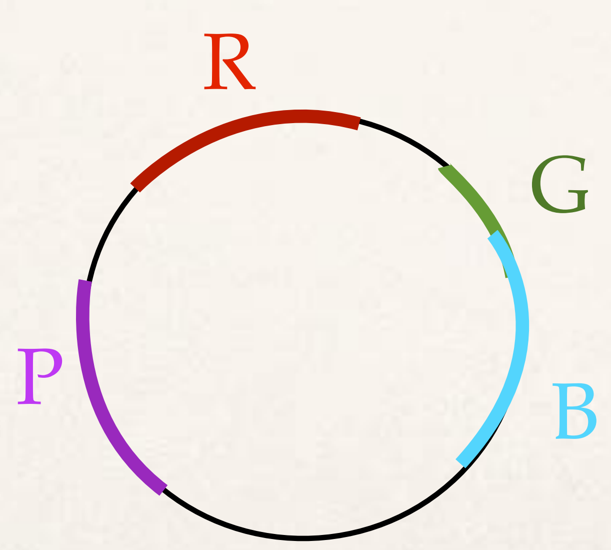

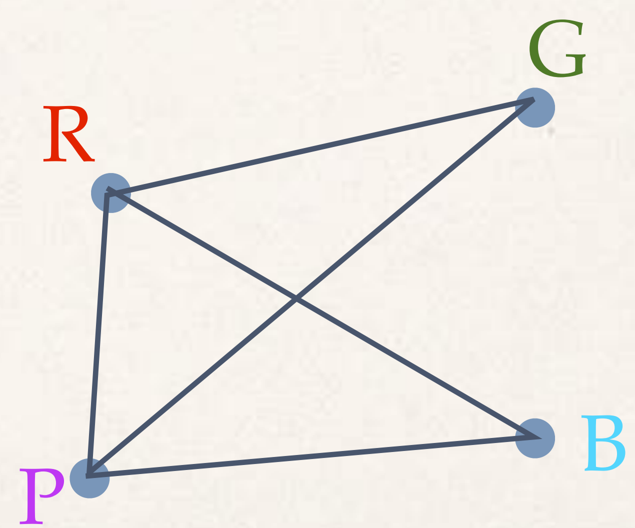

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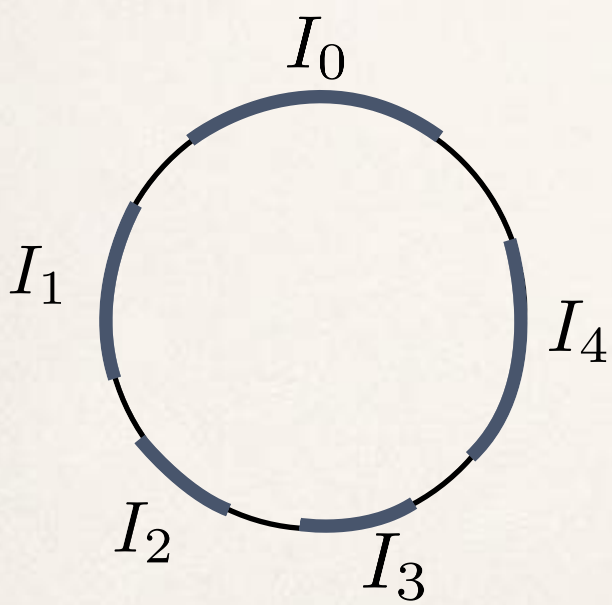
Proof

Suppose $\omega(H) = 2n + 1$



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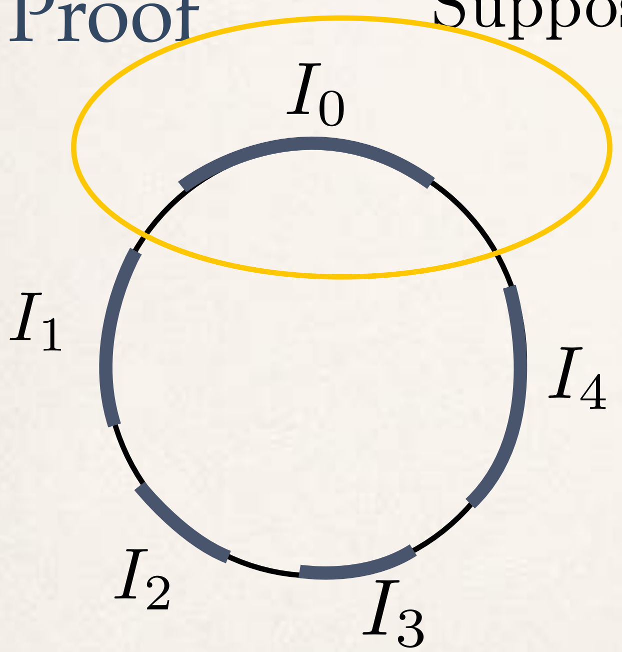

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Assign color yellow to I_0
together with all the ones intersecting I_0

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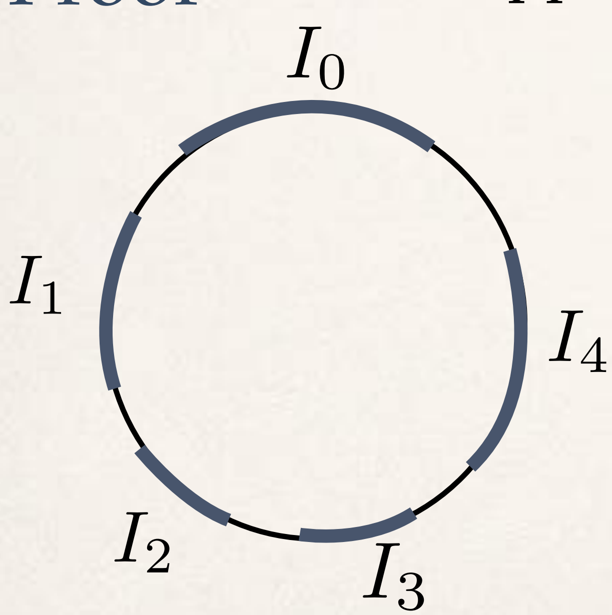

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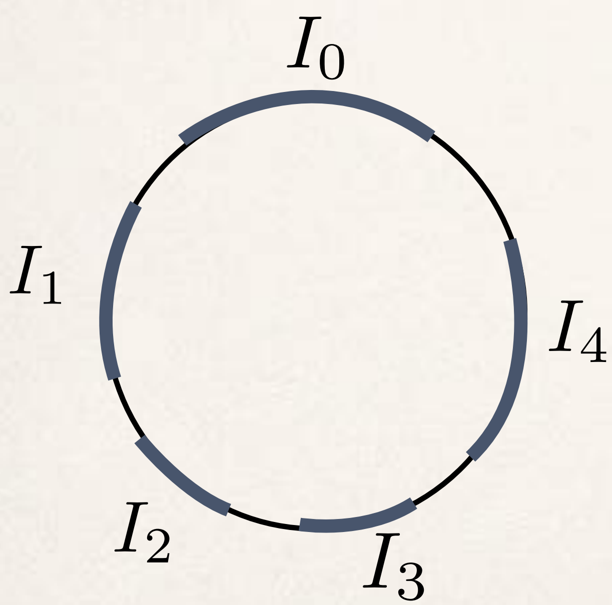

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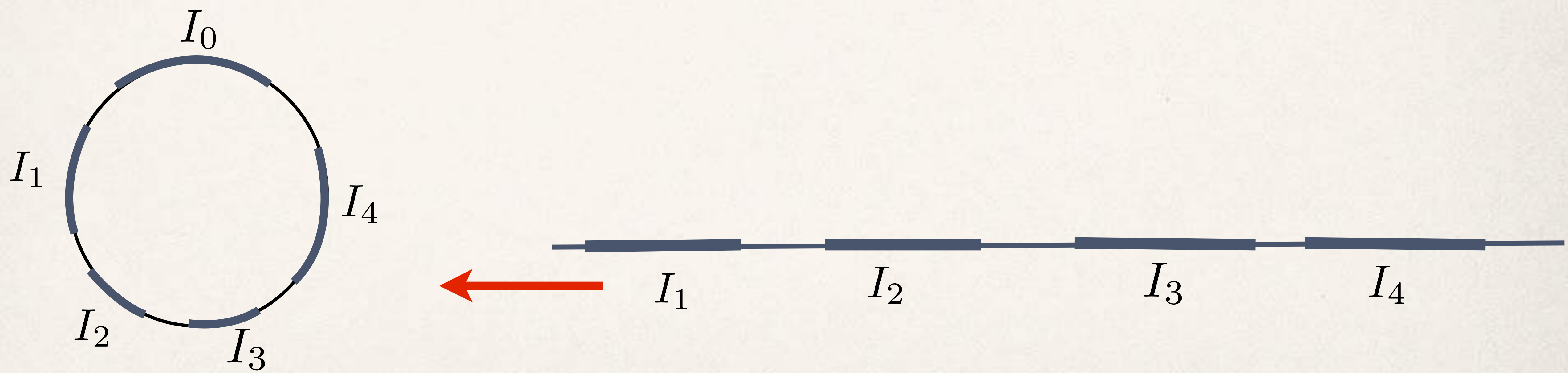

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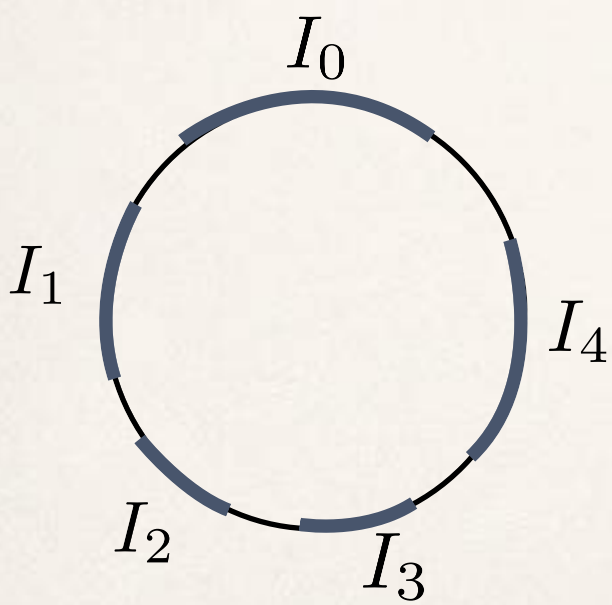

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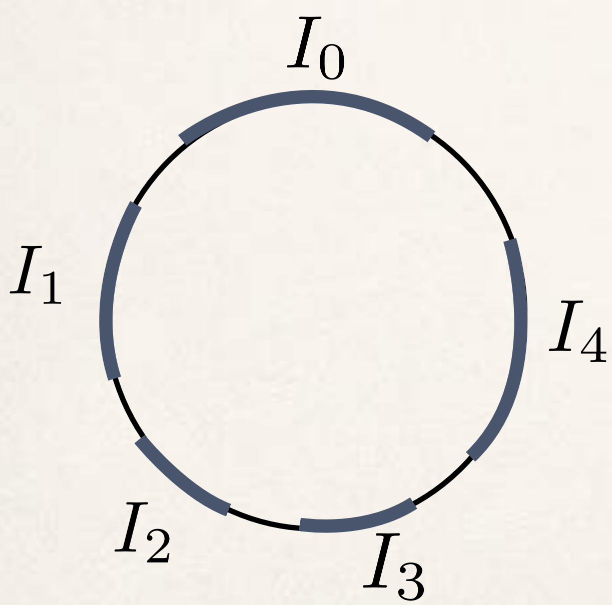

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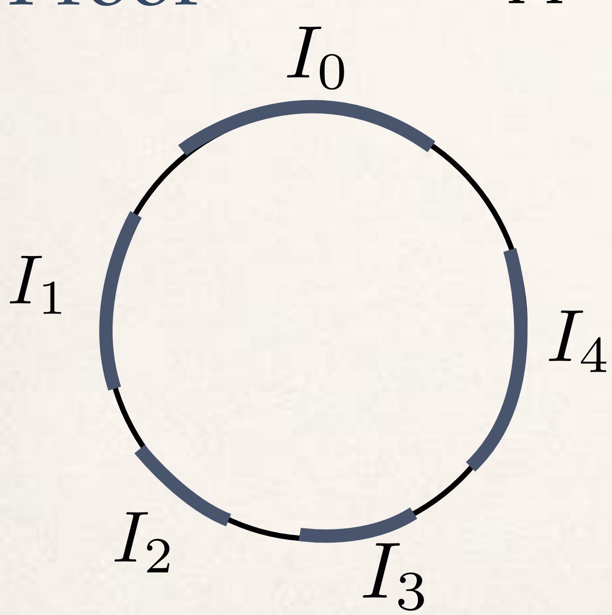

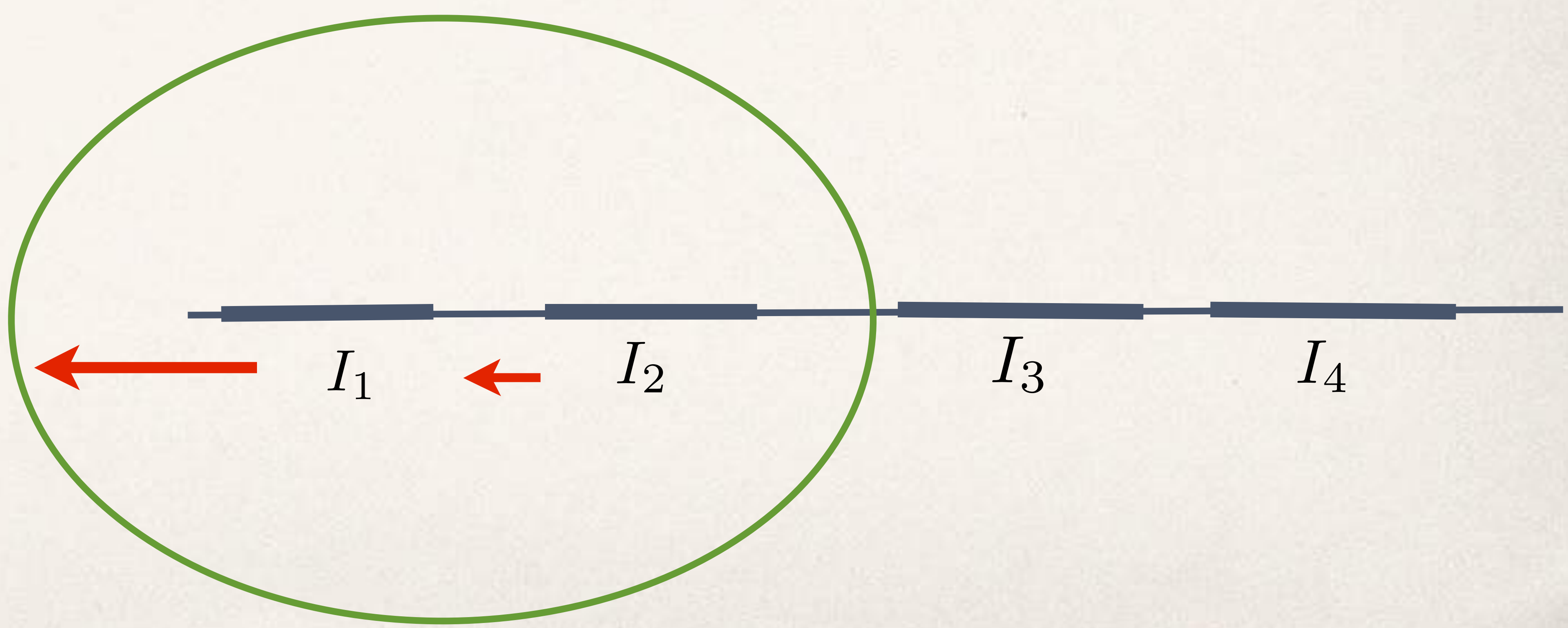

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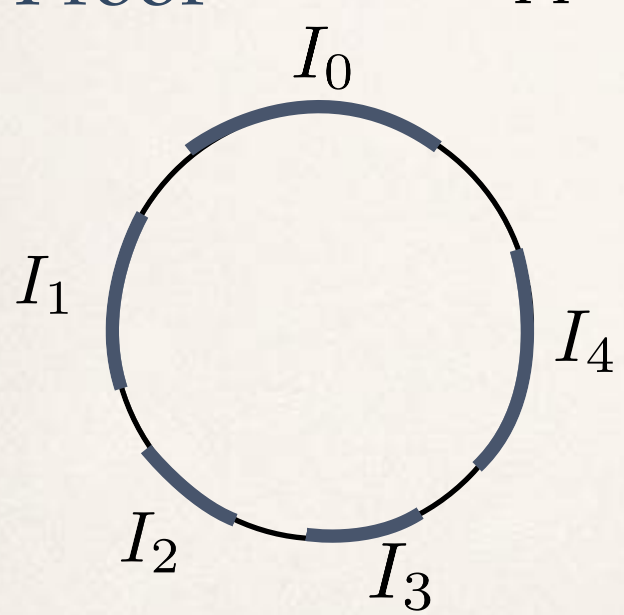

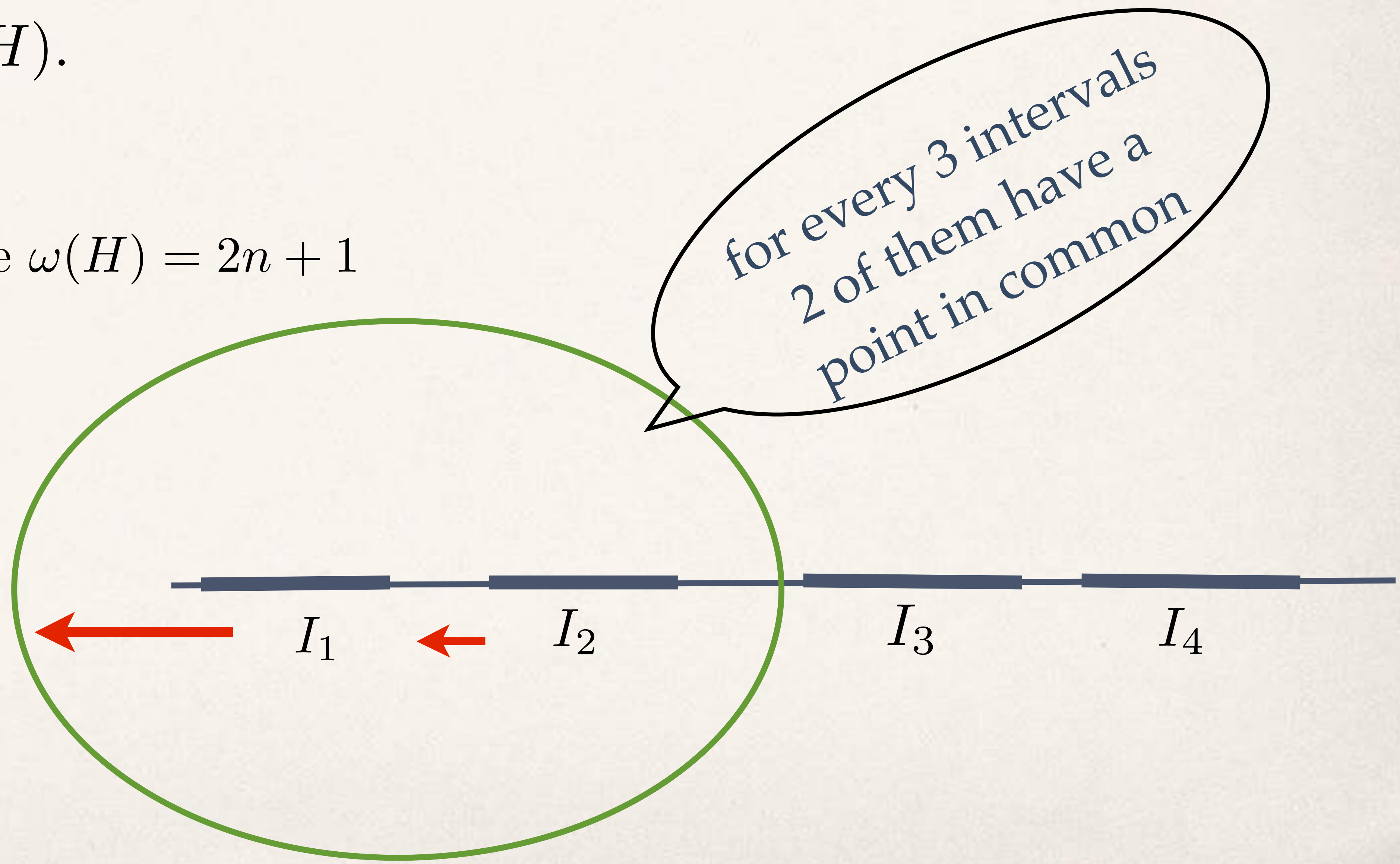

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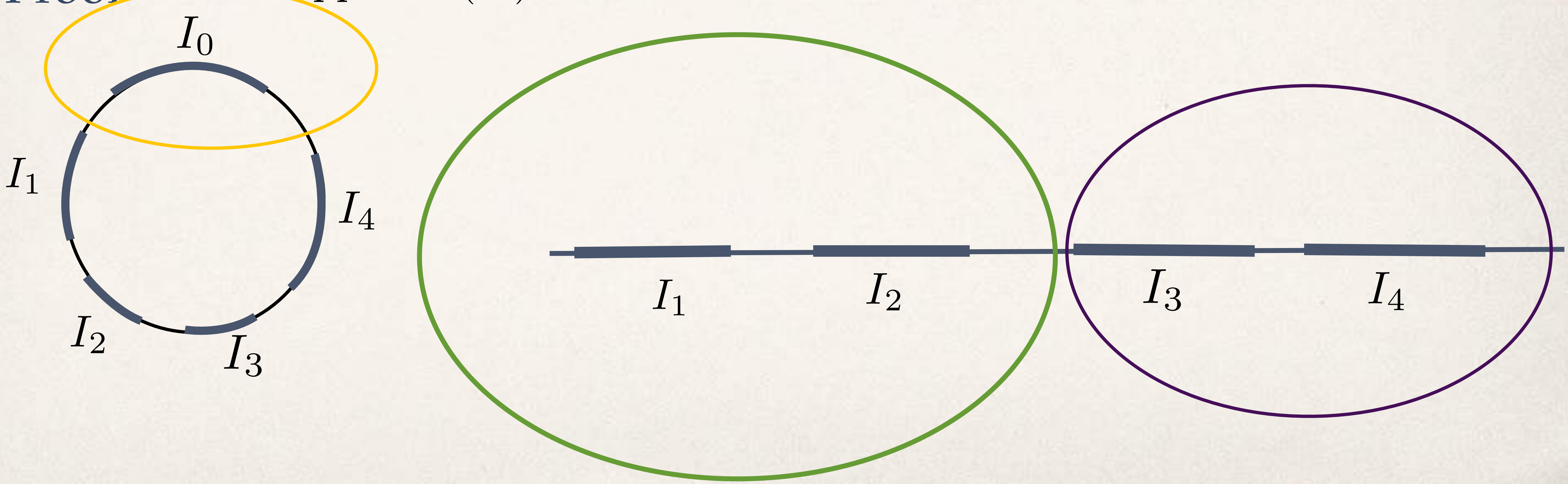

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Some well known families of graphs:

Bipartite graphs

Intersection graphs of intervals

comparability graphs

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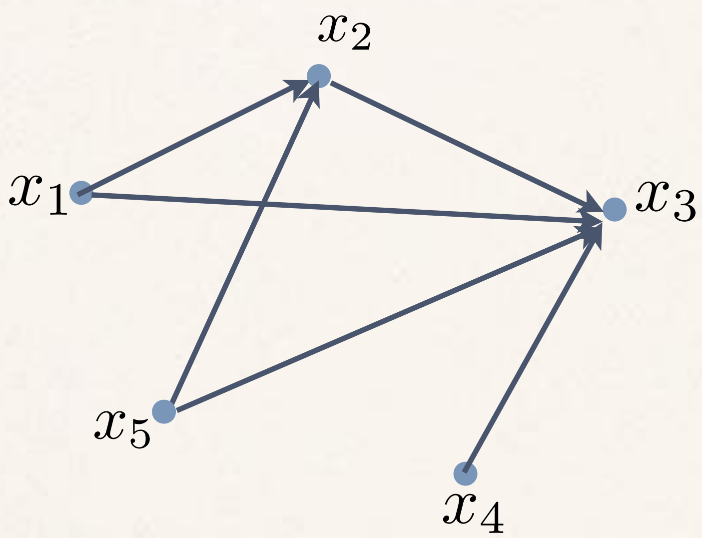

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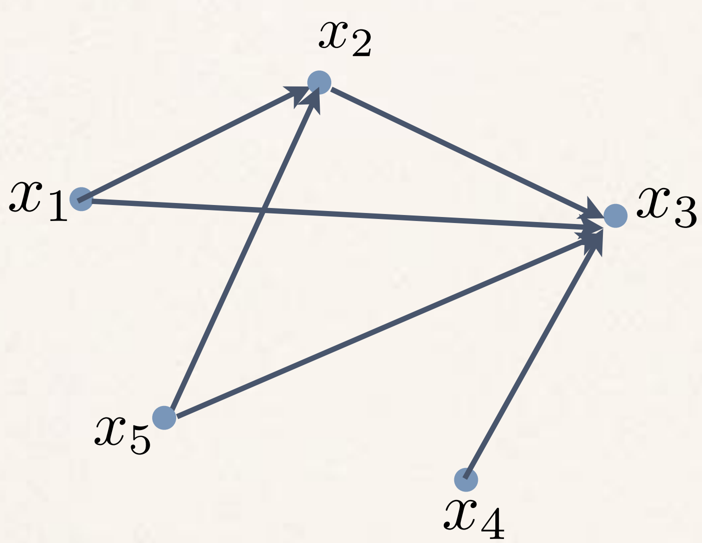

Partial order set

$x<y$ and $y<z$

implies x<z

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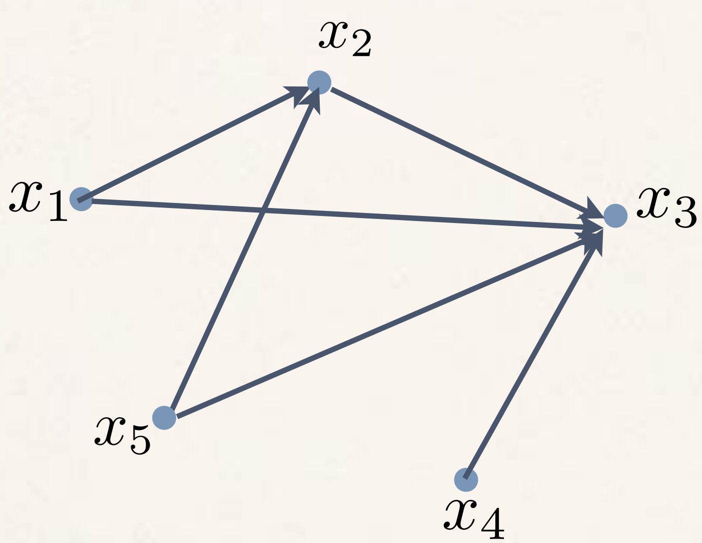
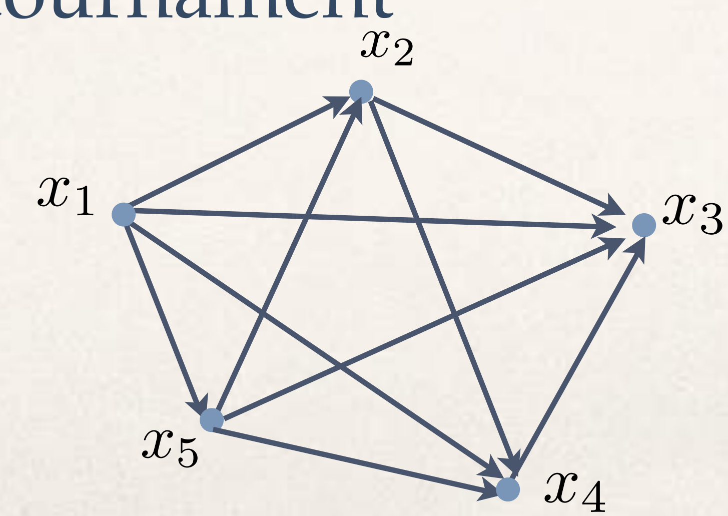


Transitive tournament



Partial order set

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Total order

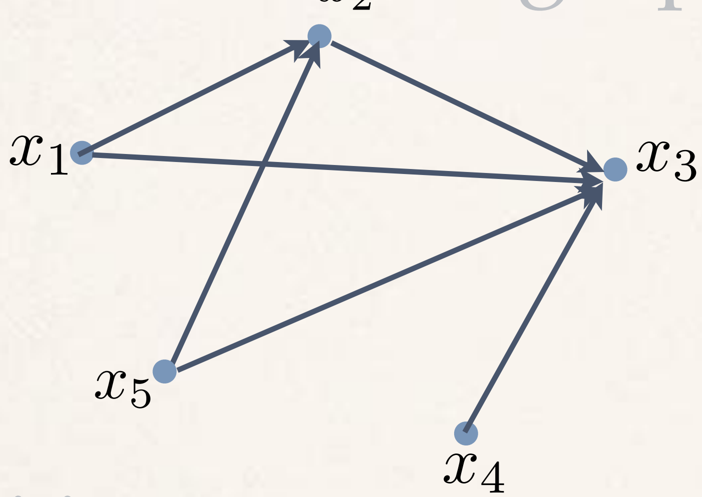
implies $x<z$

Linear order

Transitive oriented graphs

$$X := \{x_1, x_2, x_3, x_4, x_5\}$$

Transitive oriented graph



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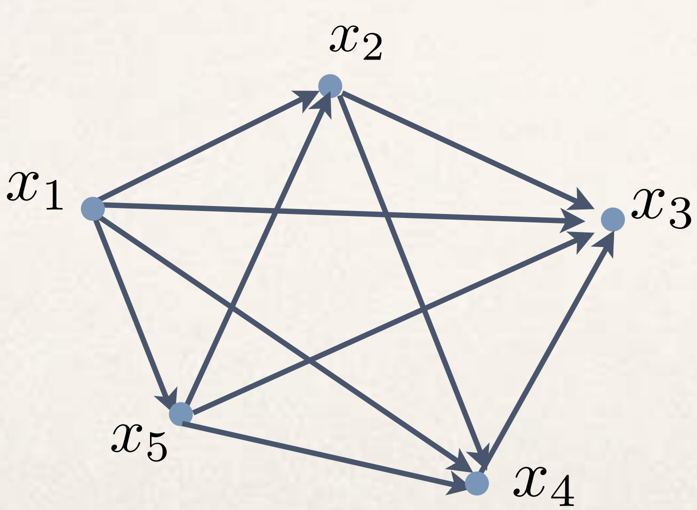

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$x_1 < x_2 < x_3$, $x_5 < x_2$, $x_4 < x_3$

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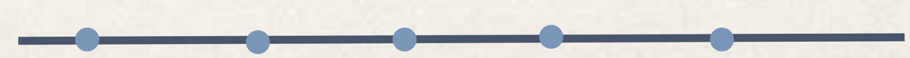
Linear order

$x_1 < x_5 < x_2 < x_4 < x_3$



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If in addition T is *total*

then T is called a *complete cyclic order*.

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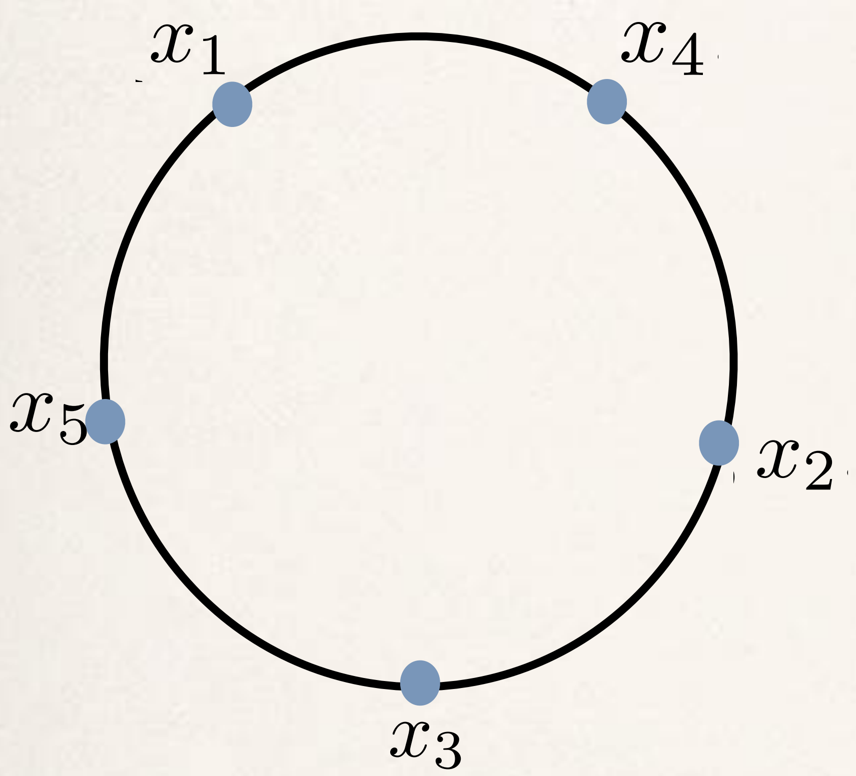
Alles, P., Nesetril, P.J., Poljak(1991)

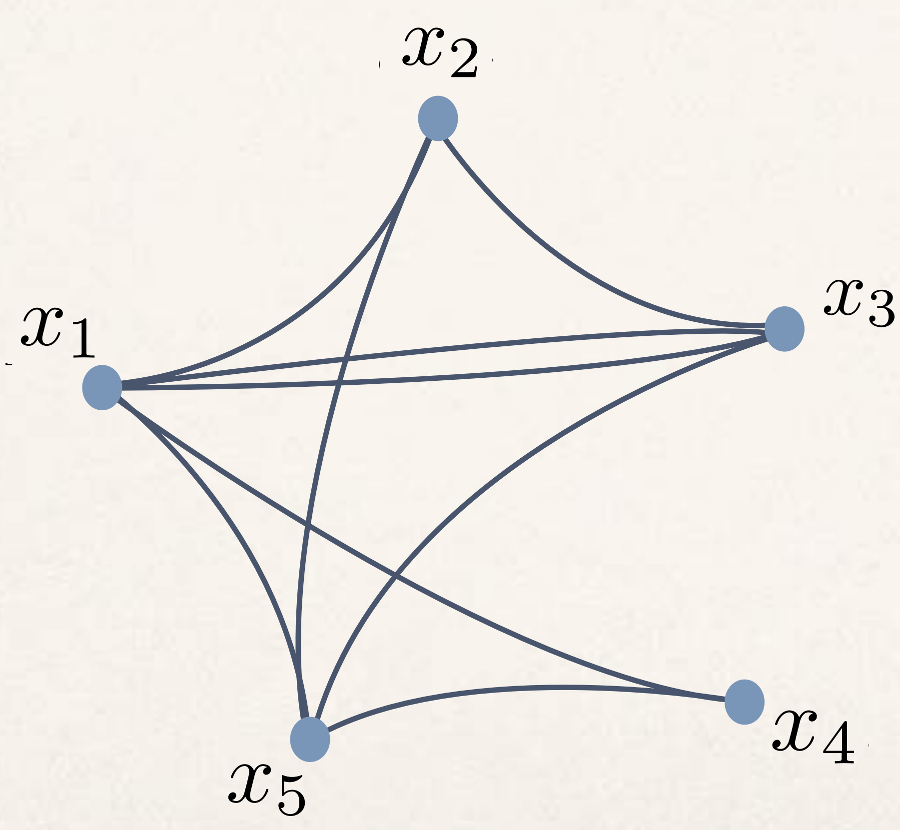
Example: Ciclic Permutations

$$X := \{x_1, x_2, x_3, x_4, x_5\}$$



Ciclic order

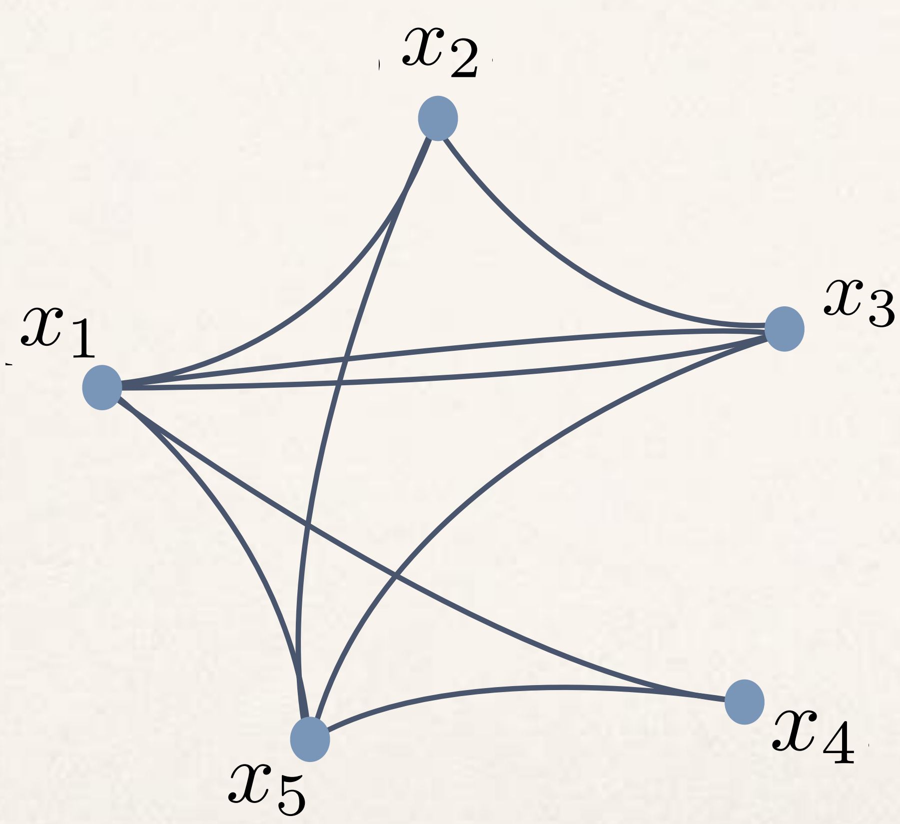
Oriented hypergraph?



Oriented hypergraph?

Start with a 3-hypergraph

uniform 3-hypergraph



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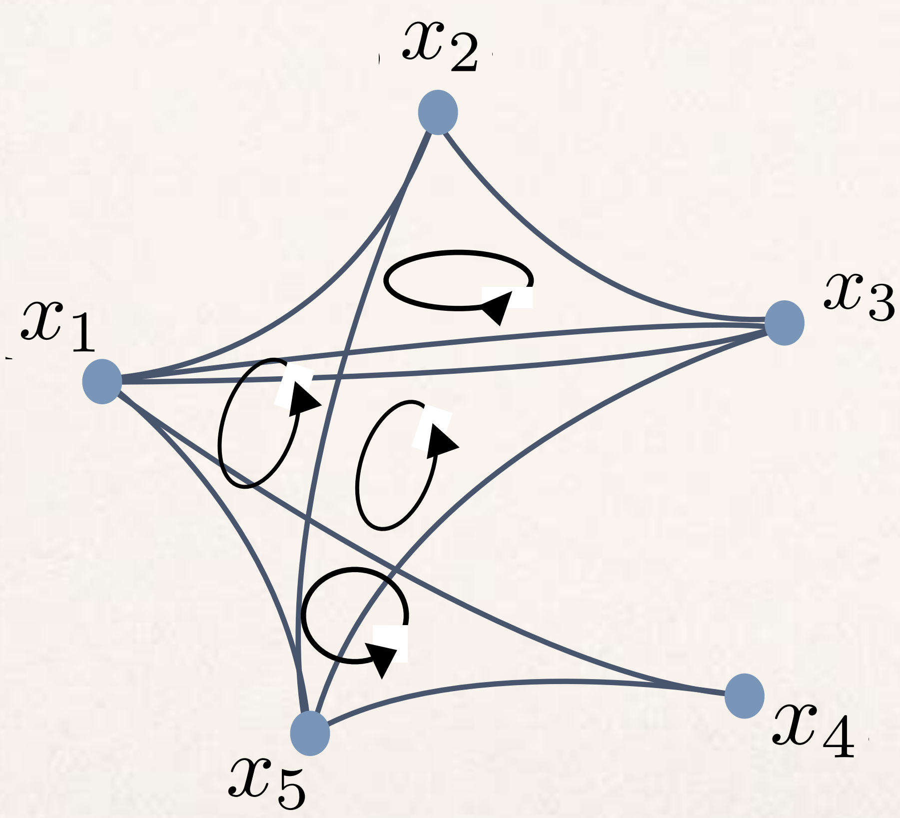

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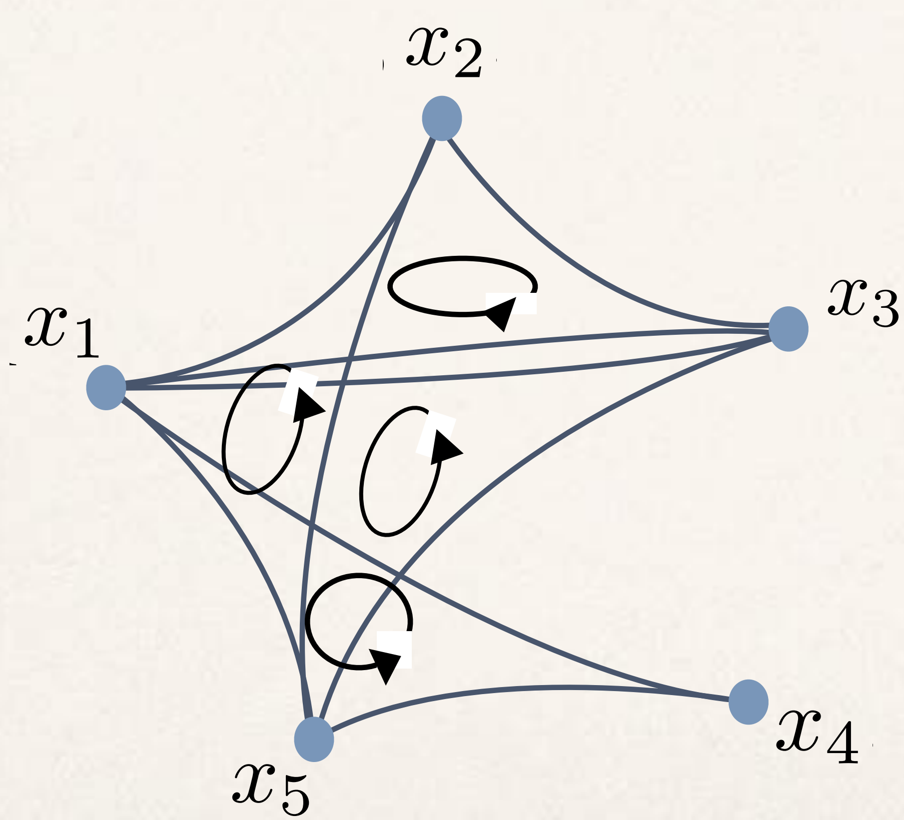

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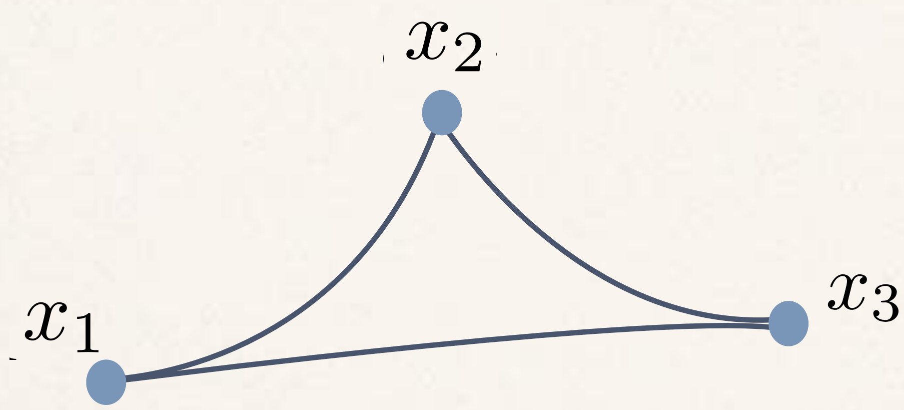

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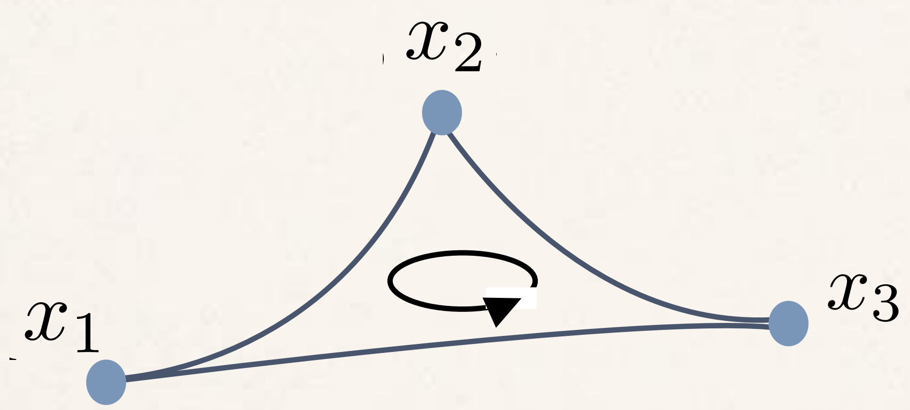

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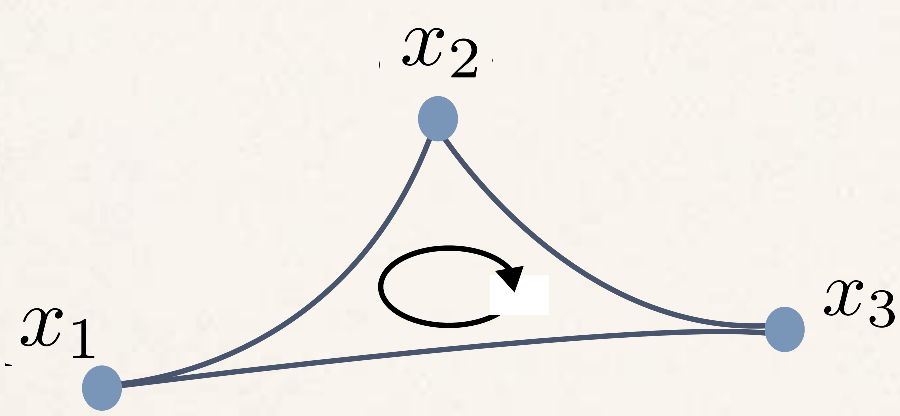

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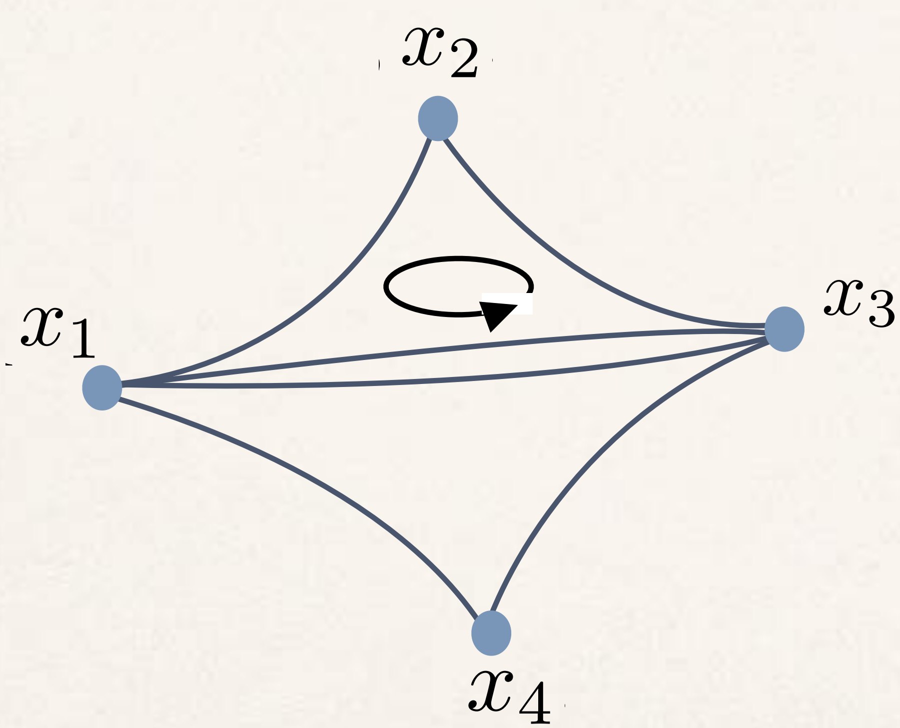

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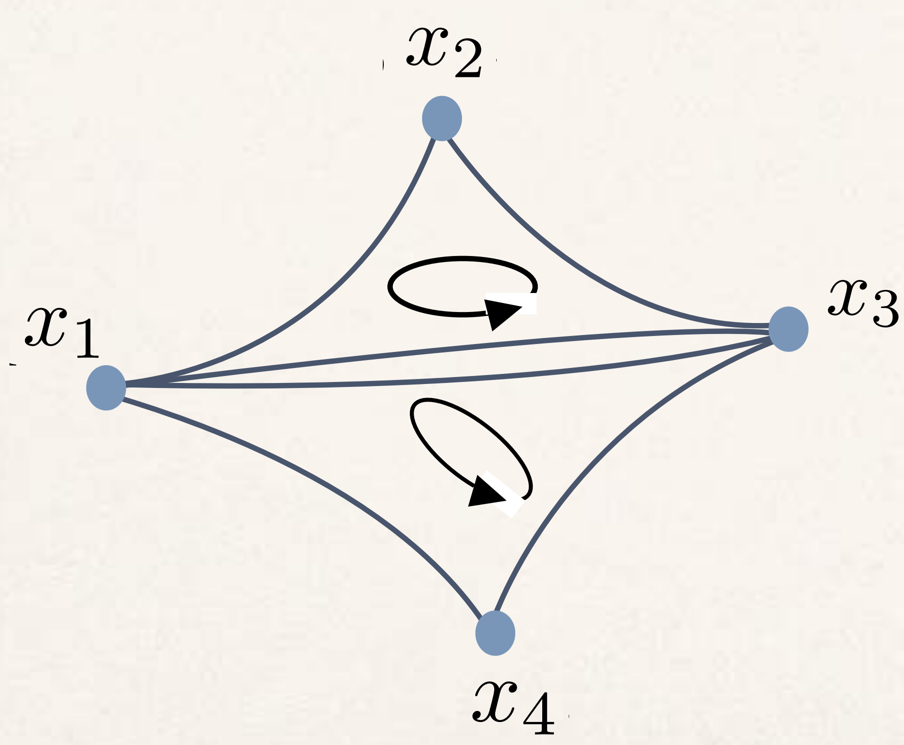

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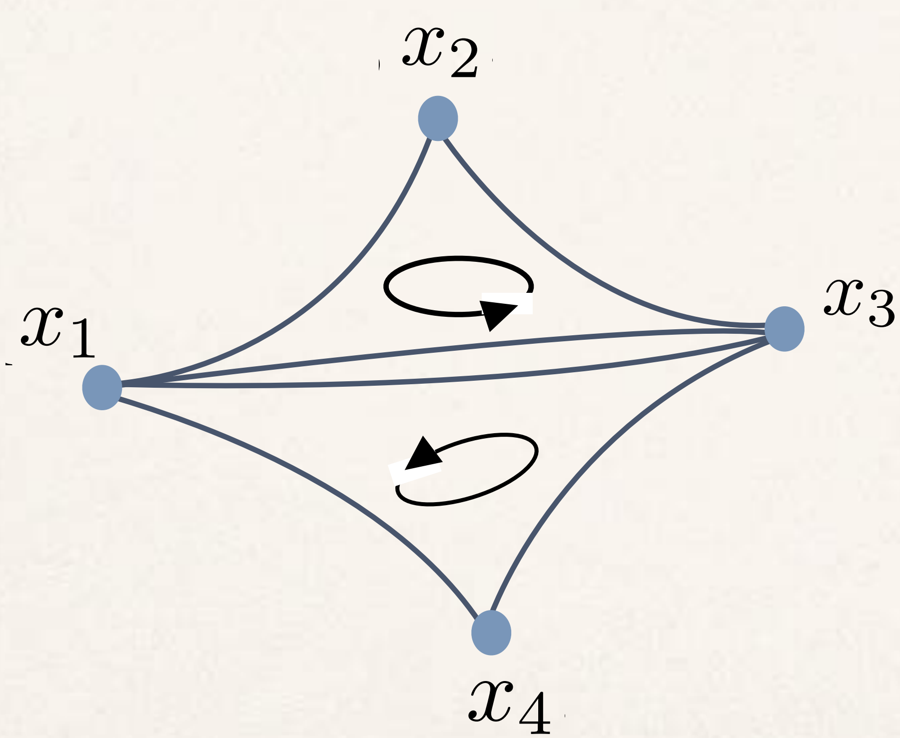
uniform 3-hypergraph



What is an orientation?

what is transitivity?

Oriented hypergraph?

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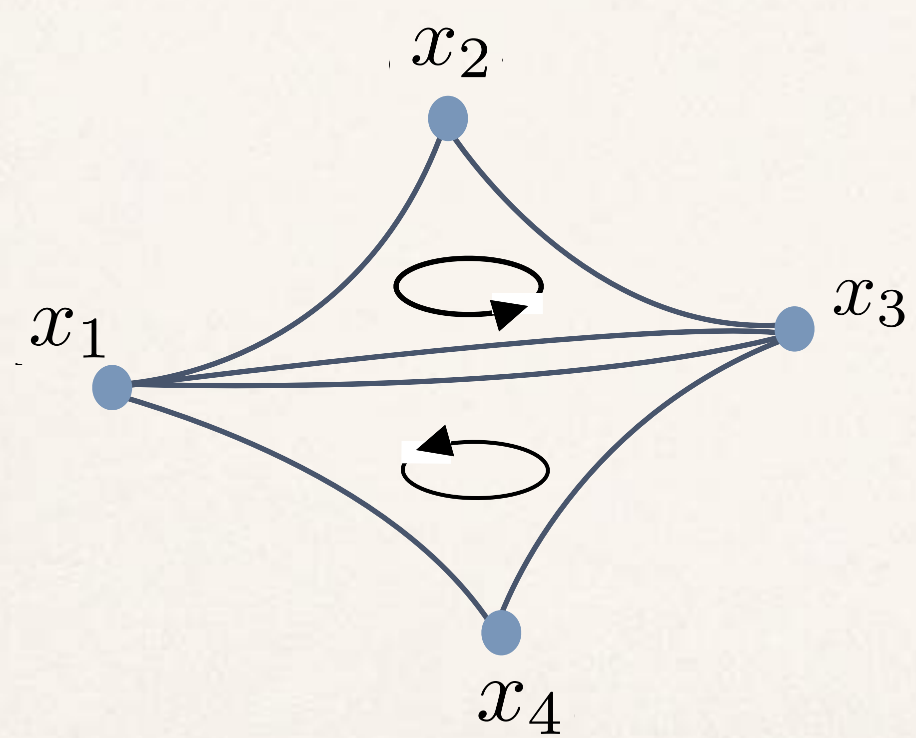

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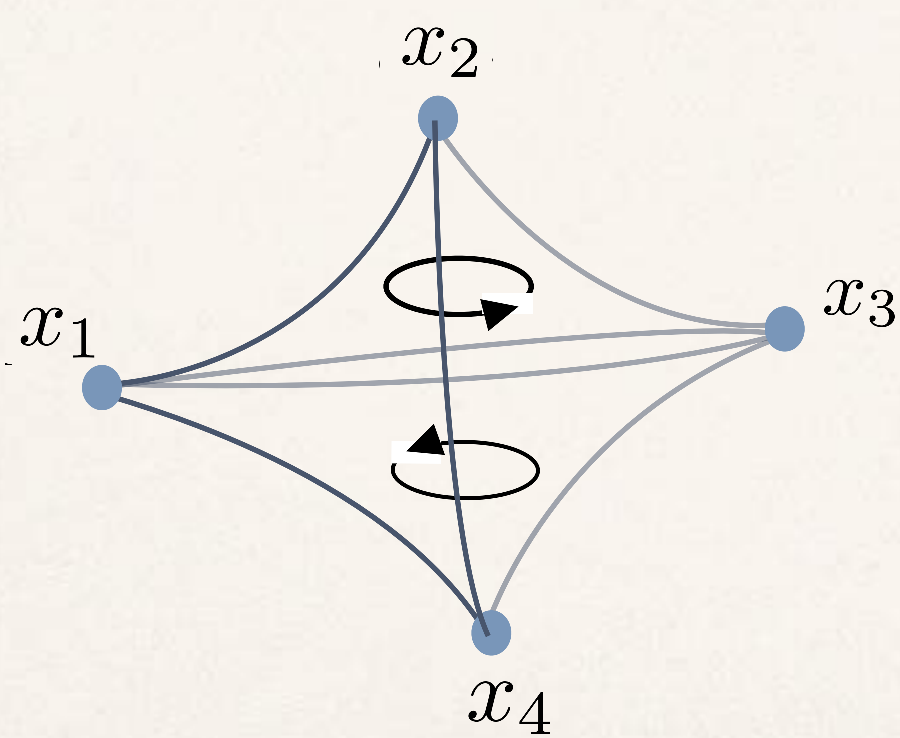

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Transitive oriented hypergraph?

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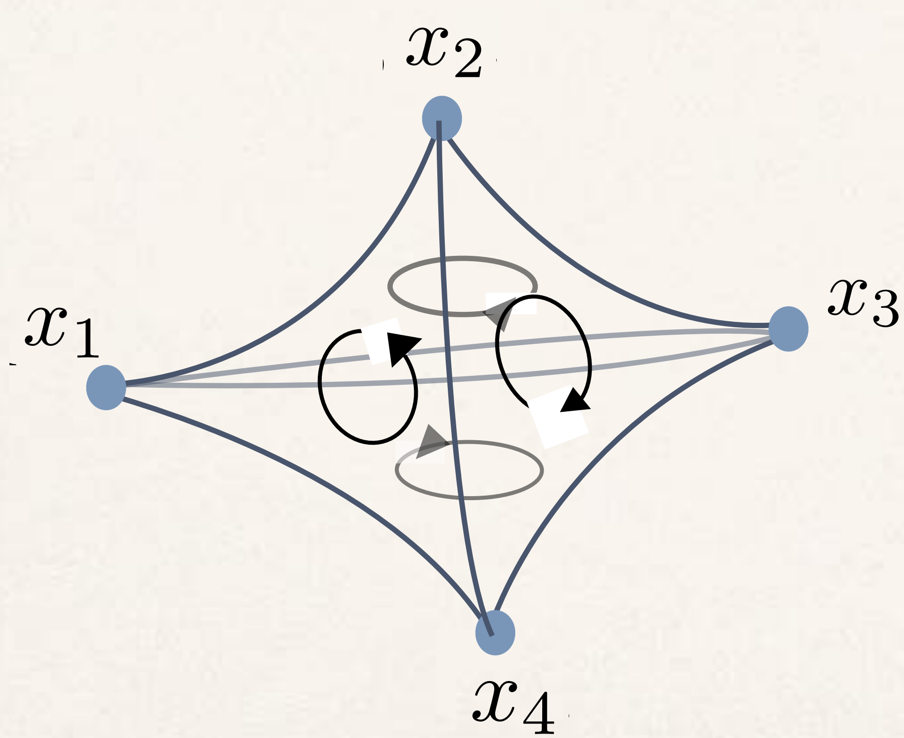
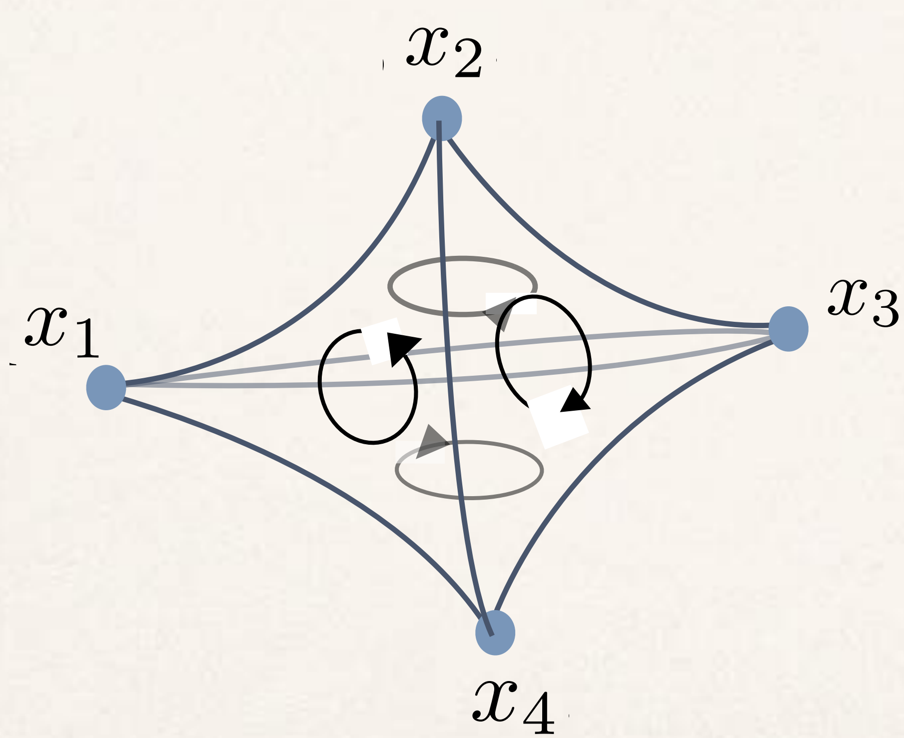


What is an orientation?

what is transitivity?

Transitive oriented hypergraphs.

Observations: Every oriented 3-subhypergraph of an oriented 3-hypergraph is oriented



Comparability 3-hypergraphs.

A *comparability* 3–*hypergraphs* is the class of non oriented 3–hypergraphs, which can be transitively oriented

Transitive oriented hypergraphs.

Observations:

There is a natural correspondence between partial cyclic orders and transitive oriented 3-hypergraphs

Transitive oriented hypergraphs.

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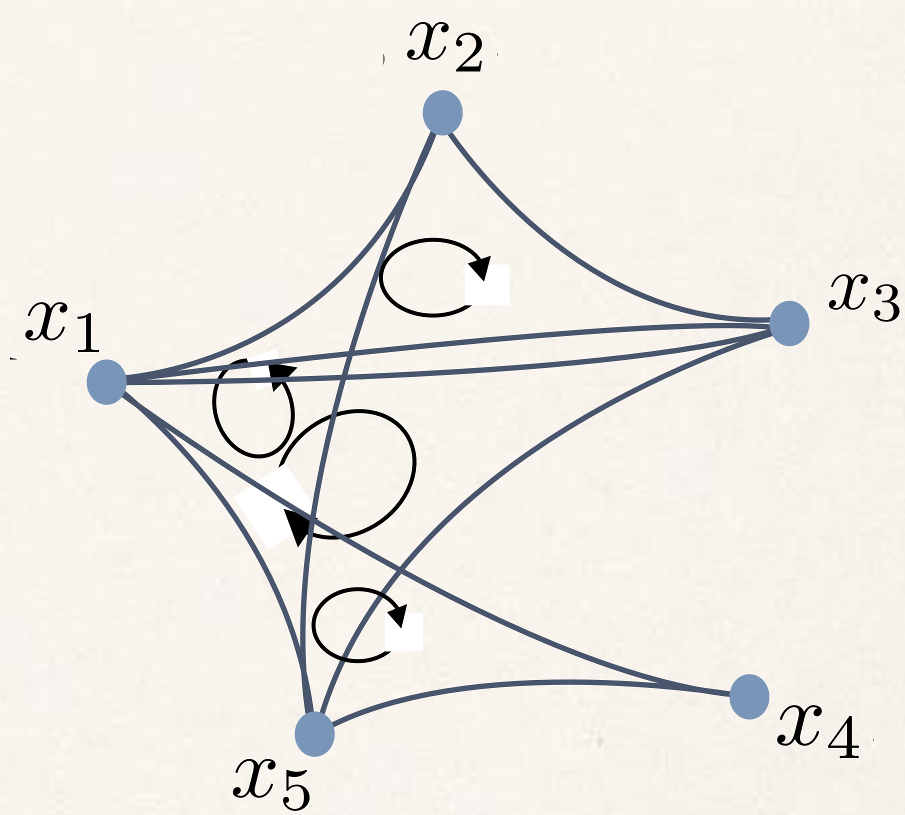


(x_1, x_2, x_3), (x_1, x_5, x_2), (x_5, x_3, x_2)
(x_1, x_3, x_5) and (x_1, x_4, x_5)

partial cyclic order

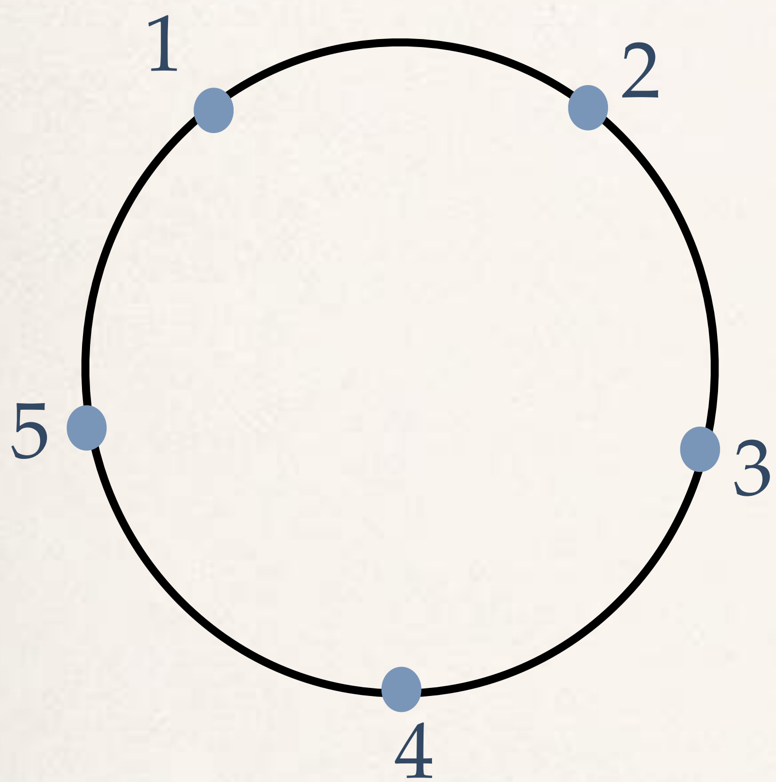
There is a natural correspondence between partial cyclic orders and transitive oriented 3-hypergraphs

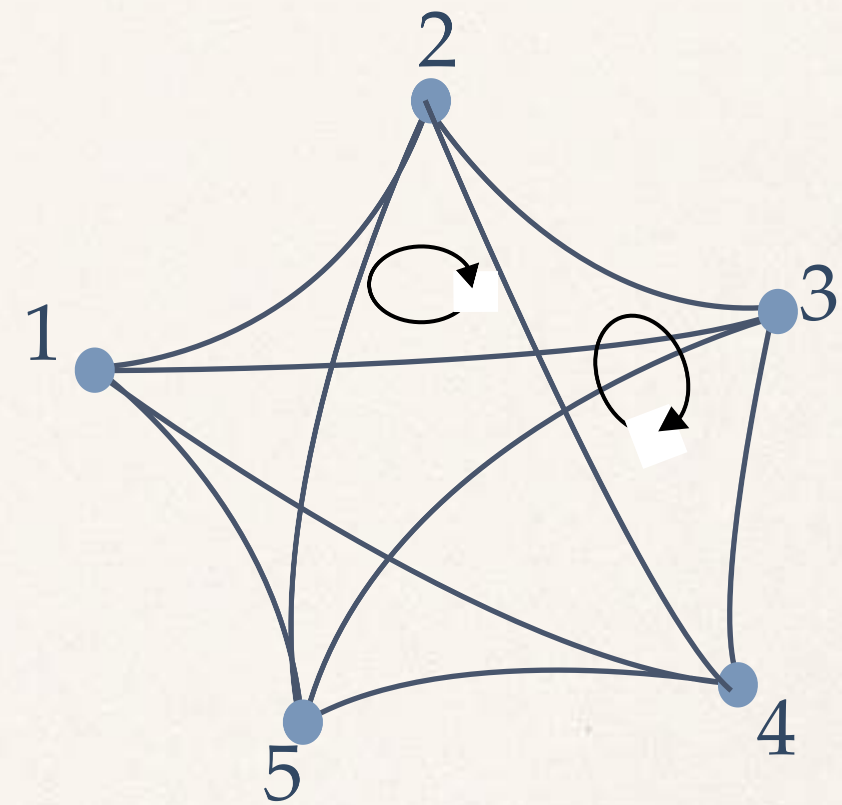
Transitive oriented hypergraphs.

A transitive oriented 3–hypergraph, H, with $E(H) = \binom{V(H)}{3}$ is called a 3–*hypertournament*.

Transitive oriented hypergraphs.

$$X = \{1, 2, 3 \ldots n\}$$





Let TT_n^3 be the oriented 3–hypergraph with $V(TT_n^3) = [n]$ and $E(H) = \binom{[n]}{3}$, where the orientation of each edge is the one induced by the cyclic ordering $(1\,2\,\ldots\,n)$

Transitive oriented hypergraphs.

Theorem 0.1 *Every transitive* 3*–hypertournament on* n *vertices is isomorphic to* TT_n^3.

(1991) Peter Alles, Peter Jaroslav Nesetril and Svatopluk Poljak.

Transitive oriented hypergraphs.

An oriented 3–hypergraph H which is a spanning subhypergraph of TT_n^3 is called *self-transitive* if it is transitive and its complement is also transitive.

Theorem 0.2 *H is an oriented cyclic permutation 3–hypergraph if and only if H is self transitive.*

Perfection?

Perfection?

Clearly complete graphs satisfy $\chi(K_n^3) = \left\lceil \frac{n}{2} \right\rceil$

then for any 3–hypergraph the following equation holds:

$$\left\lceil \frac{\omega(H)}{2} \right\rceil \leq \chi(H).$$

Perfection?

Is it true that for comparability 3-hypergraphs $\chi(H) = \left\lceil \frac{\omega(H)}{2} \right\rceil$?

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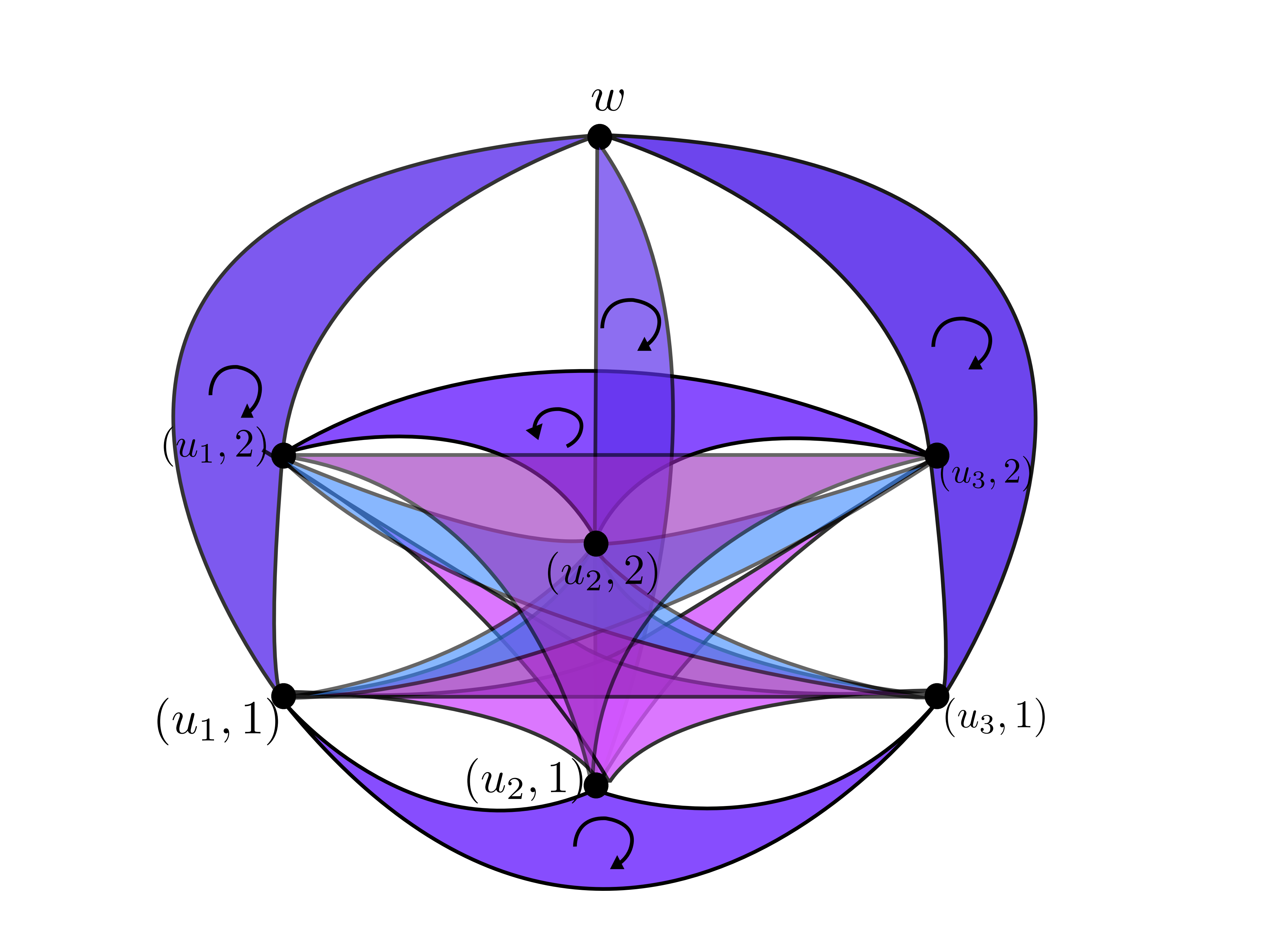
No!

Perfection?

Is it true that for comparability 3-hypergraphs $\chi(H) = \left\lceil \frac{\omega(H)}{2} \right\rceil$?

We exhibit a family of comparability 3–hypergraphs for which the difference, $\chi(H) - \left\lceil \frac{\omega(H)}{2} \right\rceil$, is arbitrarily large.

Perfection?



Perfection?

If H is a cyclic permutation 3-hypergraph is true that $\chi(H) = \lceil w(H)/2 \rceil$?

No.....

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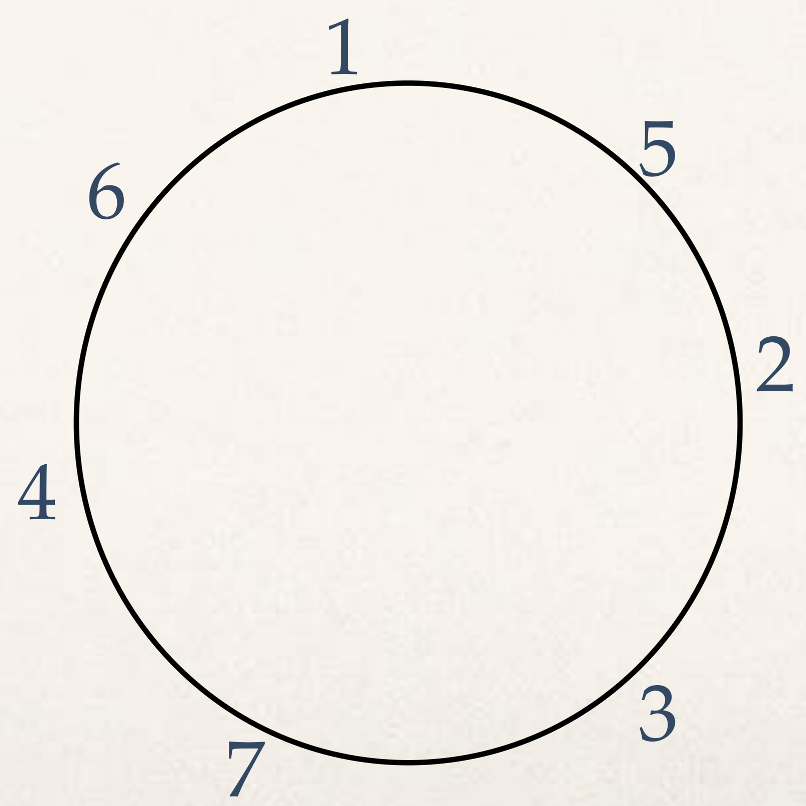

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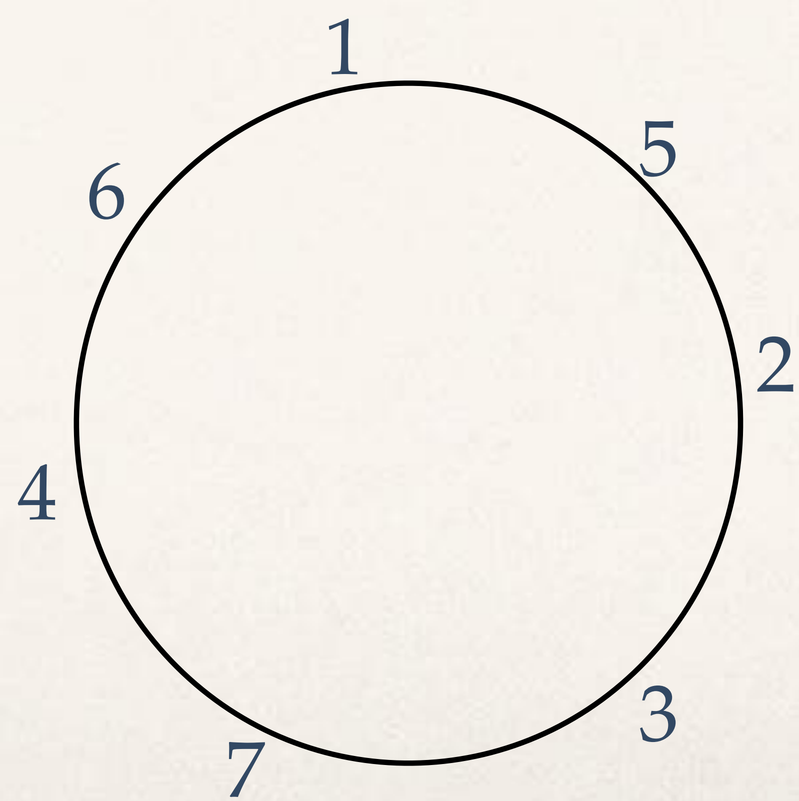

$\omega(H) = 4$ but $\chi(H) = 3$

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Theorem: Let H be a cyclic permutation 3–hypergraph, then $\chi(H) \leq \omega(H) - 1$. Furthermore, this bound is tight.

Thanks for your attention!

- Köszönöm !