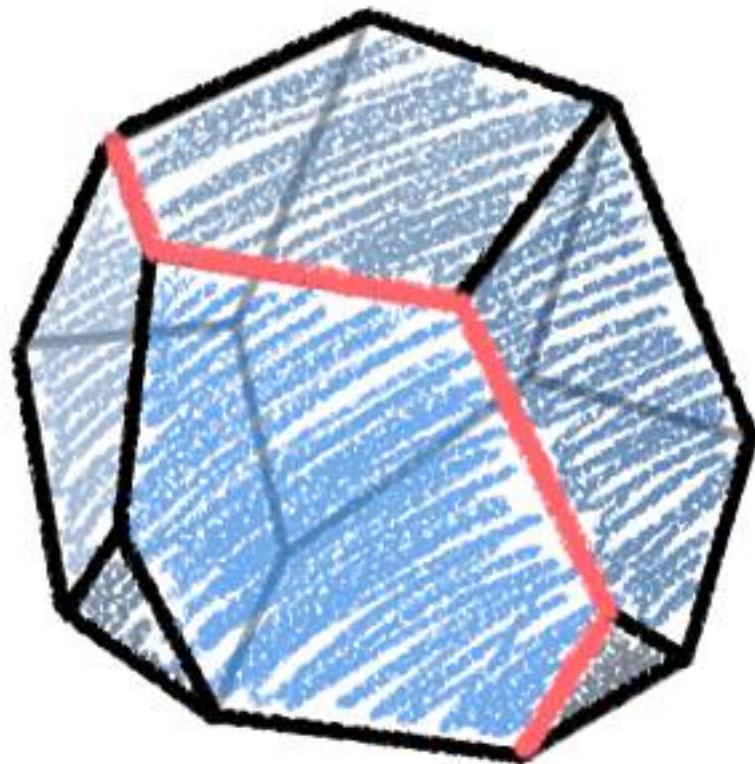


On the geometry of linear optimization



Antoine Deza, McMaster

based on joint works with
Tamás Terlaky, Lehigh
Feng Xie, Microsoft
Yuriy Zinchenko, Calgary





KöMaL arcképcsarnok

Keresés

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[2010-2011](#)

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[**\(1971-1972\) >>**](#)

1970-1971

[1. tábló](#) | [2. tábló](#) | [3. tábló](#) | [4. tábló](#)



Oláh Vera



Holló Vilmos



Vladár Károly



Juhász György



Ábrahám László



Párkány Katalin



Traply Endre



Kéméri Viktória



Bartolits István



Bezdek Károly



Terlaky Tamás



Szabados György



Császár Gyula



Pálffy László



Burda Magdalna



Tegze Miklós



Rövid Kálmán



Tarjányi László



Bartha Miklós



Bara Tamás



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8 találat:

[Bezdek Ádám](#) (2007-2008)

[Bezdek András](#) (1971-1972)

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[Bezdek András](#) (1973-1974)

[Bezdek András](#) (1974-1975)

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[Bezdek Károly](#) (1972-1973)

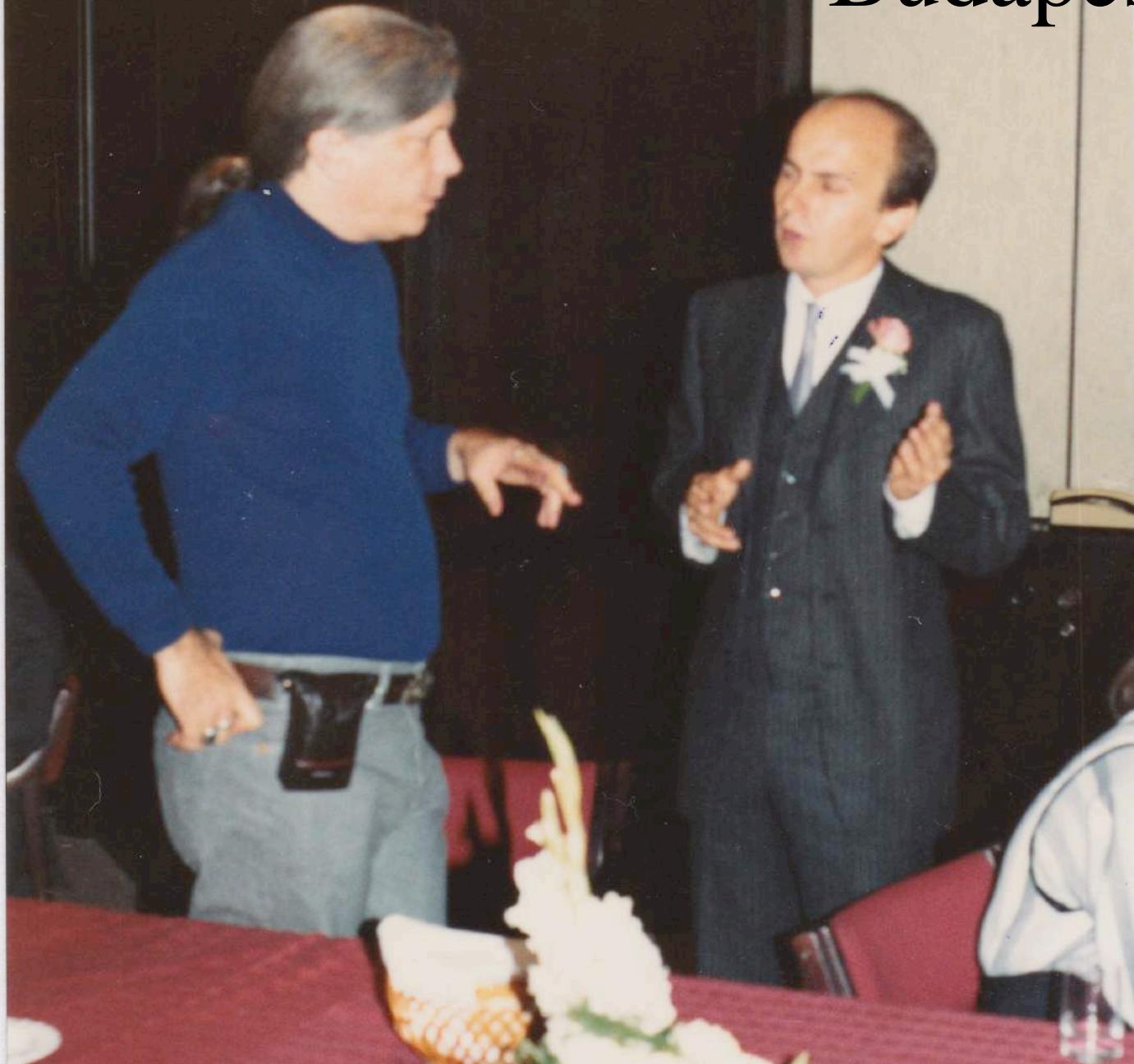


Budapest 1986



Mark, Mate, Daniel, New York, 2014

Budapest 1986



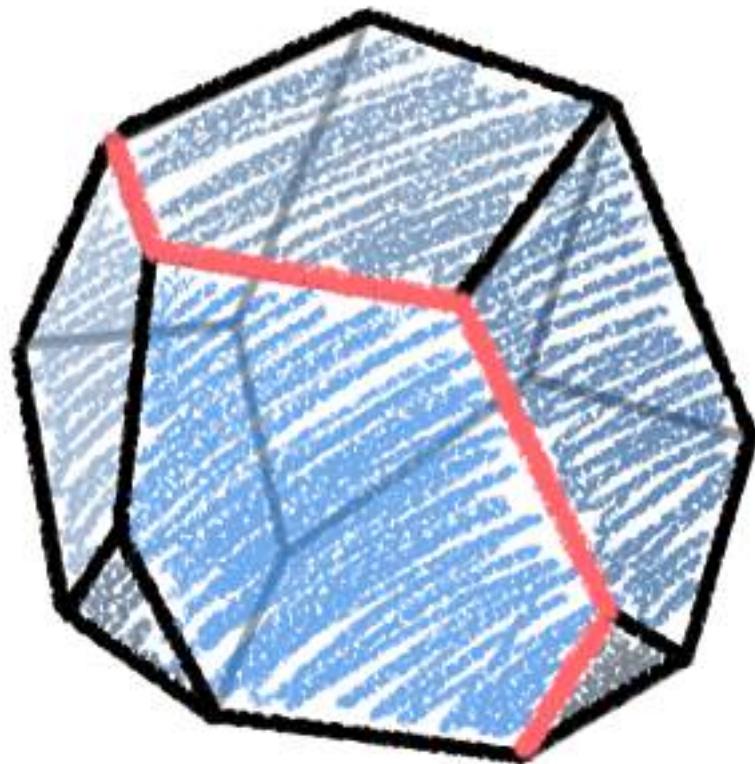


Banff 2015



Fields Institute, Toronto 2011

On the geometry of linear optimization



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Linear Optimization?

Given an n -dimensional vector \mathbf{b} and an $n \times d$ matrix \mathbf{A}
find, in any, a d -dimensional vector \mathbf{x} such that :

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} \geq 0$$

linear algebra

linear optimization

Application: Carathéodory Theorem warranties the **existence** of a proper set of points, linear optimization warranties the set can be **found efficiently**

The Fields Institute for Research in Mathematical Sciences

Károly Bezdek
Antoine Deza
Yinyu Ye
Editors



Discrete Geometry and Optimization

Polytopes & Arrangements : Diameter & Curvature

Motivations and algorithmic issues

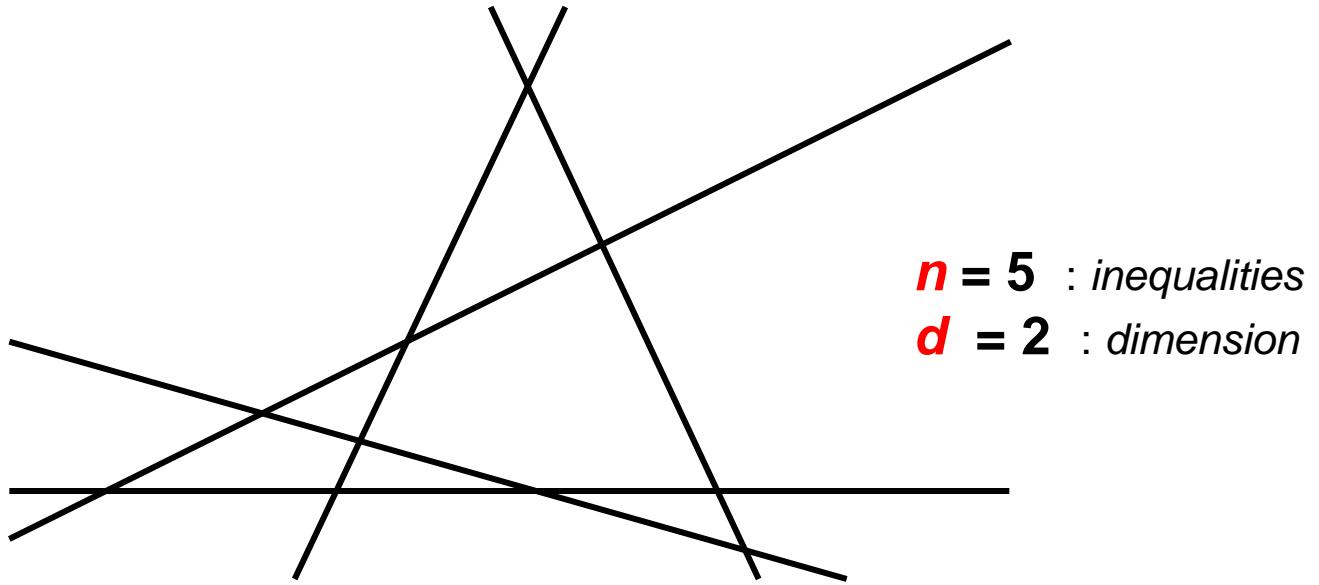
Diameter (of a polytope) :

lower bound for the number of iterations
for the **simplex method** (*pivoting methods*)

Curvature (of the central path associated to a polytope) :

large curvature indicates large number of iterations
for (*path following*) **interior point methods**

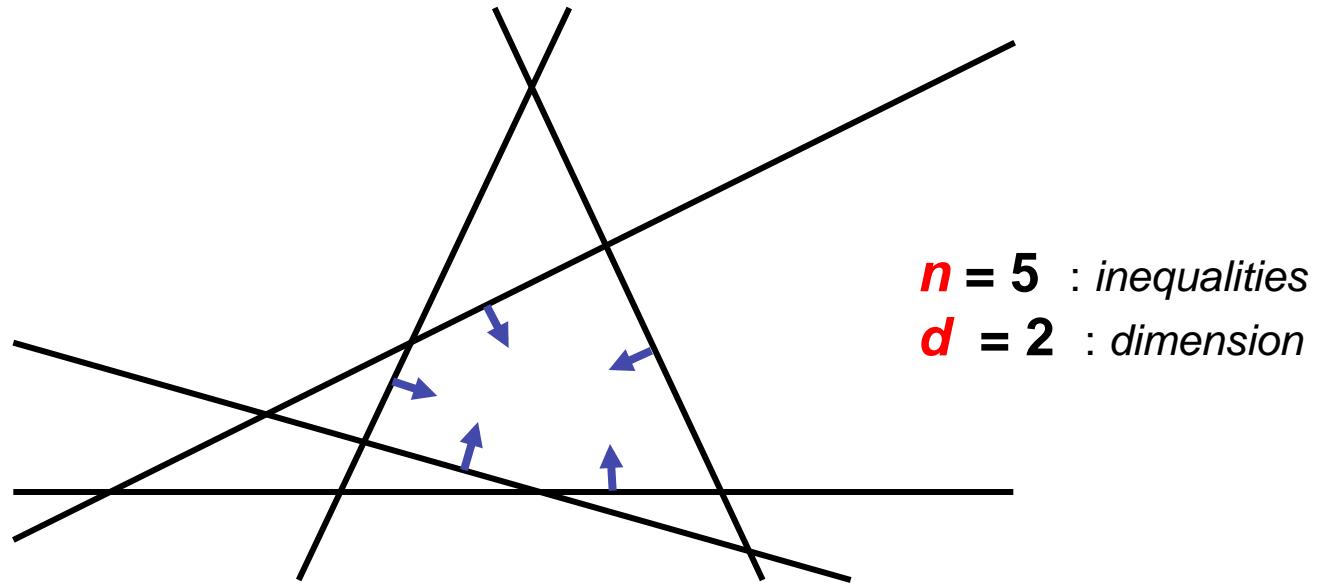
Polytopes & Arrangements : Diameter & Curvature



Polytope P defined by n inequalities in dimension d

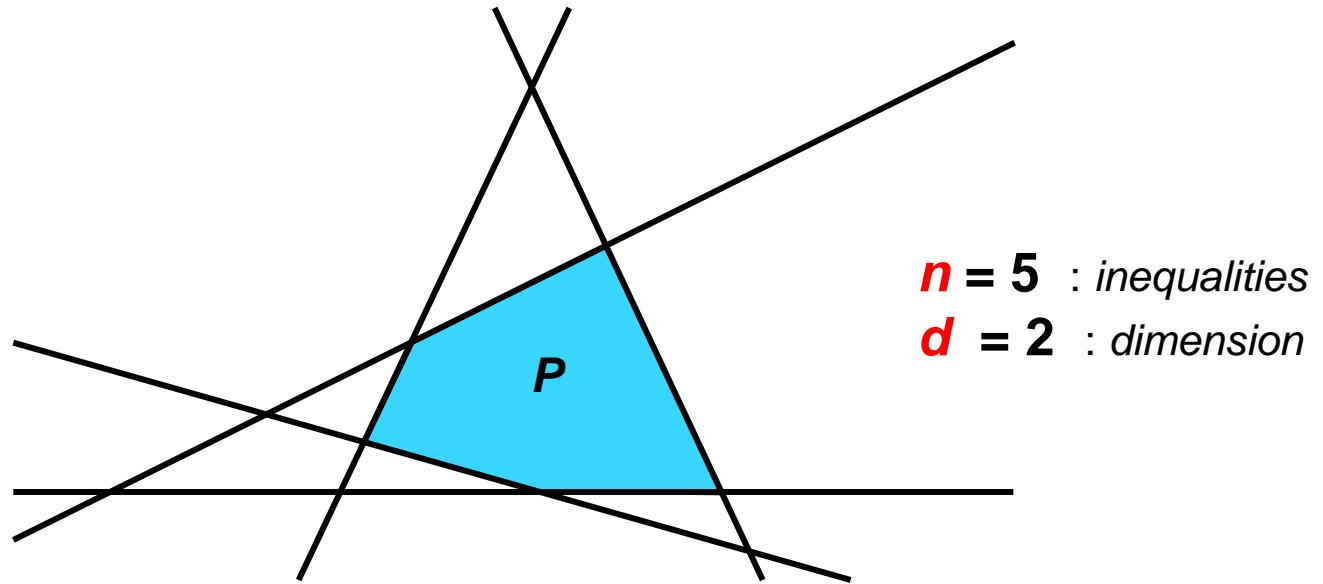
❖ polytope : *bounded polyhedron*

Polytopes & Arrangements : Diameter & Curvature



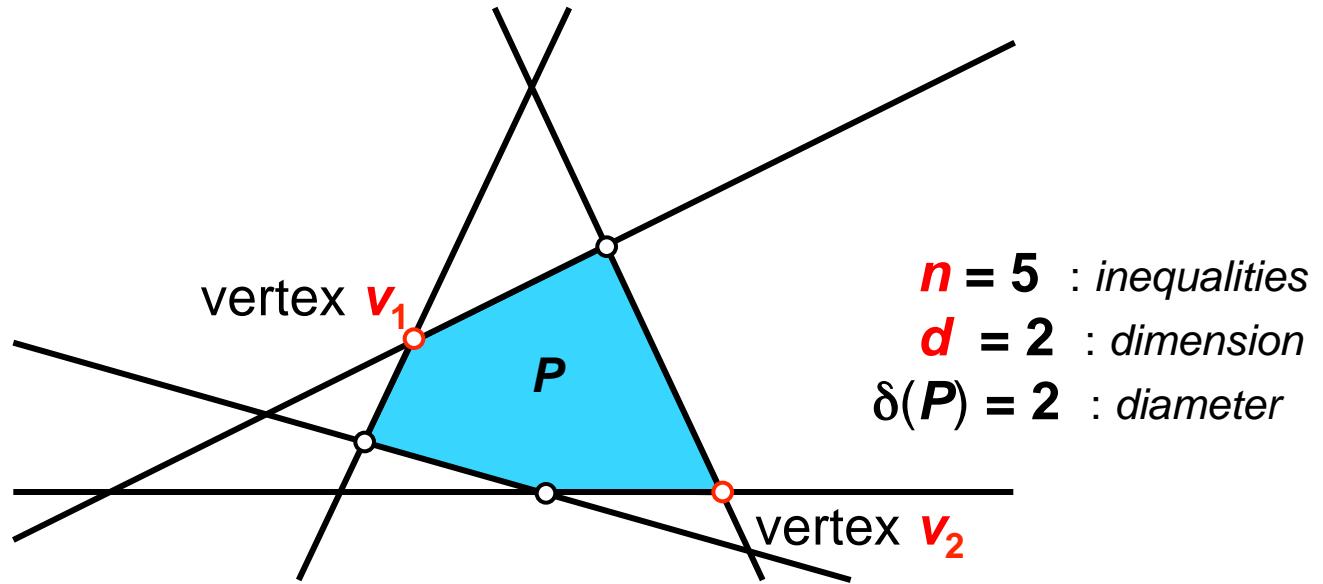
Polytope P defined by n inequalities in dimension d

Polytopes & Arrangements : Diameter & Curvature



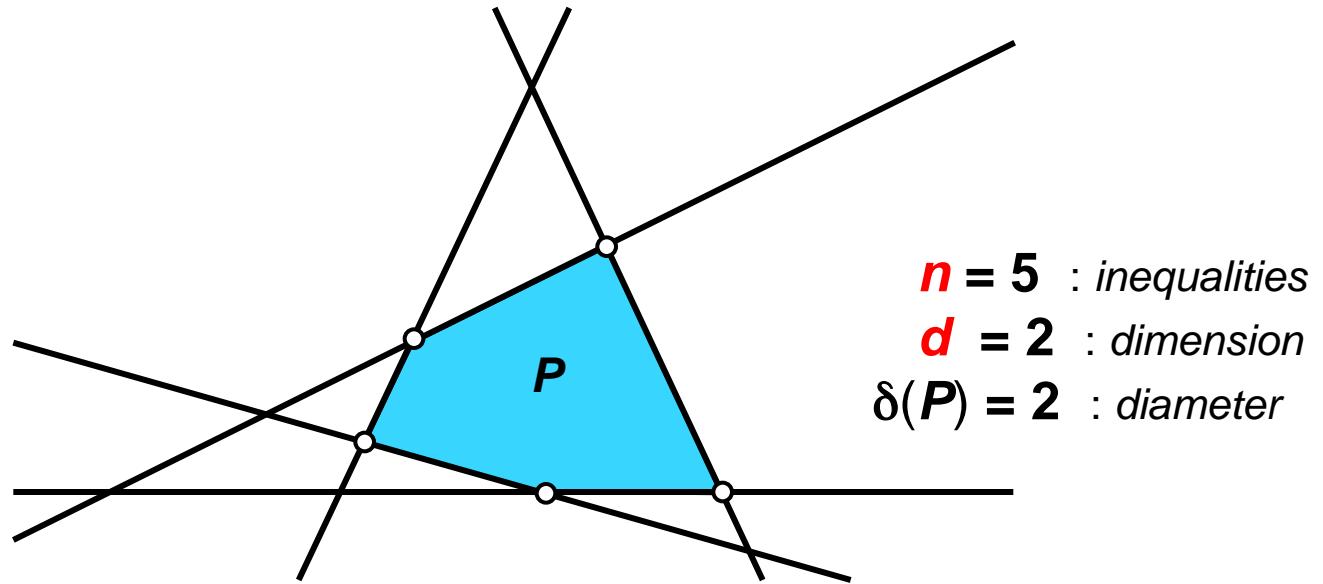
Polytope P defined by n inequalities in dimension d

Polytopes & Arrangements : Diameter & Curvature



Diameter $\delta(P)$: smallest number such that **any two vertices** (v_1, v_2) can be connected by a **path with at most $\delta(P)$ edges**

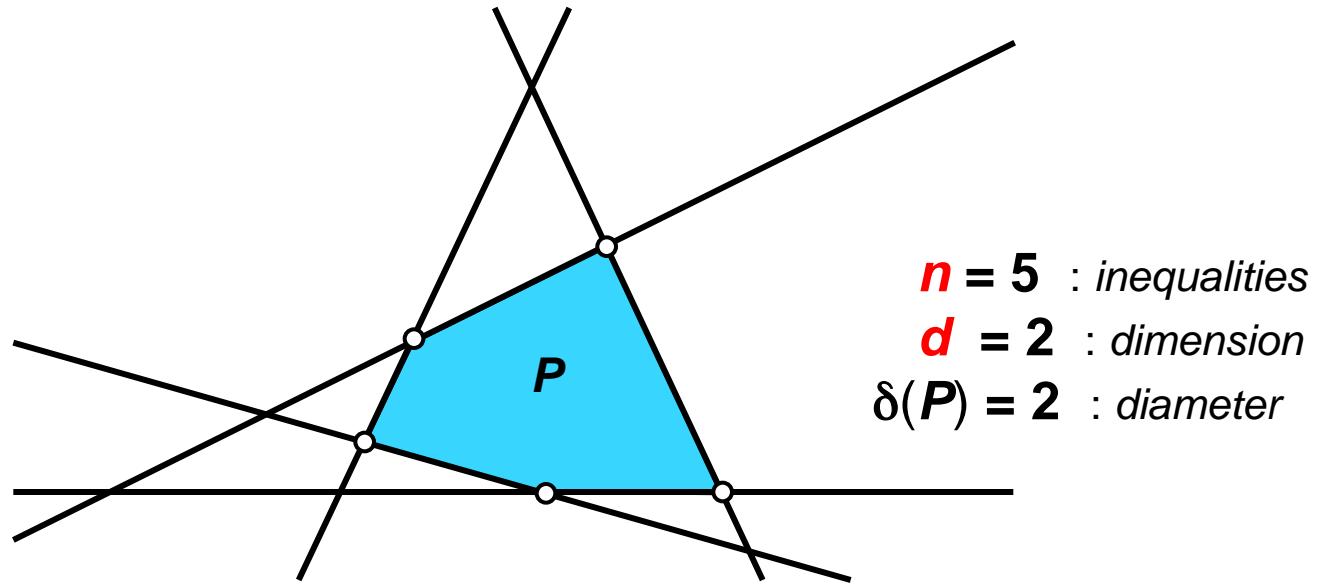
Polytopes & Arrangements : Diameter & Curvature



Diameter $\delta(P)$: smallest number such that any two vertices can be connected by a path with at most $\delta(P)$ edges

Hirsch Conjecture (1957): $\delta(P) \leq n - d$

Polytopes & Arrangements : Diameter & Curvature

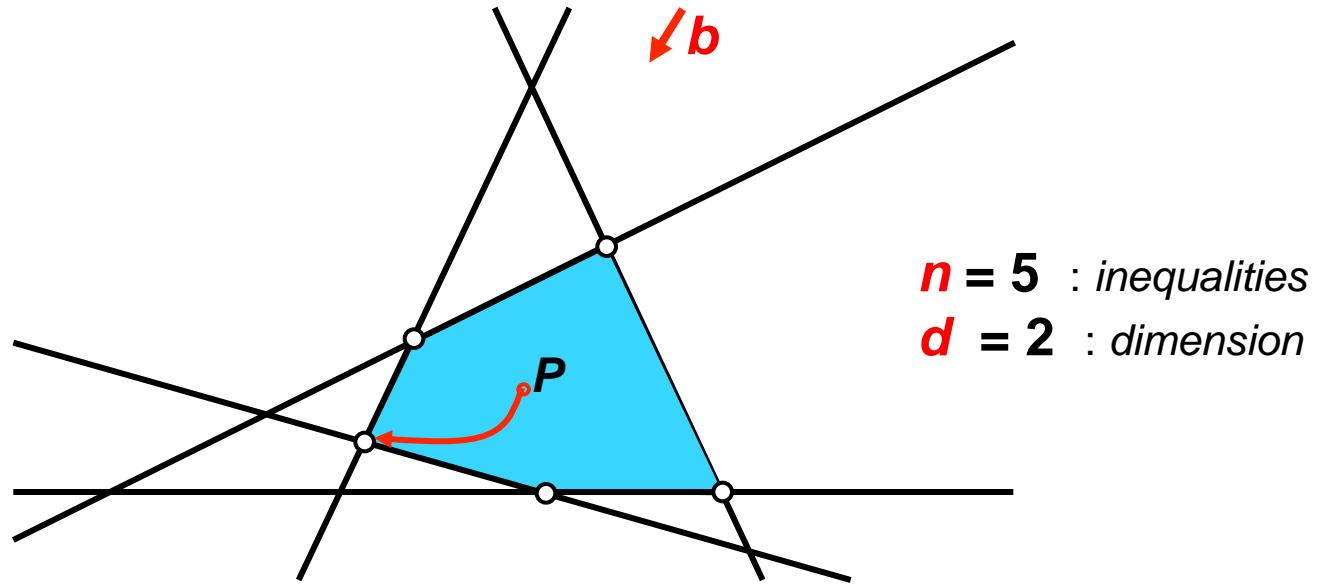


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Hirsch Conjecture (1957): $\delta(P) \leq n - d$

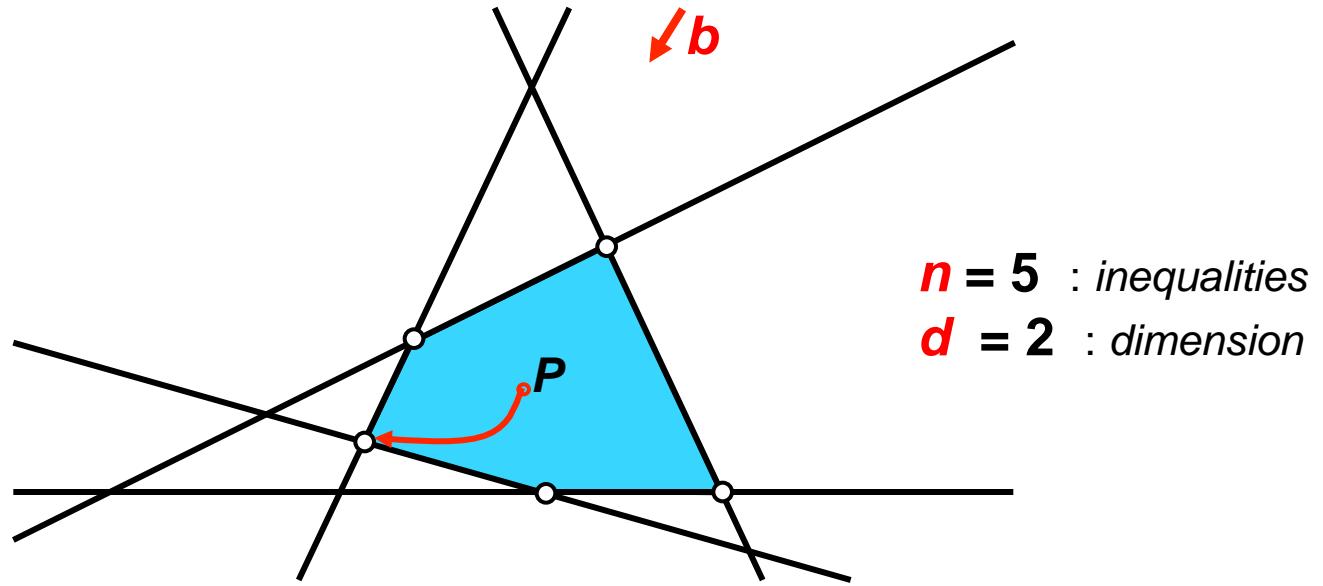
➤ **disproved** by Santos (2012) using ($d, 2d$) construction

Polytopes & Arrangements : Diameter & Curvature



$\lambda^b(P)$: total curvature of the primal central path of $\max\{b^T y : y \in P\}$

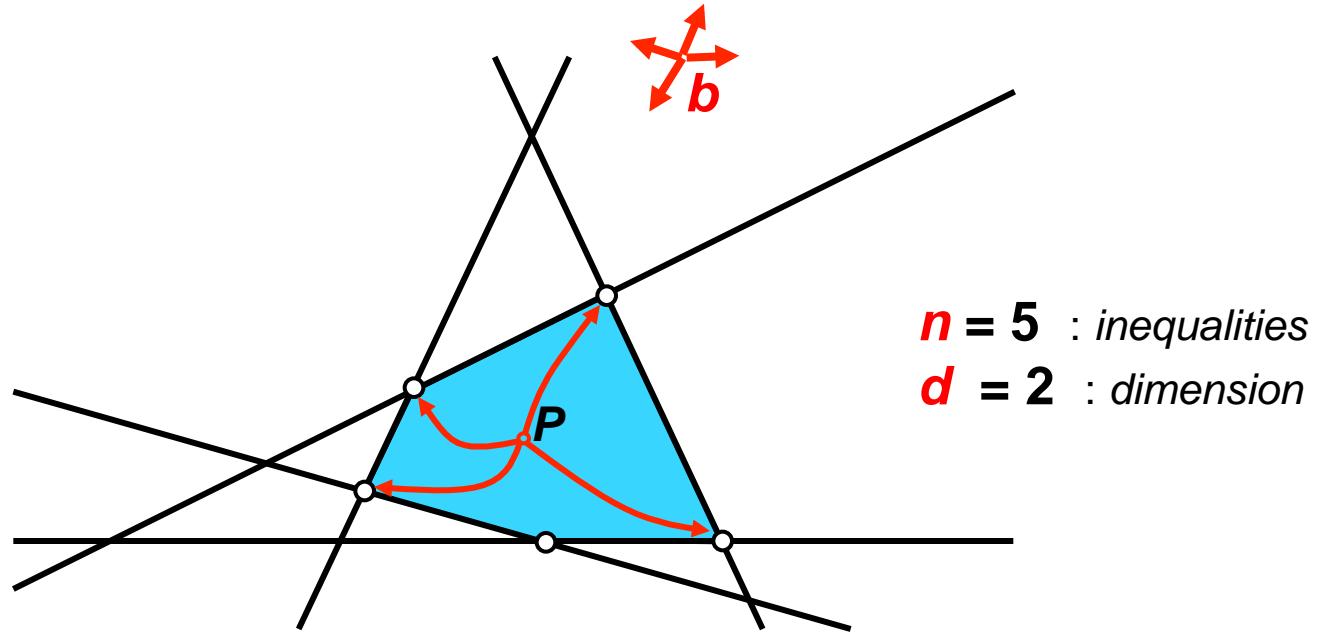
Polytopes & Arrangements : Diameter & Curvature



$\lambda^{\mathbf{b}}(\mathcal{P})$: total **curvature** of the primal **central path** of $\max\{\mathbf{b}^T \mathbf{y} : \mathbf{y} \in \mathcal{P}\}$

❖ $\lambda^{\mathbf{b}}(\mathcal{P})$: redundant inequalities count

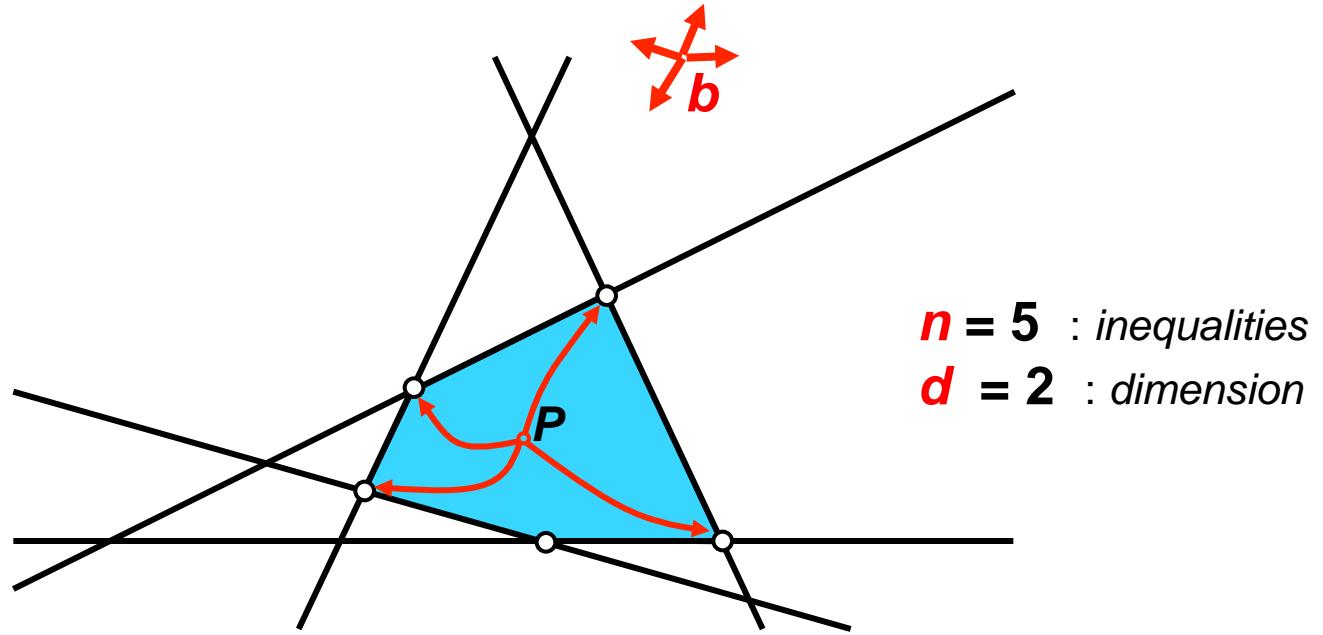
Polytopes & Arrangements : Diameter & Curvature



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$\lambda(\mathcal{P})$: largest total curvature $\lambda^{\mathbf{b}}(\mathcal{P})$ over of all possible \mathbf{b}

Polytopes & Arrangements : Diameter & Curvature

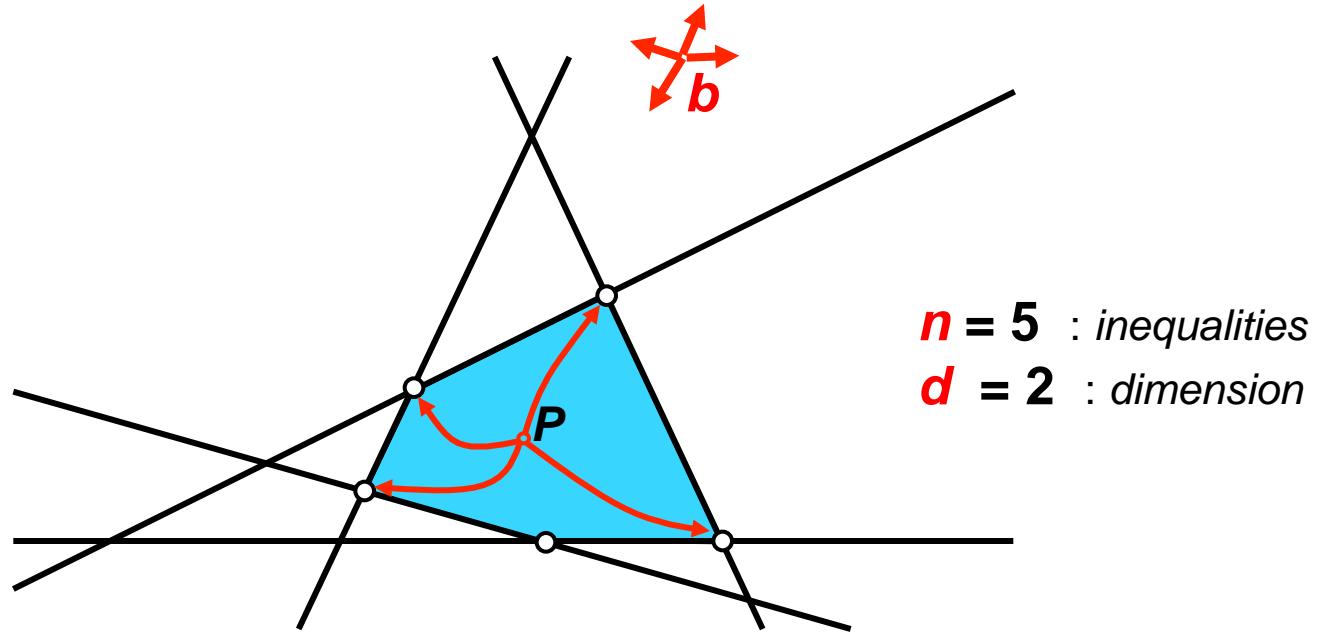


$\lambda^{\mathbf{b}}(\mathcal{P})$: total curvature of the primal central path of $\max\{\mathbf{b}^T \mathbf{y} : \mathbf{y} \in \mathcal{P}\}$

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Continuous analogue of Hirsch Conjecture: $\lambda(\mathcal{P}) = O(n)$
(D.-Terlaky-Zinchenko 2008)

Polytopes & Arrangements : Diameter & Curvature



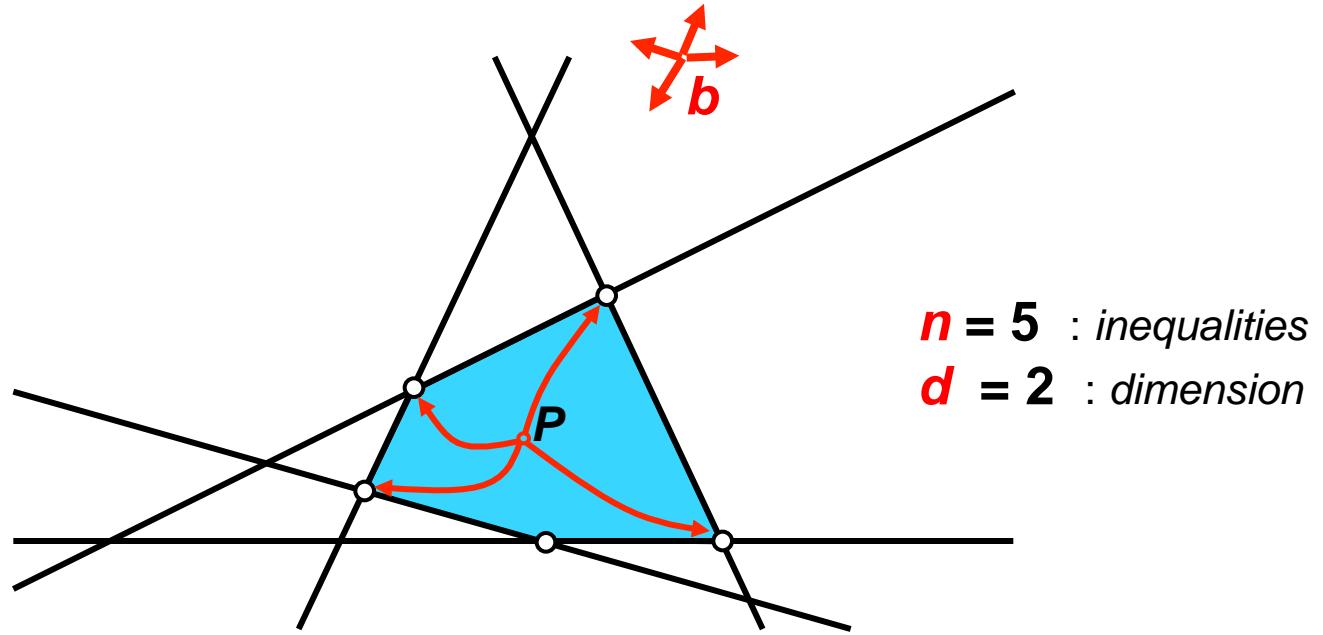
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❖ Dedieu-Shub hypothesis (2005): $\lambda(\mathcal{P}) = O(d)$

Polytopes & Arrangements : Diameter & Curvature



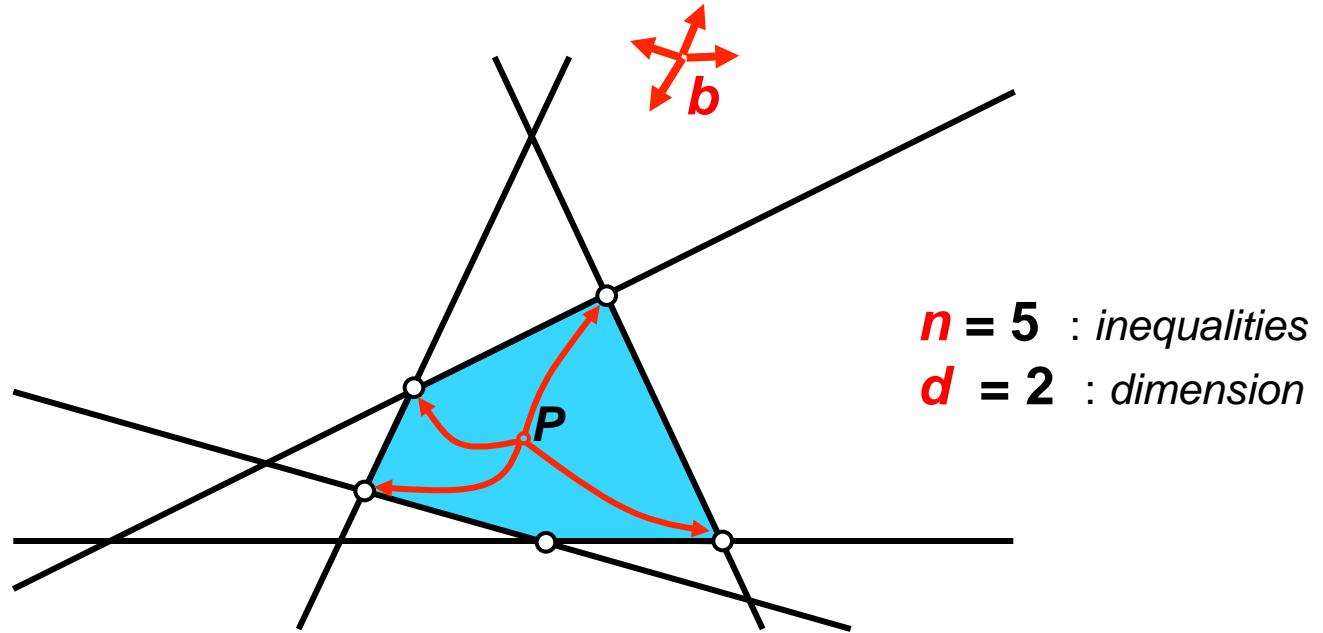
$\lambda^b(P)$: total curvature of the primal central path of $\max\{b^T y : y \in P\}$

$\lambda(P)$: largest total curvature $\lambda^b(P)$ over of all possible b

Continuous analogue of Hirsch Conjecture: $\lambda(P) = O(n)$
(D.-Terlaky-Zinchenko 2008)

- ❖ Dedieu-Shub hypothesis (2005): $\lambda(P) = O(d)$
- ❖ D.-Terlaky-Zinchenko. (2008) polytope C such that: $\lambda(C) \geq (1.5)^d$

Polytopes & Arrangements : Diameter & Curvature



$n = 5$: inequalities
 $d = 2$: dimension

$\lambda^{\mathbf{b}}(\mathbf{P})$: total curvature of the primal central path of $\max\{\mathbf{b}^T \mathbf{y} : \mathbf{y} \in \mathbf{P}\}$

$\lambda(\mathbf{P})$: largest total curvature $\lambda^{\mathbf{b}}(\mathbf{P})$ over of all possible \mathbf{b}

Continuous analogue of Hirsch Conjecture: $\lambda(\mathbf{P}) = O(n)$
(D.-Terlaky-Zinchenko 2008)

➤ **disproved** by Allamigeon et al. (2014) using $(2d, 3d)$ construction

Polytopes & Arrangements : Diameter & Curvature

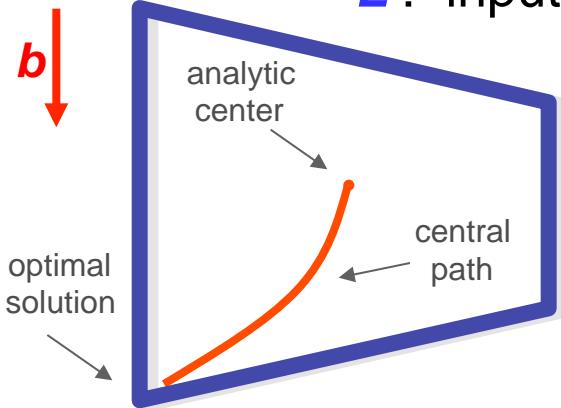
Central path following interior point methods

- start from the *analytic center*
- follow the **central path**
- converge to an *optimal solution*
- *polynomial time* algorithms for linear optimization

$O(\sqrt{nL})$: number of iterations

n : number of inequalities

L : input-data bit-length



$$\max \quad \mathbf{b}^T \mathbf{y} + \boldsymbol{\mu} \sum_i \ln(c - \mathbf{A}^T \mathbf{y})_i$$

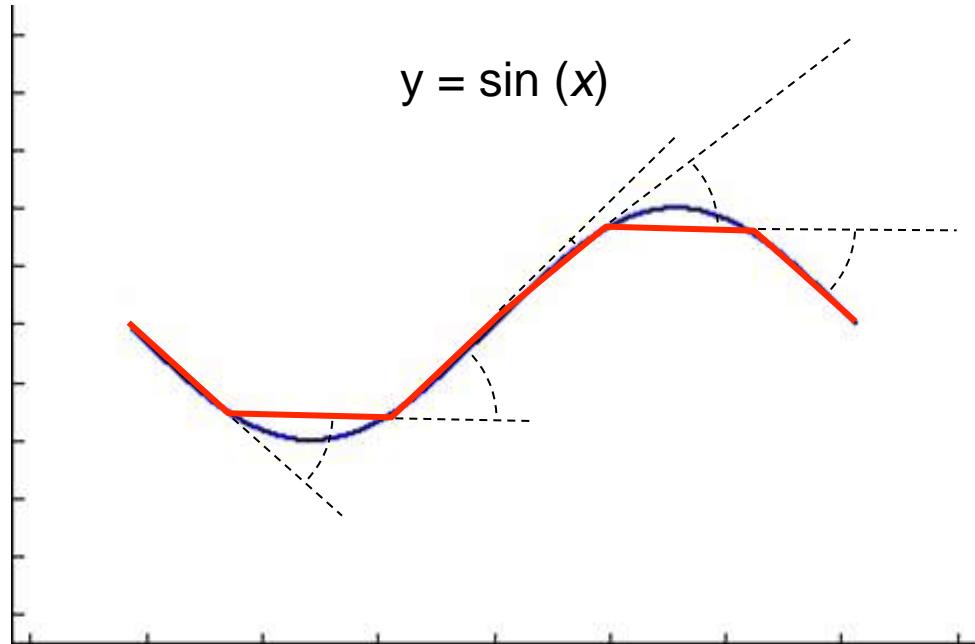
- ❖ $\boldsymbol{\mu}$: central path parameter
- ❖ $\mathbf{y} \in \mathcal{P}$: $\mathbf{A}^T \mathbf{y} \leq \mathbf{c}$

Polytopes & Arrangements : Diameter & Curvature

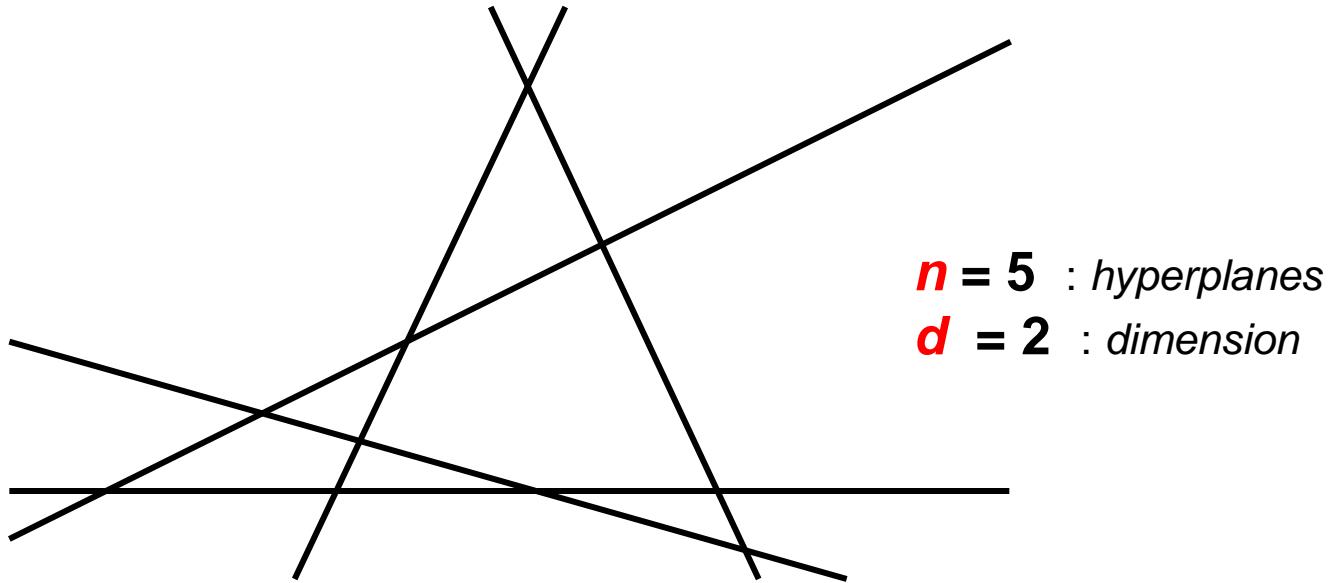
C^2 curve $\Psi : [a, b] \rightarrow \mathbb{R}^n$ parameterized by its *arc length* t (note: $\|\Psi'(t)\| = 1$)

curvature at t : $\kappa(t) = \frac{\partial^2}{\partial t^2} \Psi(t)$

total curvature: $K = \int_a^b \|\kappa(t)\| dt$

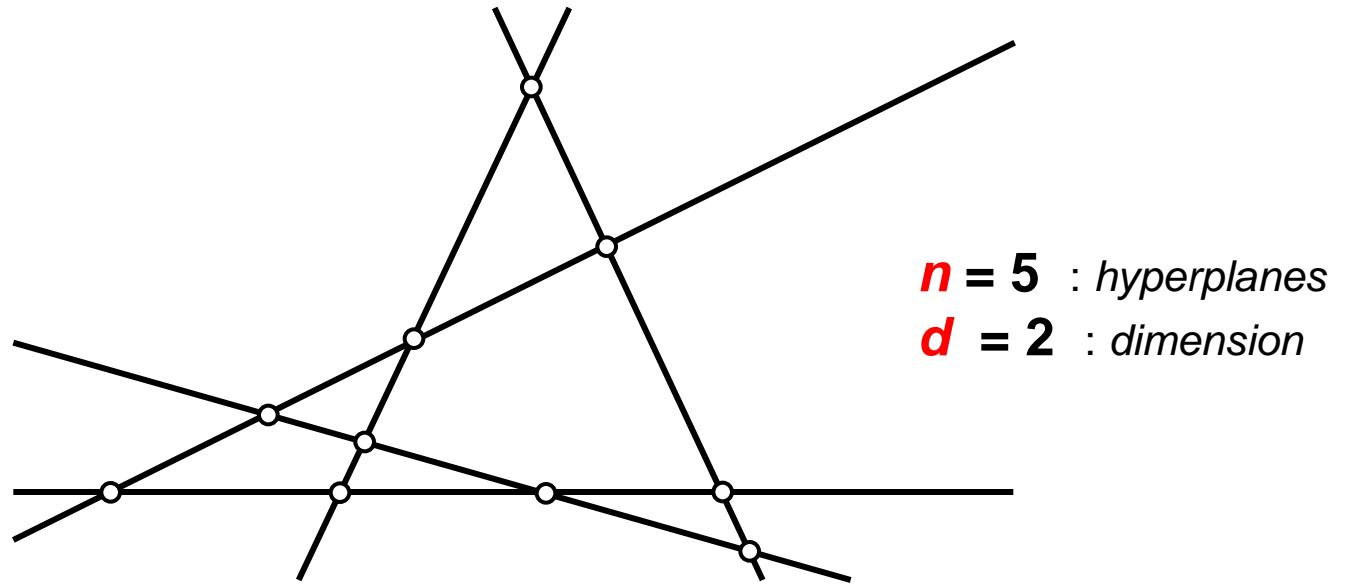


Polytopes & Arrangements : Diameter & Curvature



Arrangement A defined by **n hyperplanes** in **dimension d**

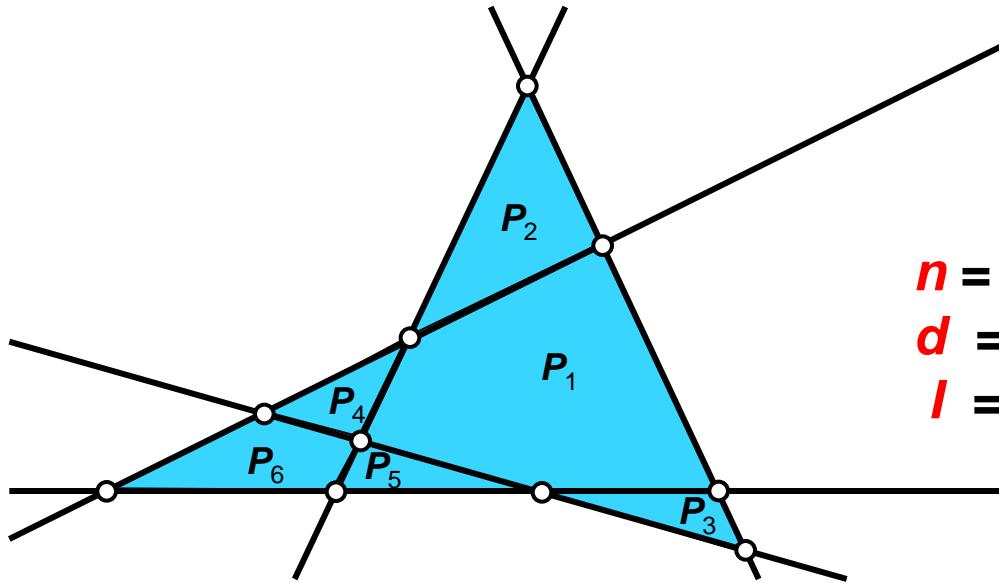
Polytopes & Arrangements : Diameter & Curvature



Simple arrangement:

$n > d$ and any d hyperplanes intersect at a unique distinct point

Polytopes & Arrangements : Diameter & Curvature



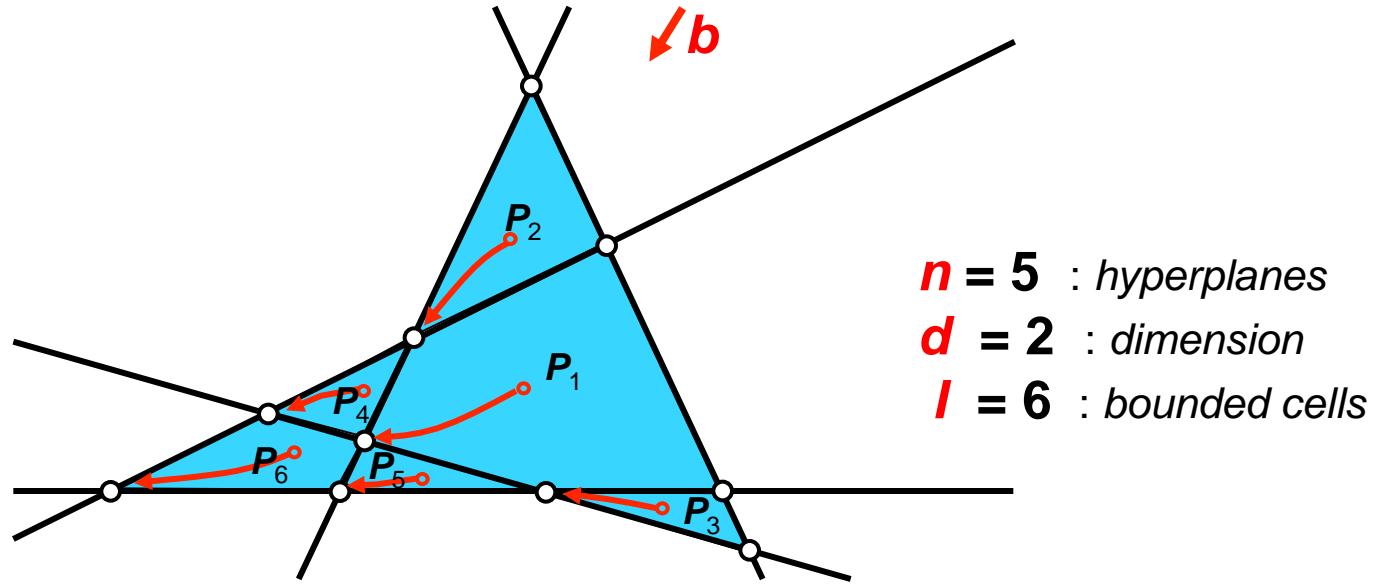
$n = 5$: hyperplanes

$d = 2$: dimension

$I = 6$: bounded cells

For a simple arrangement, the number of **bounded cells** $I = \binom{n-1}{d}$

Polytopes & Arrangements : Diameter & Curvature



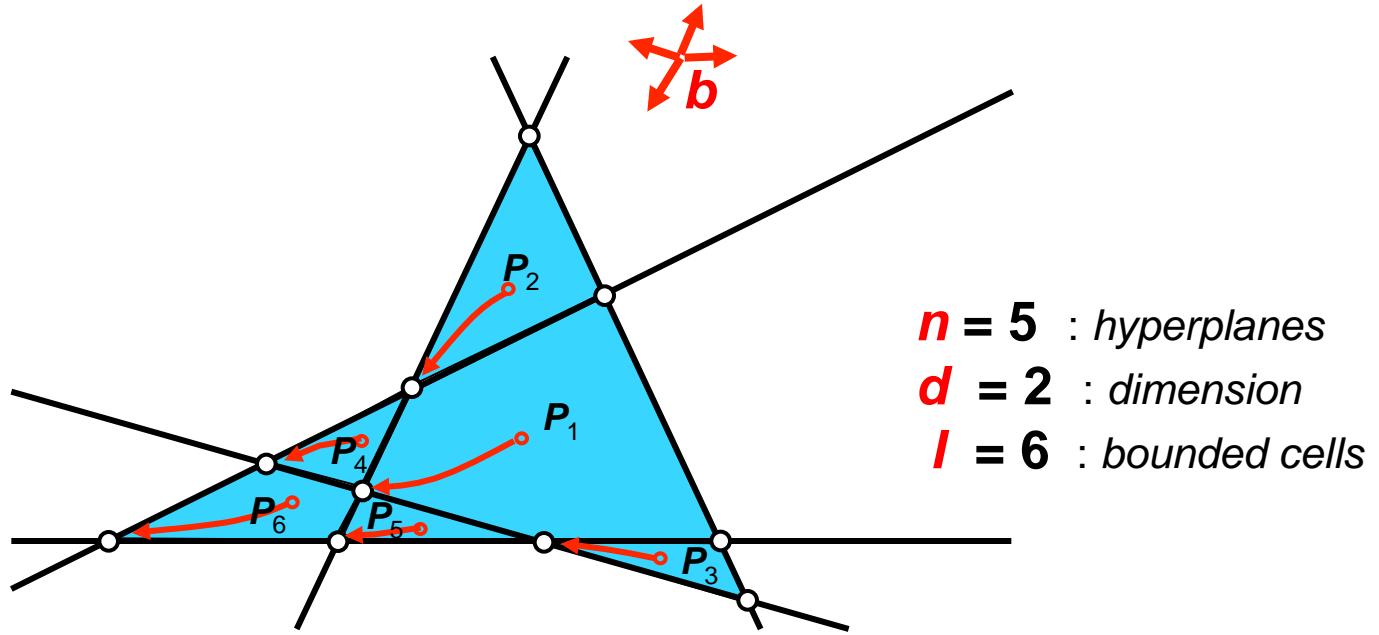
$\lambda^{\textcolor{red}{b}}(\mathbf{A})$: average value of $\lambda^{\textcolor{red}{b}}(\mathbf{P}_i)$ over the bounded cells \mathbf{P}_i of \mathbf{A} :

$$\lambda^{\textcolor{red}{b}}(\mathbf{A}) = \frac{\sum_{i=1}^{i=\mathbf{I}} \lambda^{\textcolor{red}{b}}(P_i)}{\mathbf{I}}$$

with $\mathbf{I} = \binom{\textcolor{red}{n}-1}{\textcolor{red}{d}}$

❖ $\lambda^{\textcolor{red}{b}}(\mathbf{P}_i)$: redundant inequalities count

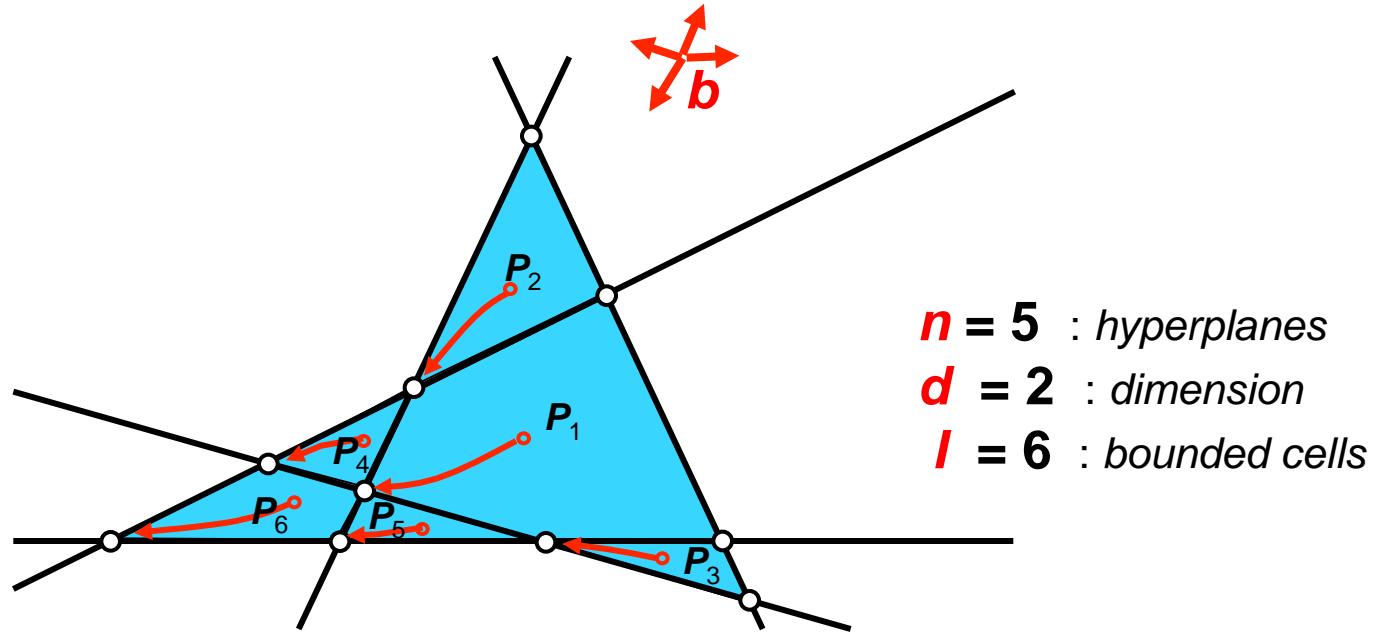
Polytopes & Arrangements : Diameter & Curvature



$\lambda^{\mathbf{b}}(\mathcal{A})$: average value of $\lambda^{\mathbf{b}}(P_i)$ over the bounded cells P_i of \mathcal{A} :

$\lambda(\mathcal{A})$: largest value of $\lambda^{\mathbf{b}}(\mathcal{A})$ over all possible \mathbf{b}

Polytopes & Arrangements : Diameter & Curvature



$\lambda^{\mathbf{b}}(\mathcal{A})$: average value of $\lambda^{\mathbf{b}}(\mathcal{P}_i)$ over the bounded cells \mathcal{P}_i of \mathcal{A} :

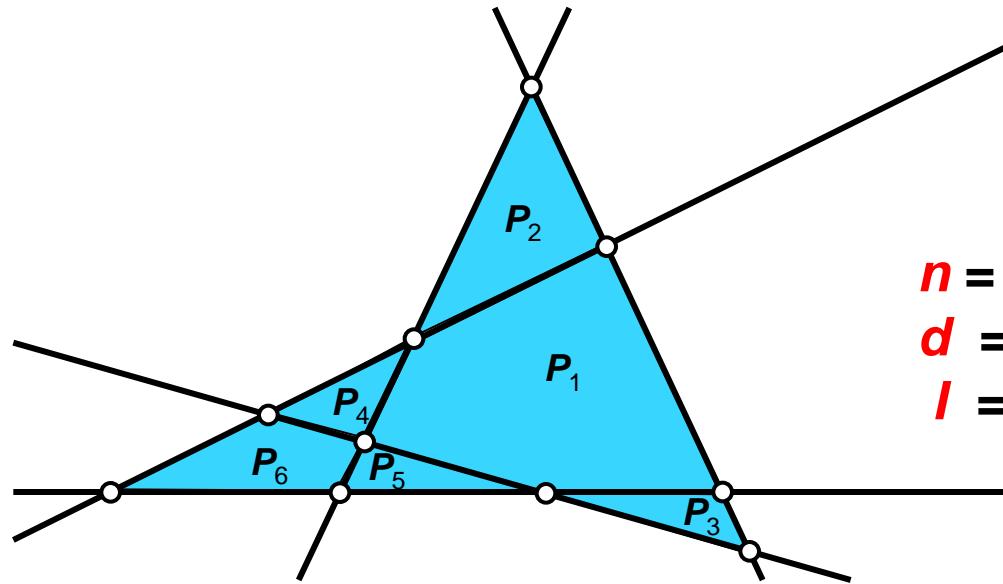
$\lambda(\mathcal{A})$: largest value of $\lambda^{\mathbf{b}}(\mathcal{A})$ over all possible \mathbf{b}

Dedieu-Malajovich-Shub (2005): $\lambda(\mathcal{A}) \leq 2\pi d$

Strengthening: De Loera-Sturmels-Vinzant (2012)

❖ \mathcal{A} : simple arrangement

Polytopes & Arrangements : Diameter & Curvature

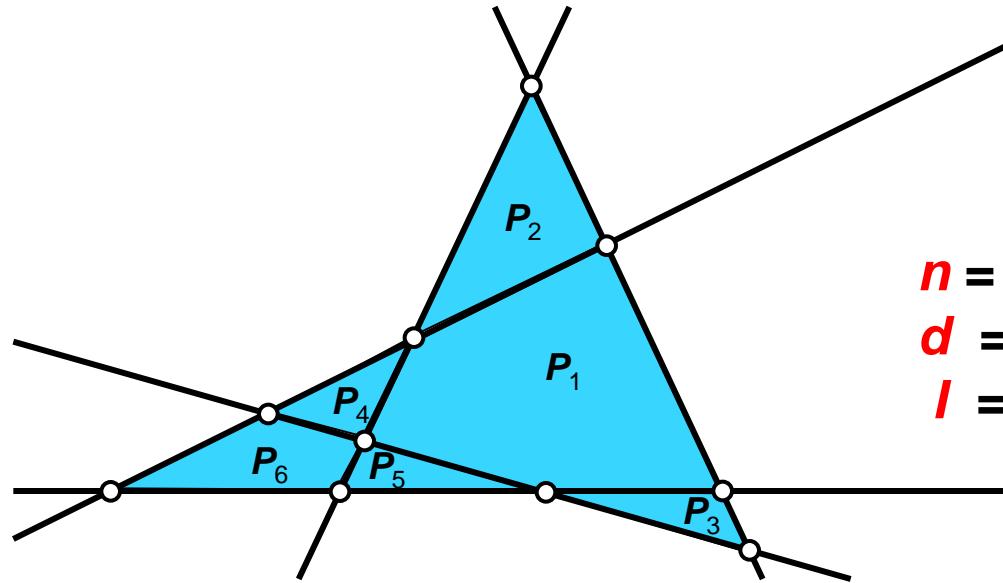


$n = 5$: hyperplanes
 $d = 2$: dimension
 $I = 6$: bounded cells

$\delta(\mathbf{A})$: average diameter of a bounded cell of \mathbf{A} :

❖ \mathbf{A} : simple arrangement

Polytopes & Arrangements : Diameter & Curvature



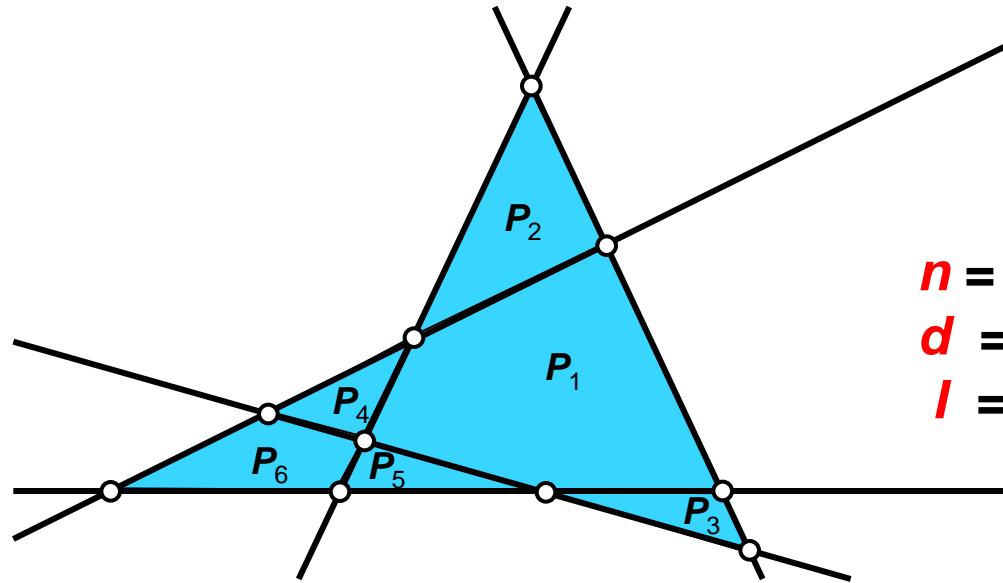
$n = 5$: hyperplanes
 $d = 2$: dimension
 $I = 6$: bounded cells

$\delta(\mathcal{A})$: average diameter of a bounded cell of \mathcal{A} :

$$\delta(\mathcal{A}) = \frac{\sum_{i=1}^{i=I} \delta(P_i)}{I} \quad \text{with } I = \binom{n-1}{d}$$

❖ $\delta(\mathcal{A})$: average diameter \neq diameter of \mathcal{A}
ex: $\delta(\mathcal{A}) = 1.333\dots$

Polytopes & Arrangements : Diameter & Curvature



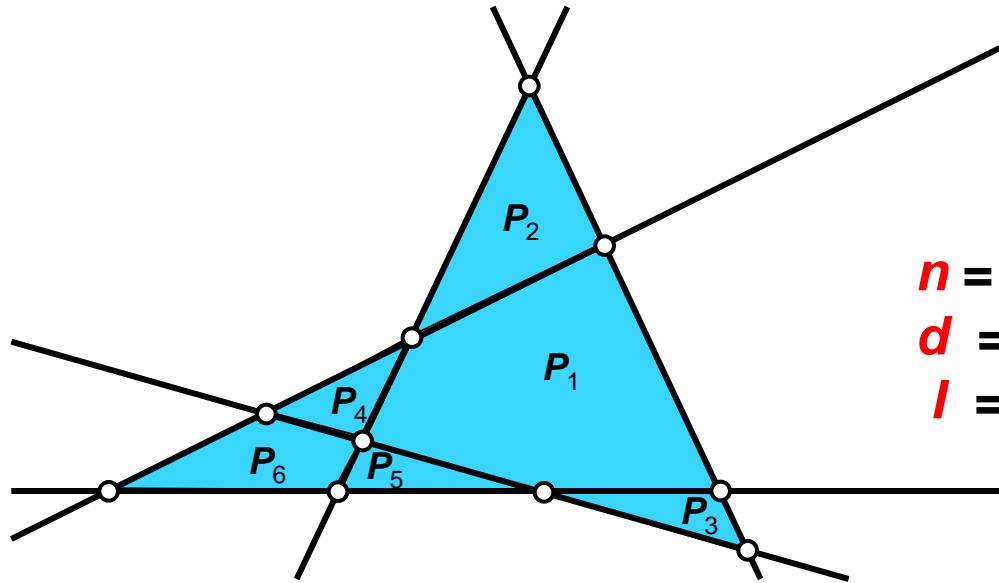
$n = 5$: hyperplanes
 $d = 2$: dimension
 $I = 6$: bounded cells

$\delta(\mathbf{A})$: average diameter of a bounded cell of \mathbf{A} :

$$\delta(\mathbf{A}) = \frac{\sum_{i=1}^{I} \delta(P_i)}{I} \quad \text{with } I = \binom{n-1}{d}$$

❖ $\delta(P_i)$: only active inequalities count

Polytopes & Arrangements : Diameter & Curvature



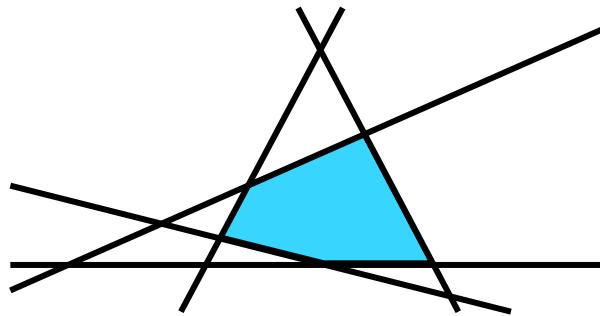
$n = 5$: hyperplanes
 $d = 2$: dimension
 $I = 6$: bounded cells

$\delta(\mathbf{A})$: average diameter of a bounded cell of \mathbf{A} :

Conjecture : $\delta(\mathbf{A}) \leq d$
(D.-Terlaky-Zinchenko 2008)

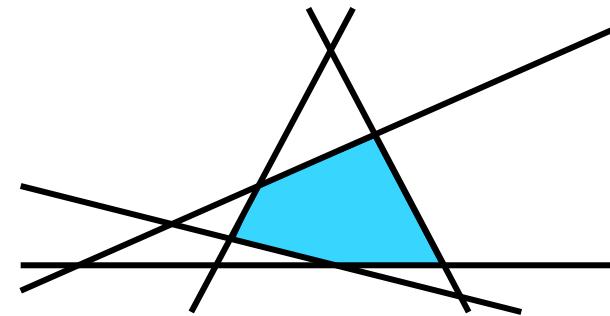
(discrete analogue of Dedieu-Malajovich-Shub result)

Polytopes & Arrangements : Diameter & Curvature



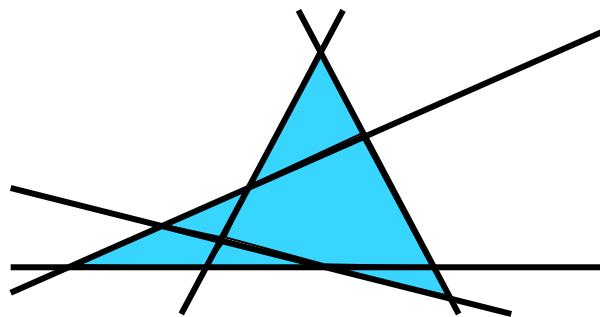
$$\delta(P) \leq n - d$$

Hirsch conjecture (1957)
[Santos \(2012\)](#)



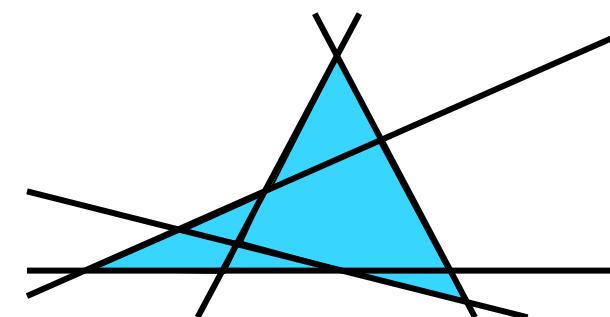
$$\lambda(P) \leq 2\pi n$$

conjecture D.-T.-Z. (2008)
[Allamigeon et al \(2014\)](#)



$$\delta(A) \leq d$$

conjecture D.-T.-Z. (2008)



$$\lambda(A) \leq 2\pi d$$

Dedieu-Malajovich-Shub
(2005)

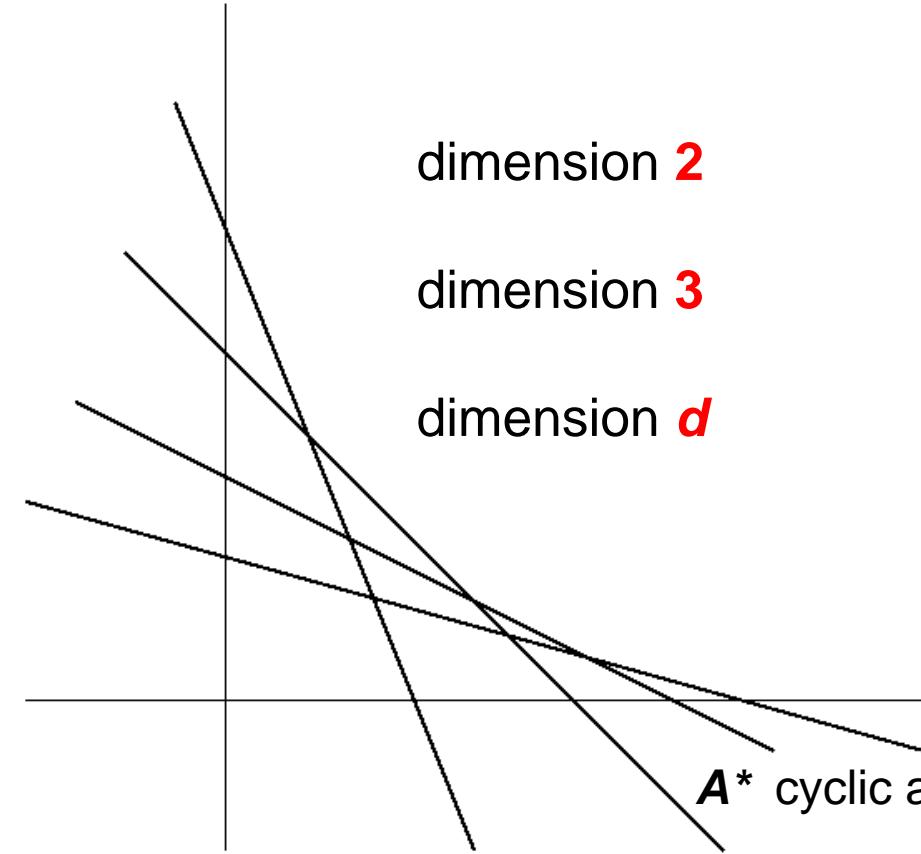
Polytopes & Arrangements : Diameter & Curvature

Links, low dimensions & substantiation

- Hirsch bound $\delta(P) \leq n - d$ implies that $\delta(A) \leq d \frac{n+1}{n-1}$
 - Hirsch conjecture *holds* for $d = 2$: $\delta(A) \leq 2 \frac{n+1}{n-1}$
 - Hirsch conjecture *holds* for $d = 3$: $\delta(A) \leq 3 \frac{n+1}{n-1}$
- Barnette / Larman (1970/74) $\delta(P) \leq n 2^d / 12$
- Kalai-Kleitman (1992) $\delta(P) \leq n^{\log d + 2}$
- Todd (2014) $\delta(P) \leq (n - d)^{\log d}$
- Sukegawa-Kitahara (2014) $\delta(P) \leq (n - d)^{\log(d-1)}$

Polytopes & Arrangements : Diameter & Curvature

Links, low dimensions & substantiation



A^* cyclic arrangement (mainly consists of cubical cells)

$$\delta(A) = \frac{2 \lceil n/2 \rceil}{(n-1)(n-2)}$$

$\delta(A)$ asymptotically equal to 3

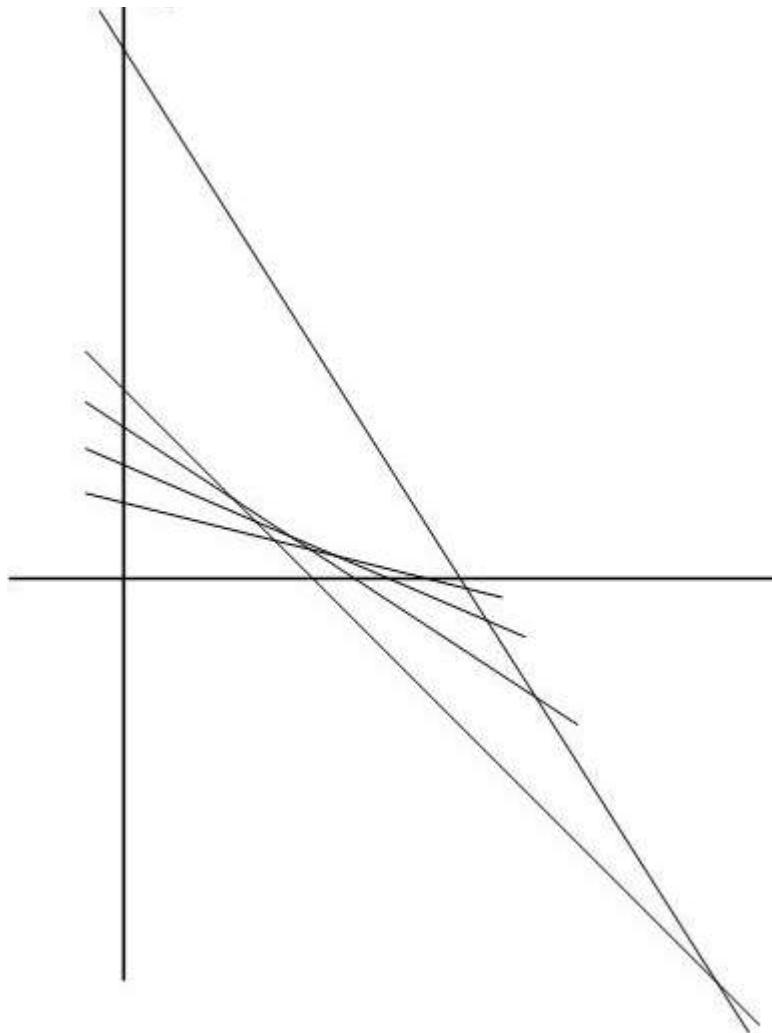
$$d \binom{n-d}{d} / \binom{n-1}{d} \leq \delta(A) \leq d ?$$

D.-Xie (2009)

- ❖ Haimovich's probabilistic analysis of shadow-vertex simplex method - Borgwardt (1987)
- ❖ Forge – Ramírez Alfonsín (2001) counting k -face cells of A^*

Polytopes & Arrangements : Diameter & Curvature

Links, low dimensions & substantiation



dimension 2

$$\delta(A) = 2 - \frac{2[n/2]}{(n-1)(n-2)}$$

maximize(diameter) amounts to
minimize(#external edge + #odd-gons)

Polytopes & Arrangements : Diameter & Curvature

Links, low dimensions & substantiation

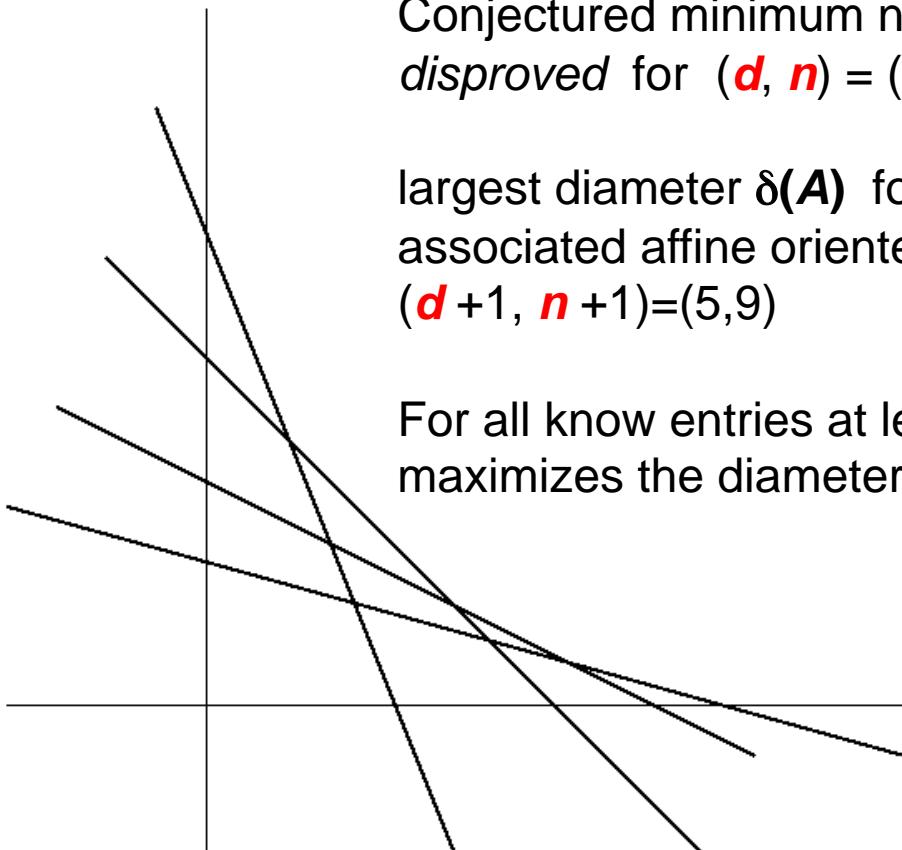
No single extension of the cyclic arrangement A^* achieve the largest diameter for $(d, n) = (3,8)$ and $(4,8)$ (no covering one for $(d, n) = (3,7)$)

Conjectured minimum number of external facets for an arrangement *disproved* for $(d, n) = (3,8)$

largest diameter $\delta(A)$ for $n \leq 8$ and $d \leq 4$ computed by examining all associated affine oriented matroids, e.g. 9,276,595 affine OM for $(d+1, n+1) = (5,9)$

For all known entries at least one arrangement *simultaneously* maximizes the diameter and minimizes the number of external facets

D.-Miyata-Moriyama-Xie (2012)



Polytopes & Arrangements : Diameter & Curvature

bounds in low dimensions up till 2011

$\delta(d, n)$		$n - d$				
		4	5	6	7	8
d	4	4	5	5	[6,7]	7+
	5	4	5	6	[7,9]	7+
	6	4	5	[6,7]	[7,9]	8+
	7	4	5	[6,7]	[7,10]	8+

$$\delta(4,10) = 5, \quad \delta(5,11) = 6 \quad \text{Goodey (1972)}$$

Polytopes & Arrangements : Diameter & Curvature

recent progresses

$\delta(d, n)$		$n - d$				
		4	5	6	7	8
d	4	4	5	5	6	7+
	5	4	5	6	[7,8]	7+
	6	4	5	6	[7,9]	8+
	7	4	5	6	[7,10]	8+

$$\delta(4,11) = \delta(6,12) = 6 \quad \text{Bremner-Schewe (2011)}$$

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recent progresses

$\delta(d, n)$		$n - d$				
		4	5	6	7	8
d	4	4	5	5	6	7
	5	4	5	6	7	[7,9]
	6	4	5	6	[7,8]	[8,11]
	7	4	5	6	[7,9]	[8,12]

$$\delta(4,12) = \delta(5,12) = 7 \quad \text{Bremner-D.-Hua-Schewe (2013)}$$

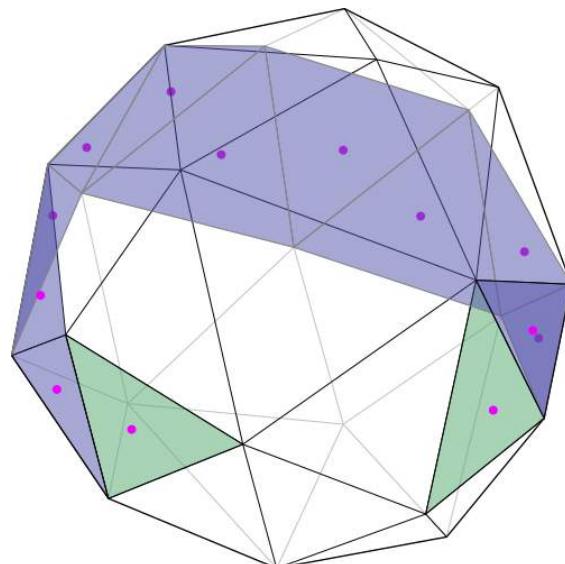
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main idea

Characterize all **combinatorial types** of paths of length **k**

Find necessary conditions for a (chirotope of a) polytope to admit an **embedding** of a **k -path** on its **boundary** (without shortcuts)

If **no** such (chirotope of a) polytope exists: $\delta(\mathbf{d}, \mathbf{n}) \neq \mathbf{k}$



Polytopes & Arrangements : Diameter & Curvature

Links, low dimensions & substantiation

- Hirsch conjecture is *tight*

- for $n > d \geq 7$ $\exists P$ such that $\delta(P) \geq n - d$

Holt-Klee (1998)

Fritzsche-Holt (1999)

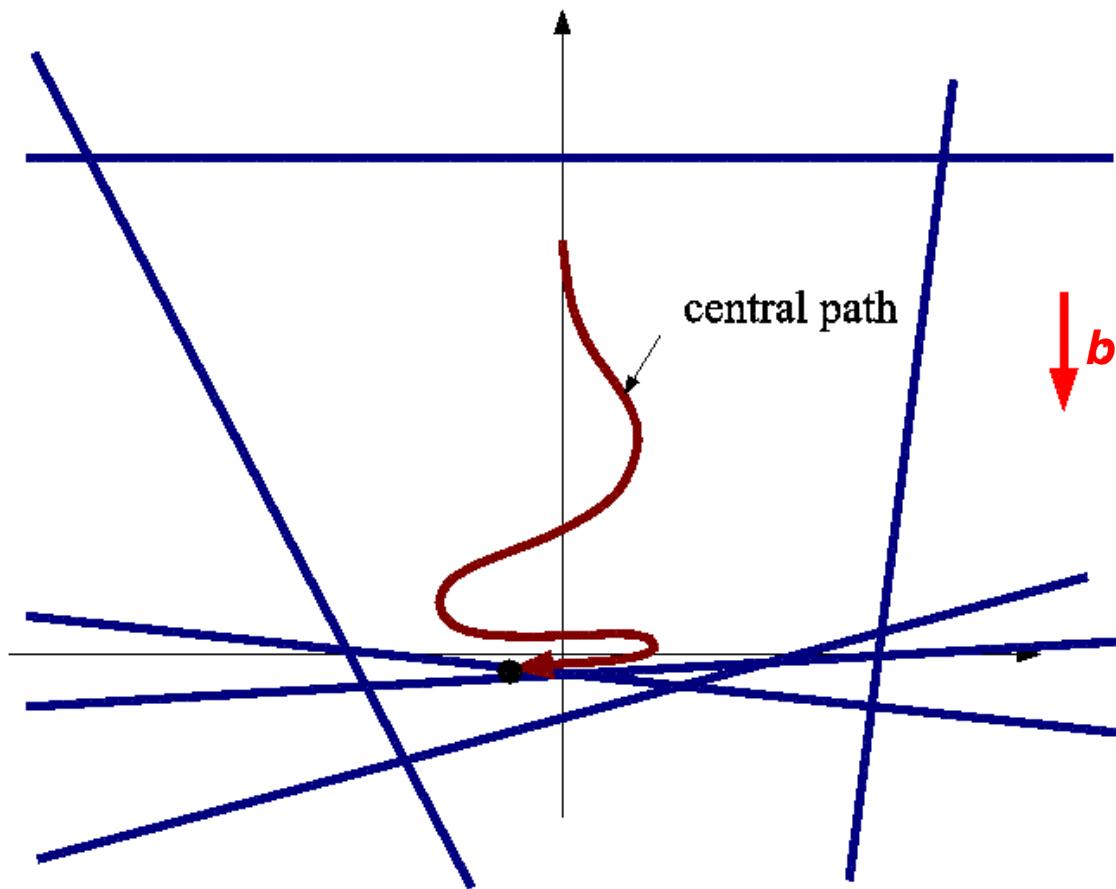
Holt (2004)

- continuous analogue of tightness:

- for $n \geq d \geq 2$ $\exists P$ such that $\liminf_{n \rightarrow \infty} \frac{\lambda^b(P)}{n} \geq \pi$

D.-T.-Z. (2008)

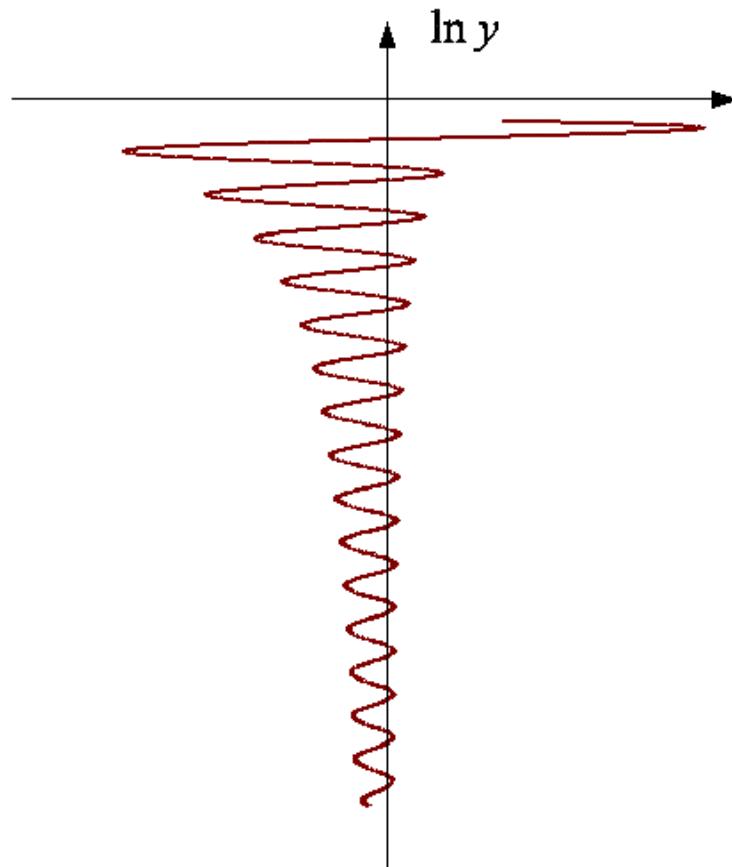
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$$P \text{ such that } \liminf_{n \rightarrow \infty} \frac{\lambda^b(P)}{n} \geq \pi$$

❖ slope: geometric decrease

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Links, low dimensions & substantiation

- Hirsch conjecture is *tight*
 - for $n > d \geq 7$ $\exists P$ such that $\delta(P) \geq n - d$
- continuous analogue of *tightness*:
 - for $n \geq d \geq 2$ $\exists P$ such that $\lambda(P) = \Omega(n)$ D.-T.-Z. (2008)
- Hirsch special case $n = 2d$ (for all d) is *equivalent* to the general case
(d -step conjecture) Klee-Walkup (1967)
- continuous analogue of *d-step equivalence*:
 - $\lambda(P) = O(n) \iff \lambda(P) = O(d)$ for $n = 2d$ D.-T.-Z. (2008)

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Proof of a continuous analogue of Klee-Walkup result (d -step conjecture)

Want: $\{\lambda(P) = O(n), \forall n\} \Leftrightarrow \{\lambda(P) = O(d) \text{ for } n = 2d, \forall d\}$

Clearly $\{\lambda(P) \leq Kn, \forall n\} \Rightarrow \{\lambda(P) \leq Kn \text{ for } n = 2d, \forall d\}$

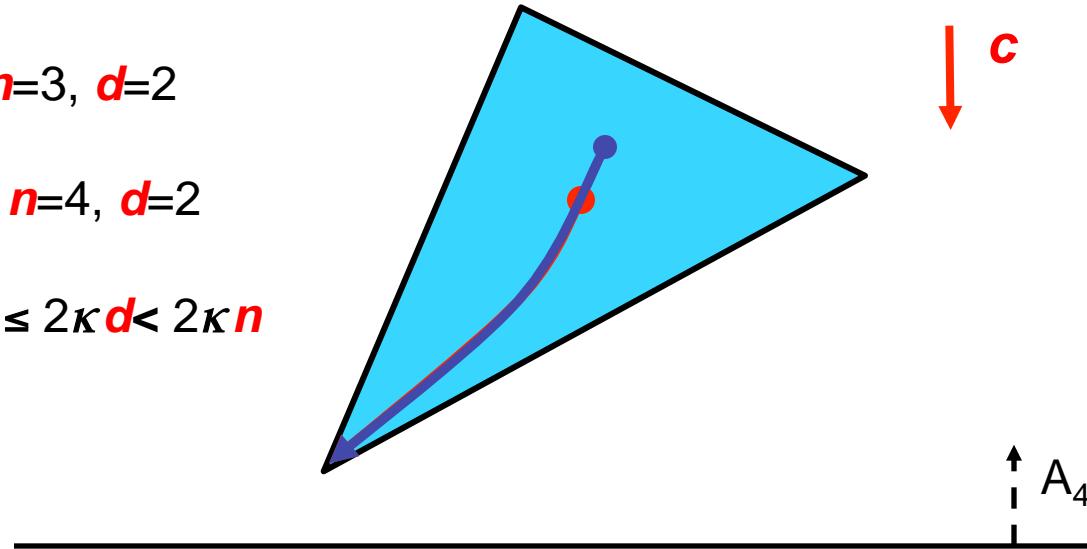
Need $\{\lambda(P) \leq Kn, \forall n\} \Leftarrow \{\lambda(P) \leq \kappa n \text{ for } n = 2d, \forall d\}$

Case (i) $d < n < 2d$

e.g., P with $n=3, d=2$

$P \rightarrow \tilde{P}$ with $n=4, d=2$

$$\lambda^c(P) \leq \lambda^c(\tilde{P}) \leq 2\kappa d < 2\kappa n$$



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Need $\{\lambda(P) \leq Kn, \forall n\} \Leftrightarrow \{\lambda(P) \leq \kappa n \text{ for } n = 2d, \forall d\}$

Case (ii) $2d < n$

e.g., P with $n=5, d=2$

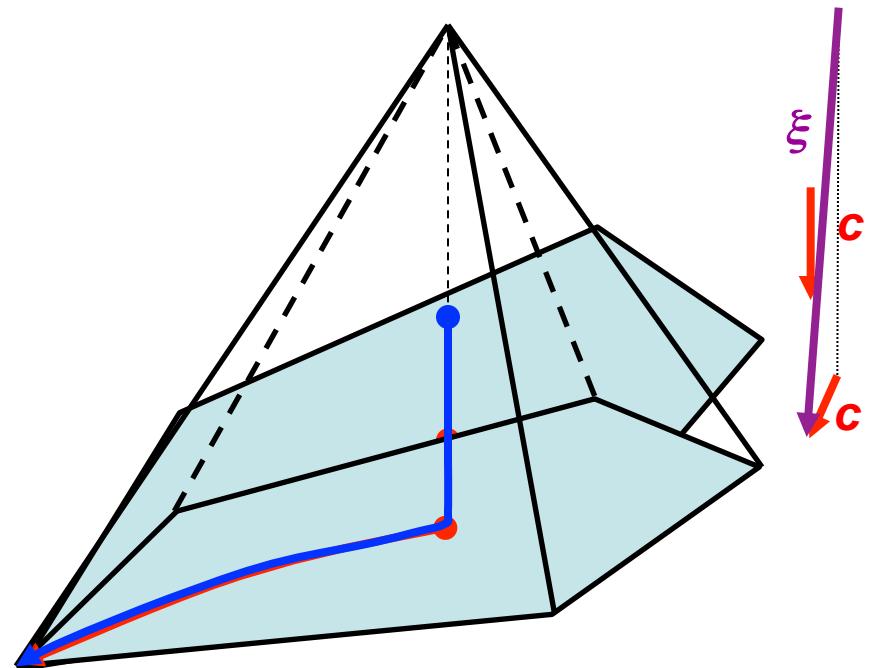
$P \rightarrow \tilde{P}$ with $n=6, d=3$

$$\lambda^c(P) \leq \lambda^\xi(\tilde{P}) \leq 2\kappa(d+1)$$

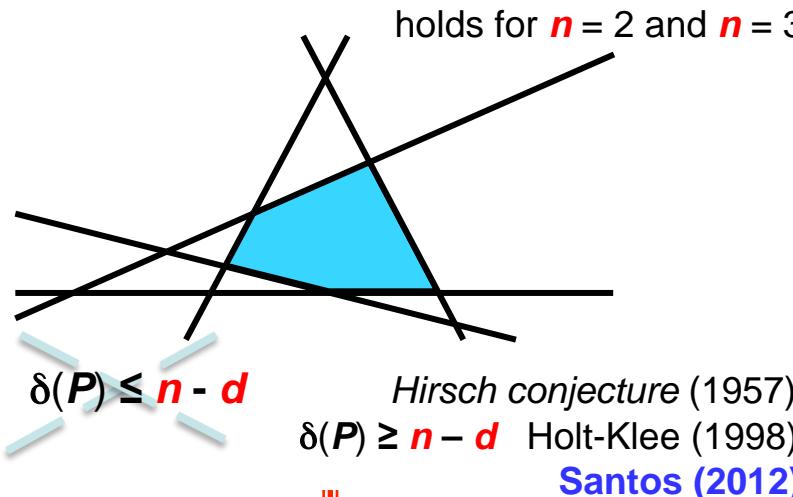
...

$$< 2\kappa(n-d)$$

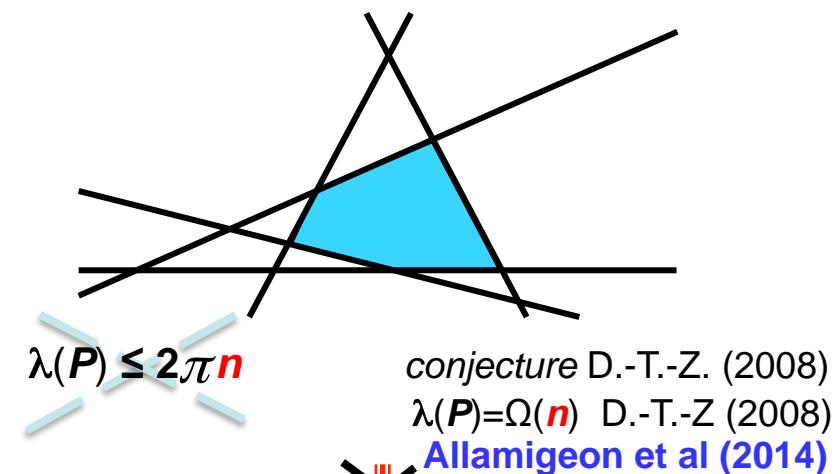
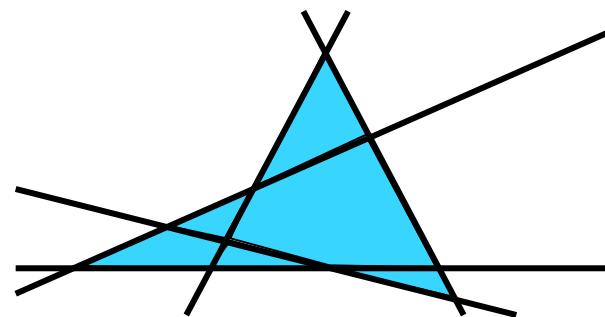
$$< 2\kappa n$$



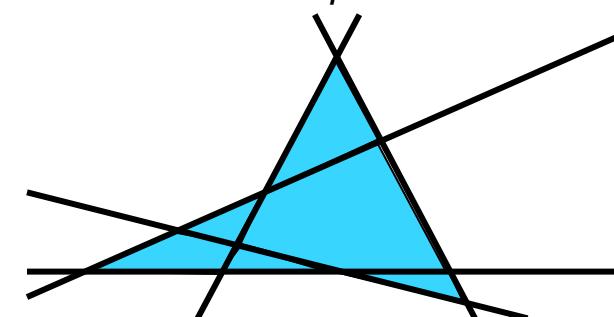
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holds for $n = 2$ and $n = 3$



redundant inequalities counts for $\lambda(P)$



$$\begin{aligned} \delta(P) \leq n - d &\Leftrightarrow \delta(P) \leq d && \text{for } n = 2d \\ \lambda(P) = O(n) &\Leftrightarrow \lambda(P) = O(d) && \text{for } n = 2d \end{aligned}$$

Klee-Walkup (1967)
D.-T.-Z. (2008)

similar results with Sonnevend curvature

Terlaky-Mut (2014)

Polytopes & Arrangements : Diameter & Curvature

Diameter (of a polytope) :

lower bound for the number of iterations
for the **simplex method** (*pivoting methods*)

lower bound : $(1 + \varepsilon)(n - d)$ **upper bound**: $(n - d)^{\log d}$

Curvature (of the central path associated to a polytope) :

large curvature indicates large number of iteration
for (*central path following*) **interior point methods**

lower bound : $2^d / d$ ($n=3d/2$) **upper bound**: $2\pi d \binom{n-1}{d}$

Allamigeon-Benchimol-Gaubert-Joswig (2014) **exponential** lower bound
for $\lambda(P)$ contrasts with the belief that a **polynomial** upper bound for $\delta(P)$
might exist ($\delta(P) \leq d(n - d) / 2$)

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✓ *thank you*

boldog születésnapot Károly!

