# On polyhedra induced by point sets and on their triangulations.

Andras Bezdek

Auburn University, Auburn, AL

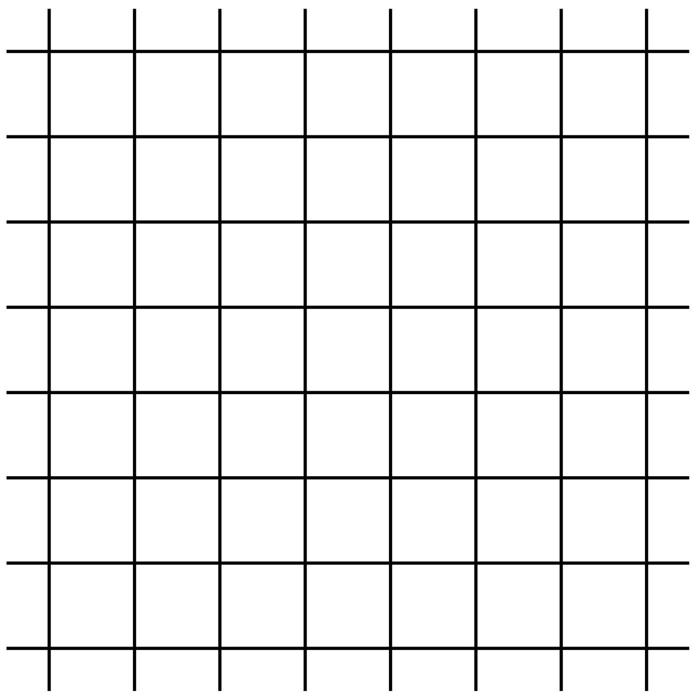
Happy 60th birthday Egon and Karoly!

# On a variant of the 'tic - tac - toe'

First player: $\bigcirc$

Second player: $\times$

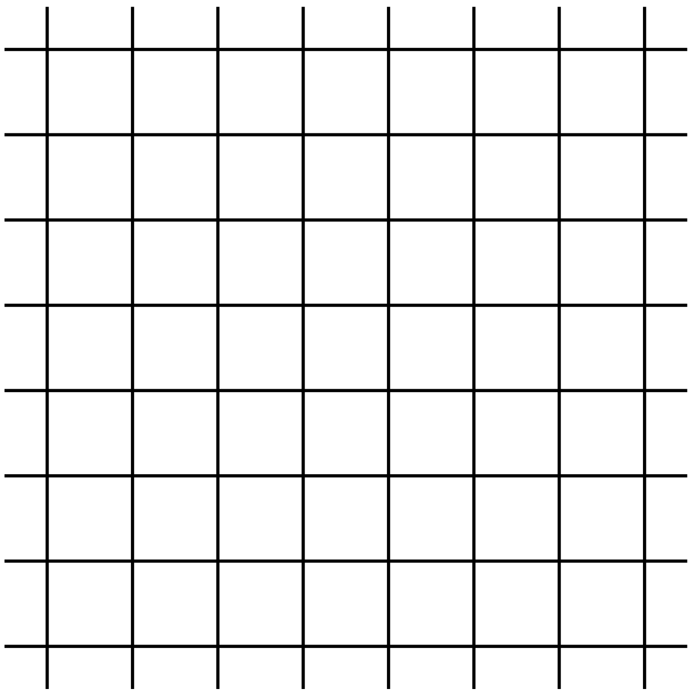
$5 - OX game$



The first player who gets 5 consecutive signs in a row (column or in a diagonal resp.) winns.

Can the second player prevent the first player from winning if 5 is replaced by a larger specific n?

$$5 - OX game$$



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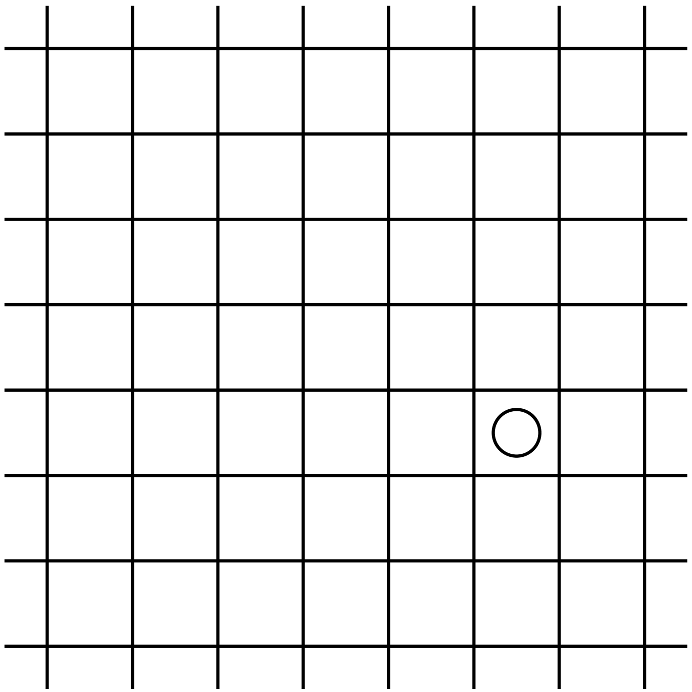

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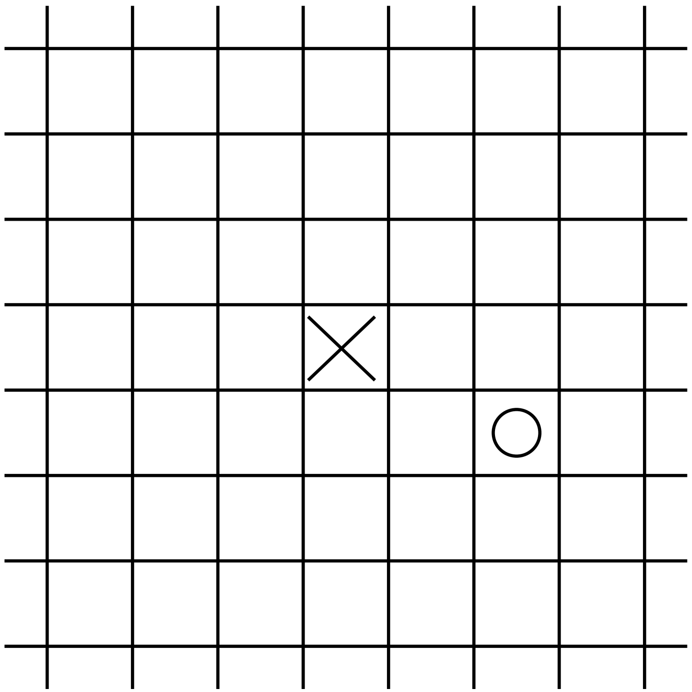

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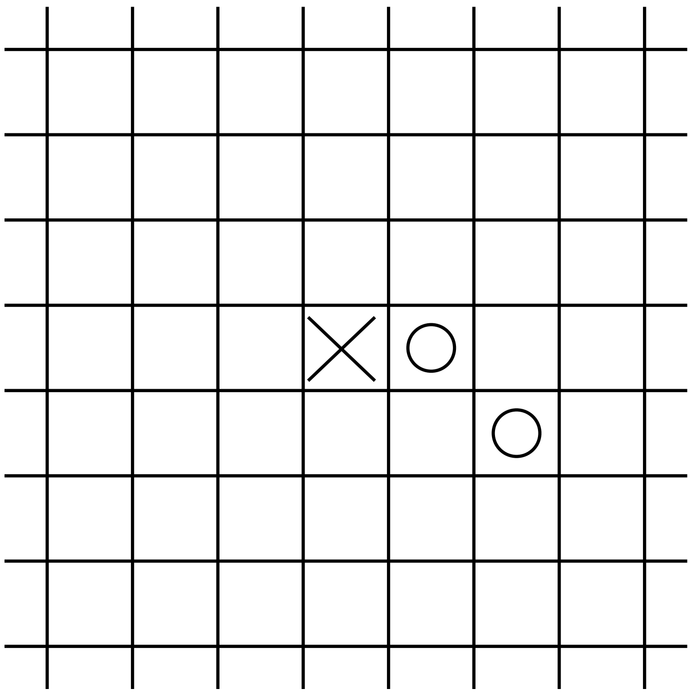

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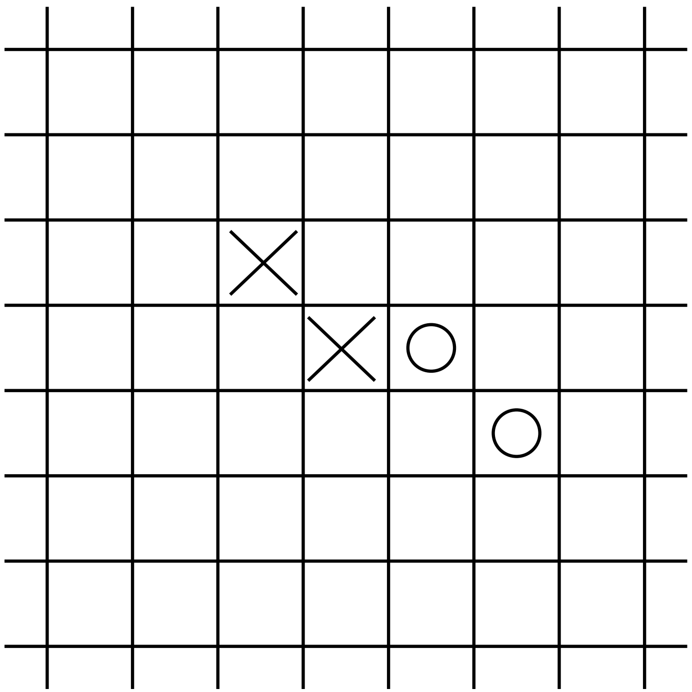

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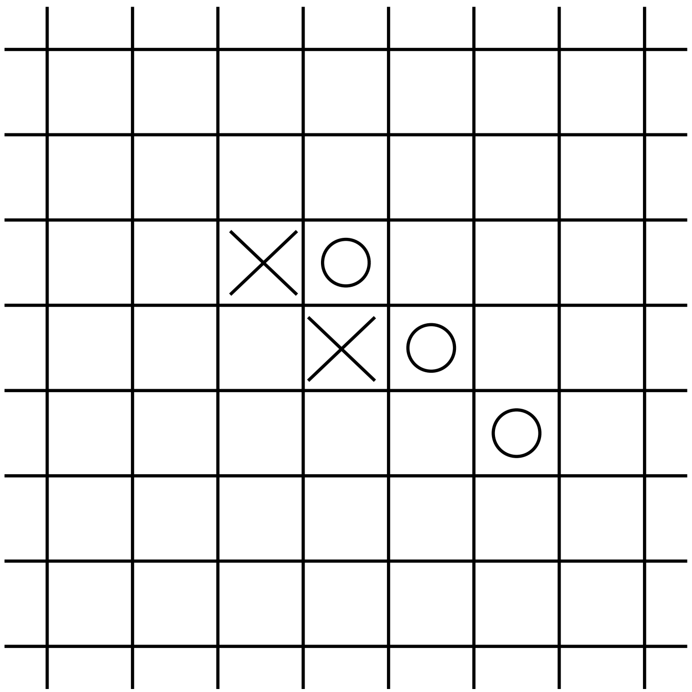

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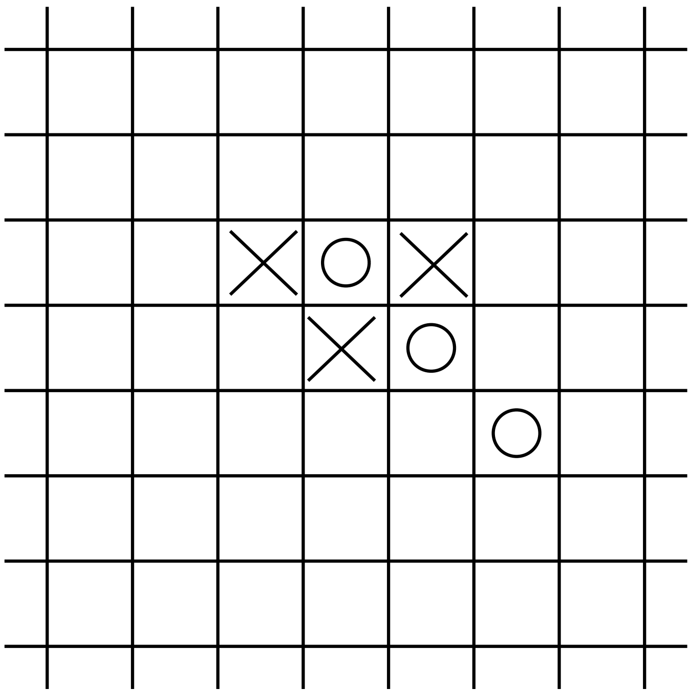

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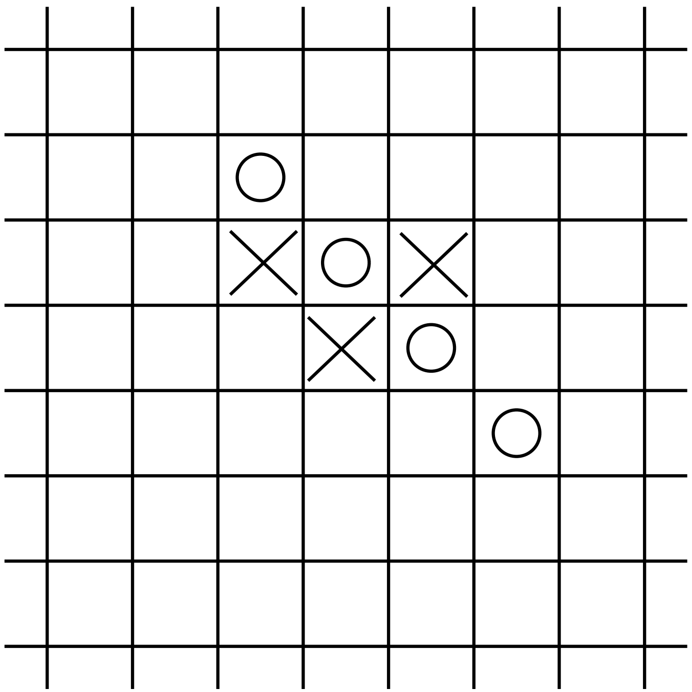

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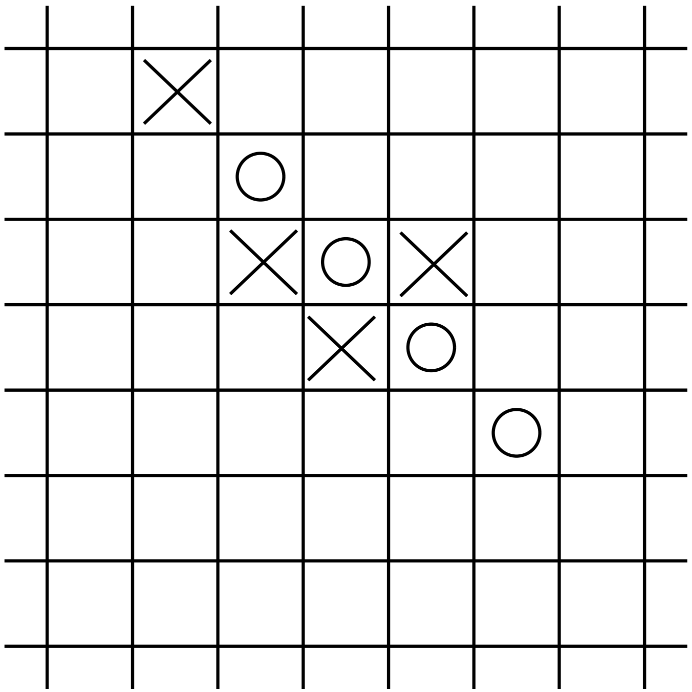

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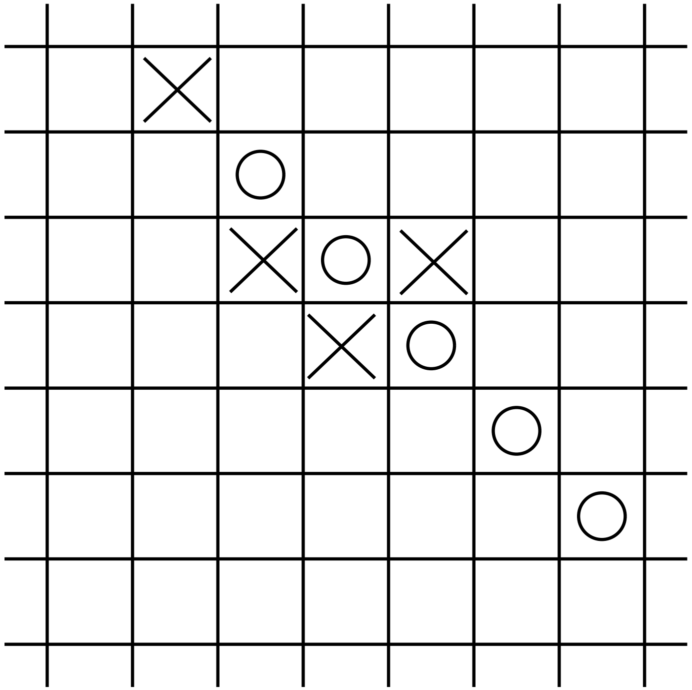

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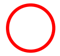
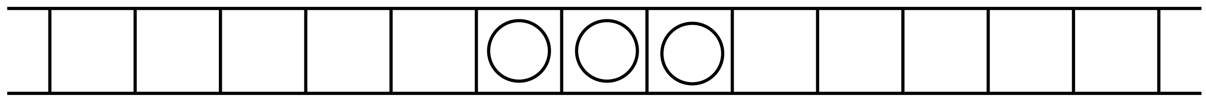
Can the second player prevent the first player from winning if 5 is replaced by a larger specific n?

First player: 

Second player: 

# Linear OX

1st player cannot have three consecutive O's.



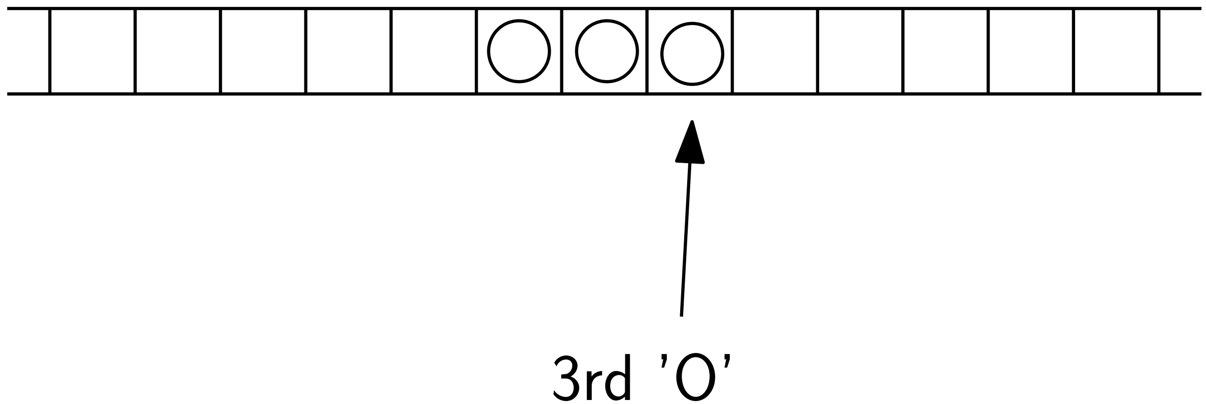
2nd player: tries to block 1st player from right, then from left ...

First player: ○

Second player: ×

# Linear OX

1st player cannot have three consecutive O's.

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First player: ○

Second player: ×

# Linear OX

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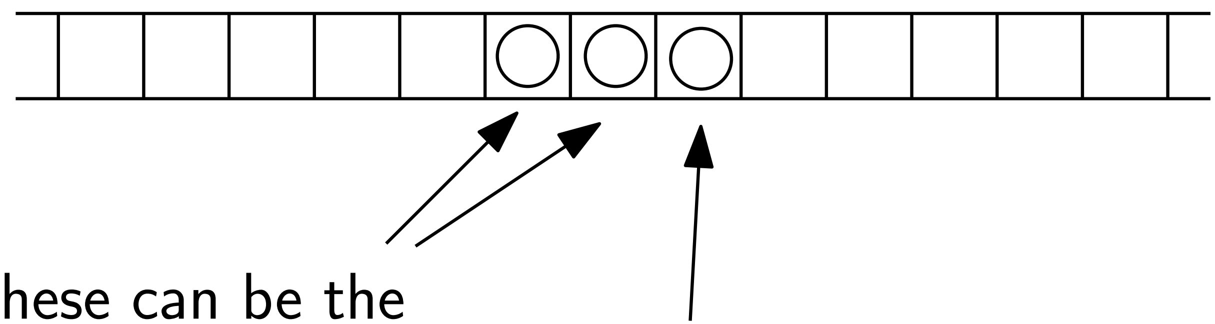


None of these can be the 2nd 'O'.

3rd 'O'

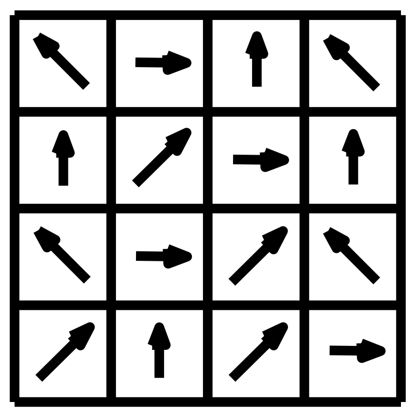
2nd player: tries to block 1st player from right, then from left ...

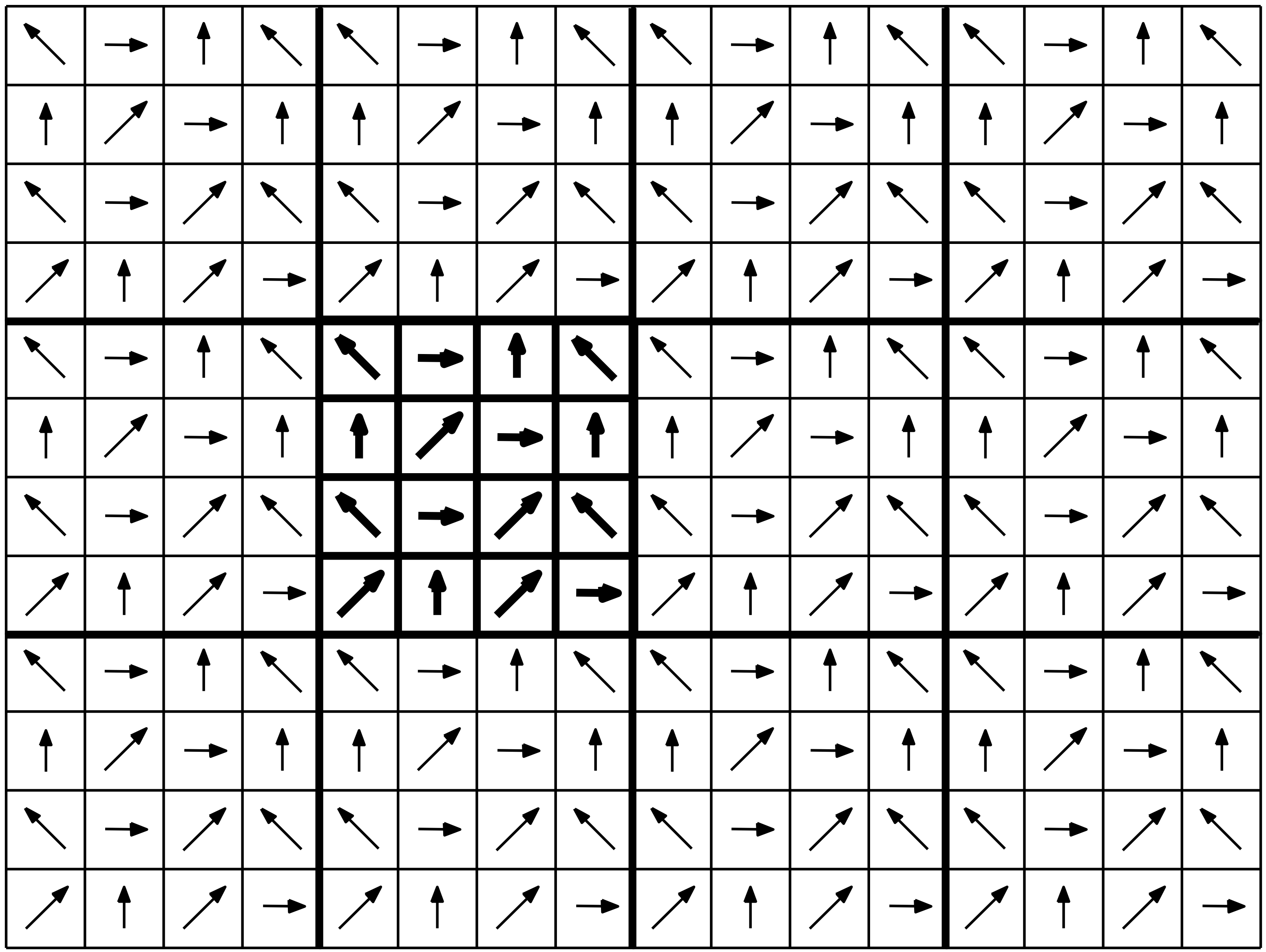
First player: $\bigcirc$

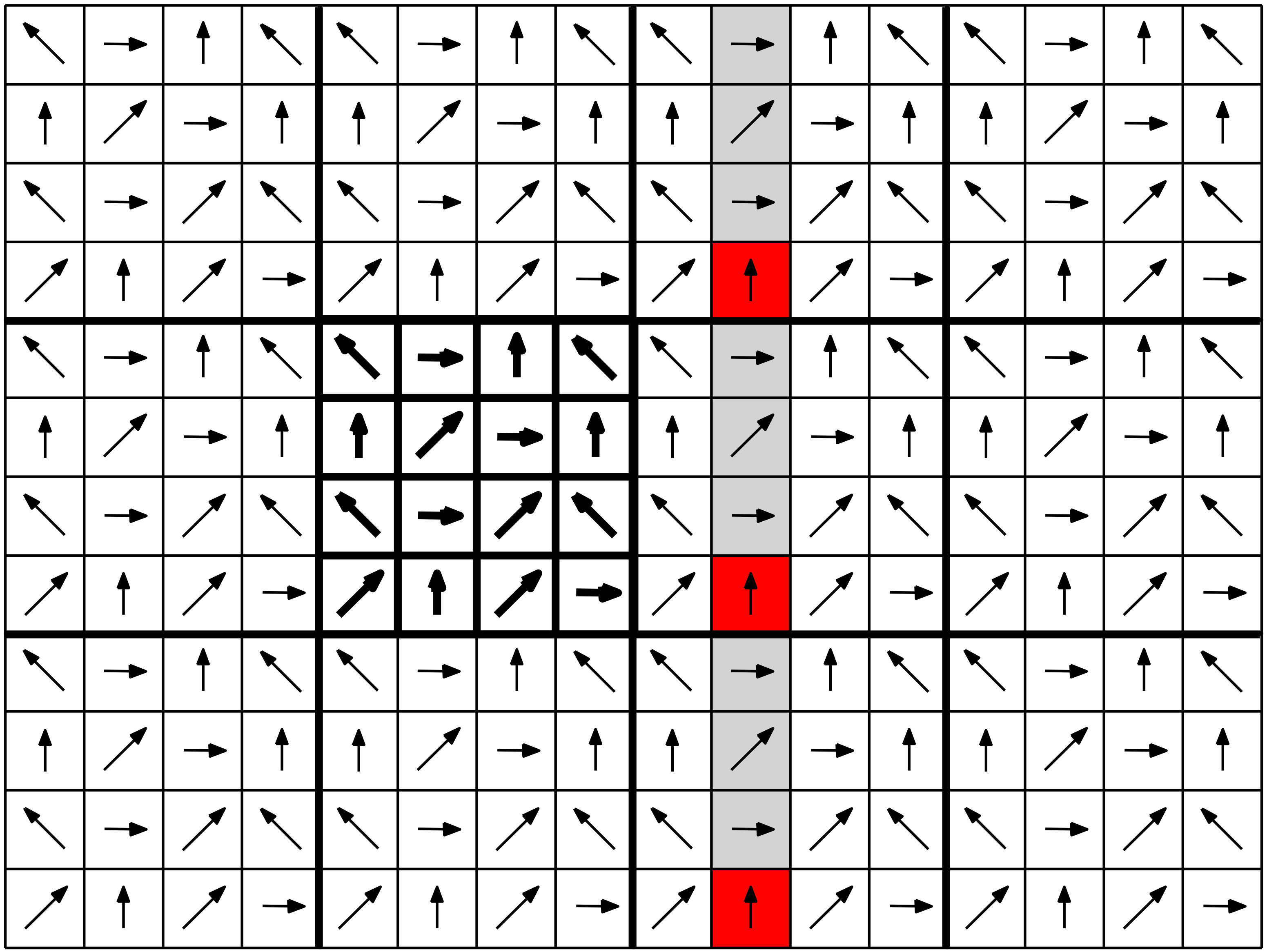
Second player: $\times$

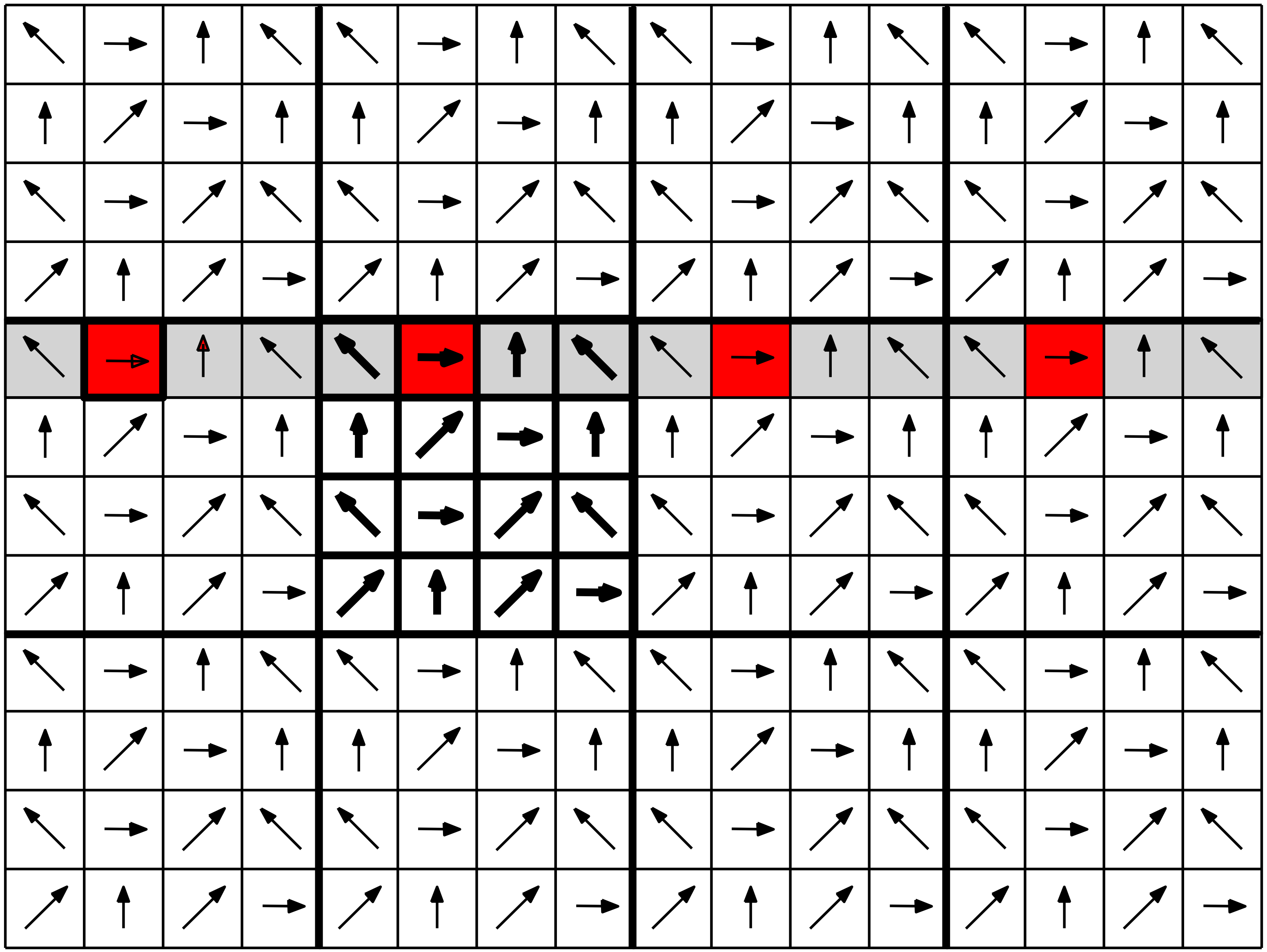
# 12 OX game

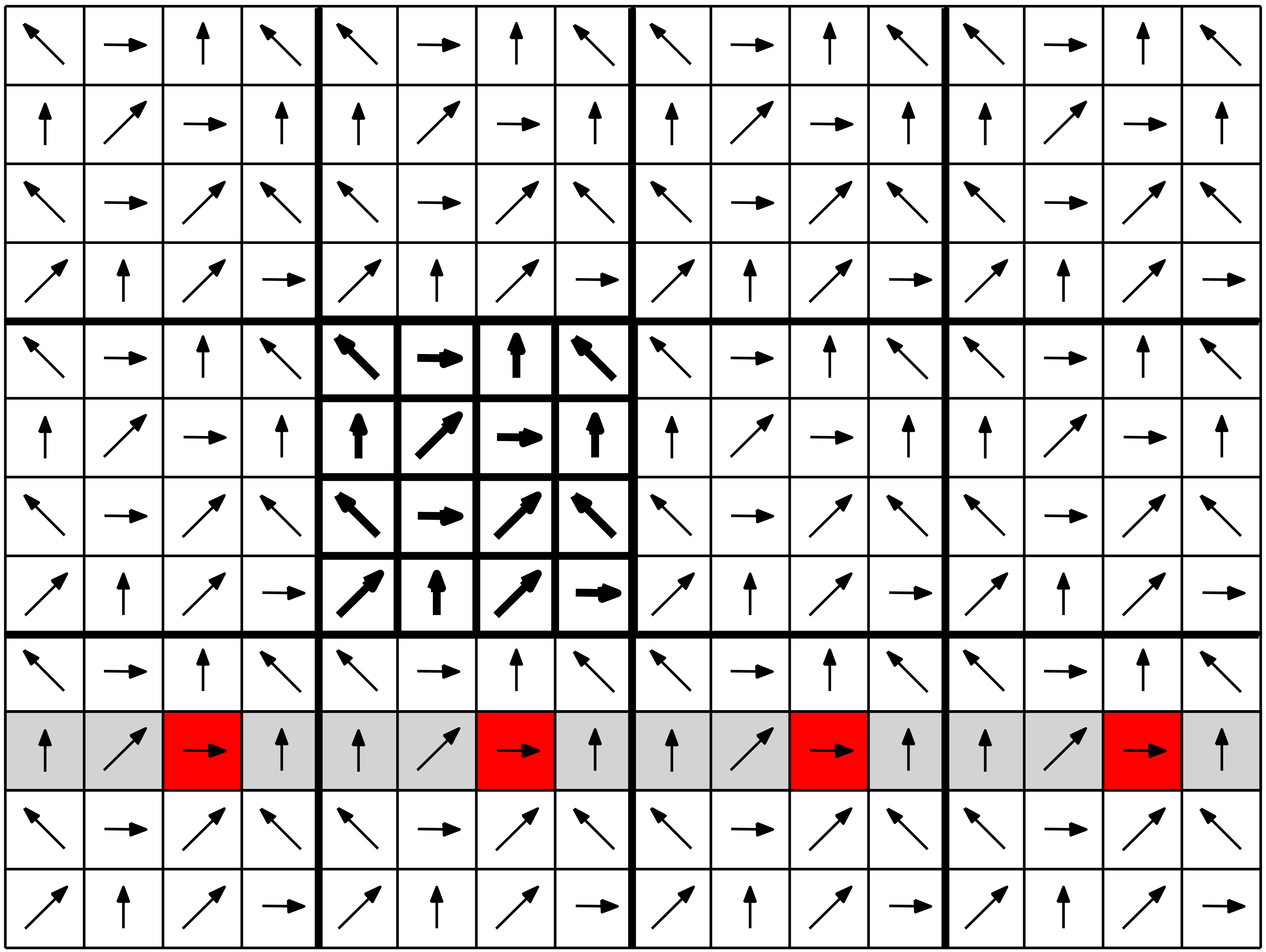
2nd player's can prevent 1st player from getting 12 consecutive O's:

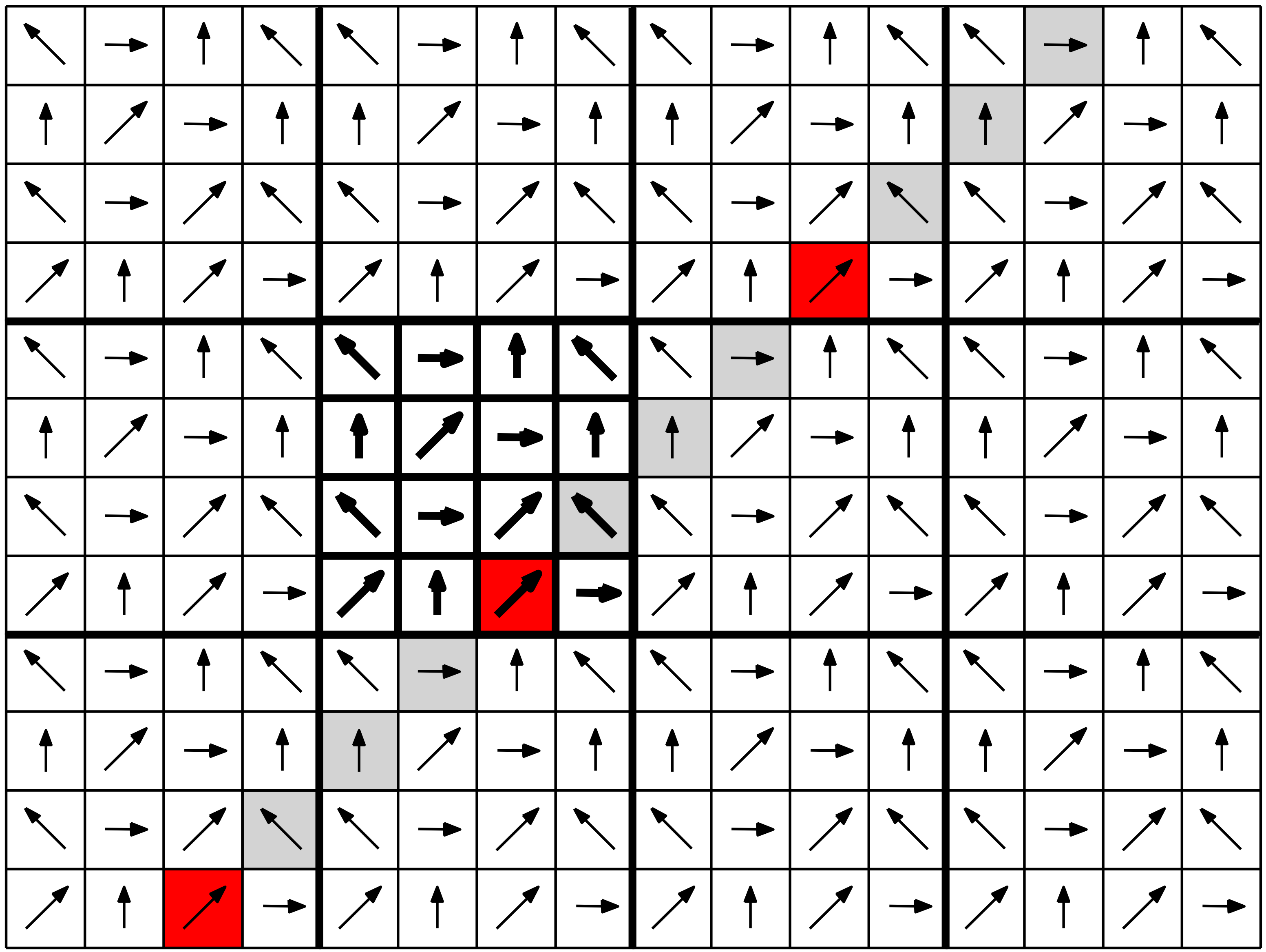


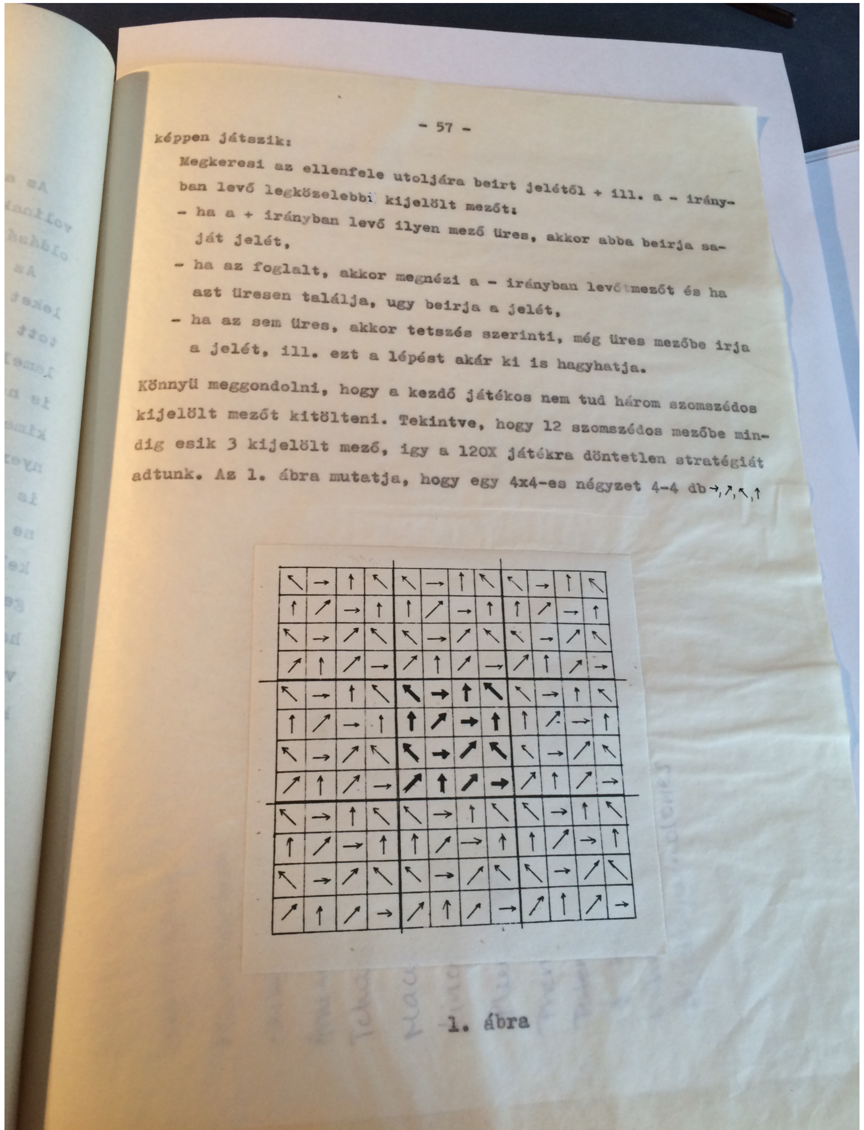












- 57 -

képpen játszik:

Megkeresi az ellenfele utoljára beirt jelétől + ill. a - irányban levő legközelebbi kijelölt mezőt:

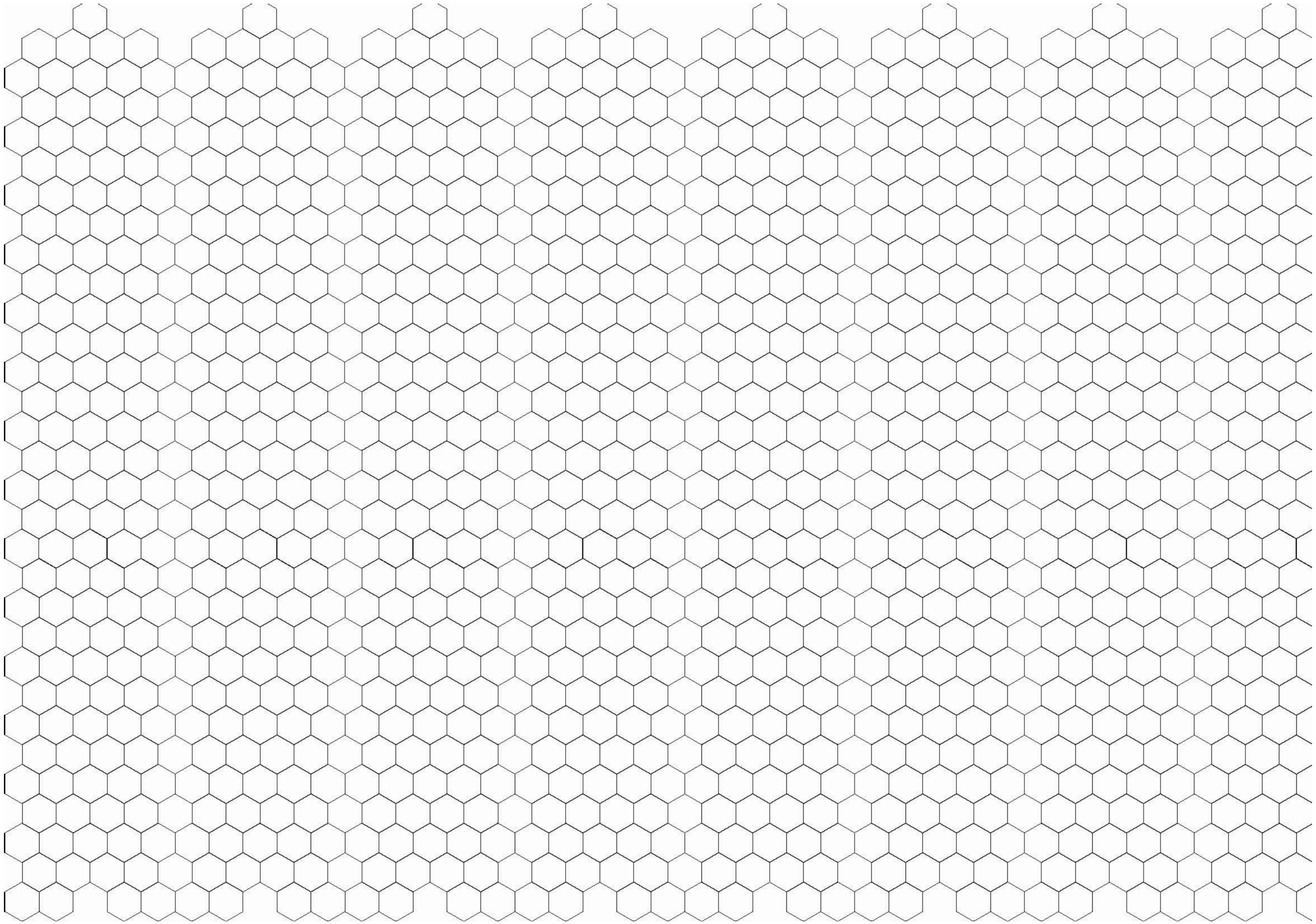
- ha a + irányban levő ilyen mező üres, akkor abba beirja saját jelét,
- ha az foglalt, akkor megnézi a - irányban levő mezőt és ha azt üresen találja, ugy beirja a jelét,
- ha az sem üres, akkor tetszés szerinti, még üres mezőbe irja a jelét, ill. ezt a lépést akár ki is hagyhatja.

Könnyü meggondolni, hogy a kezdő játékos nem tud három szomszédos kijelölt mezőt kitölteni. Tekintve, hogy 12 szomszédos mezőbe mindig esik 3 kijelölt mező, igy a 120X játékra döntetlen stratégiát adtunk. Az 1. ábra mutatja, hogy egy 4x4-es négyzet 4-4 db →,↗,↖,↑

1. ábra

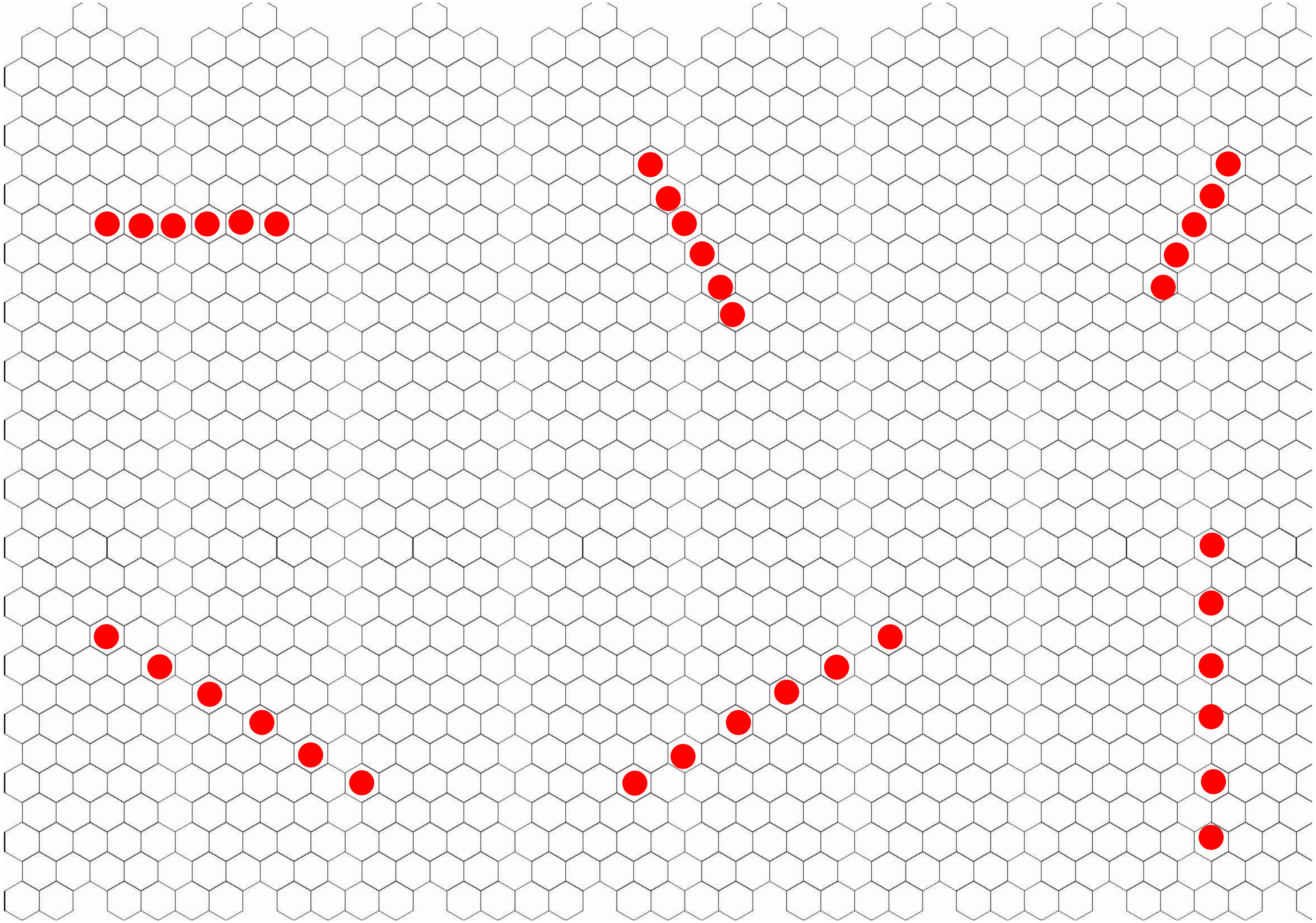
The second player prepars the gome board with a special pattern.

# n - OX game on the hexagonal grid



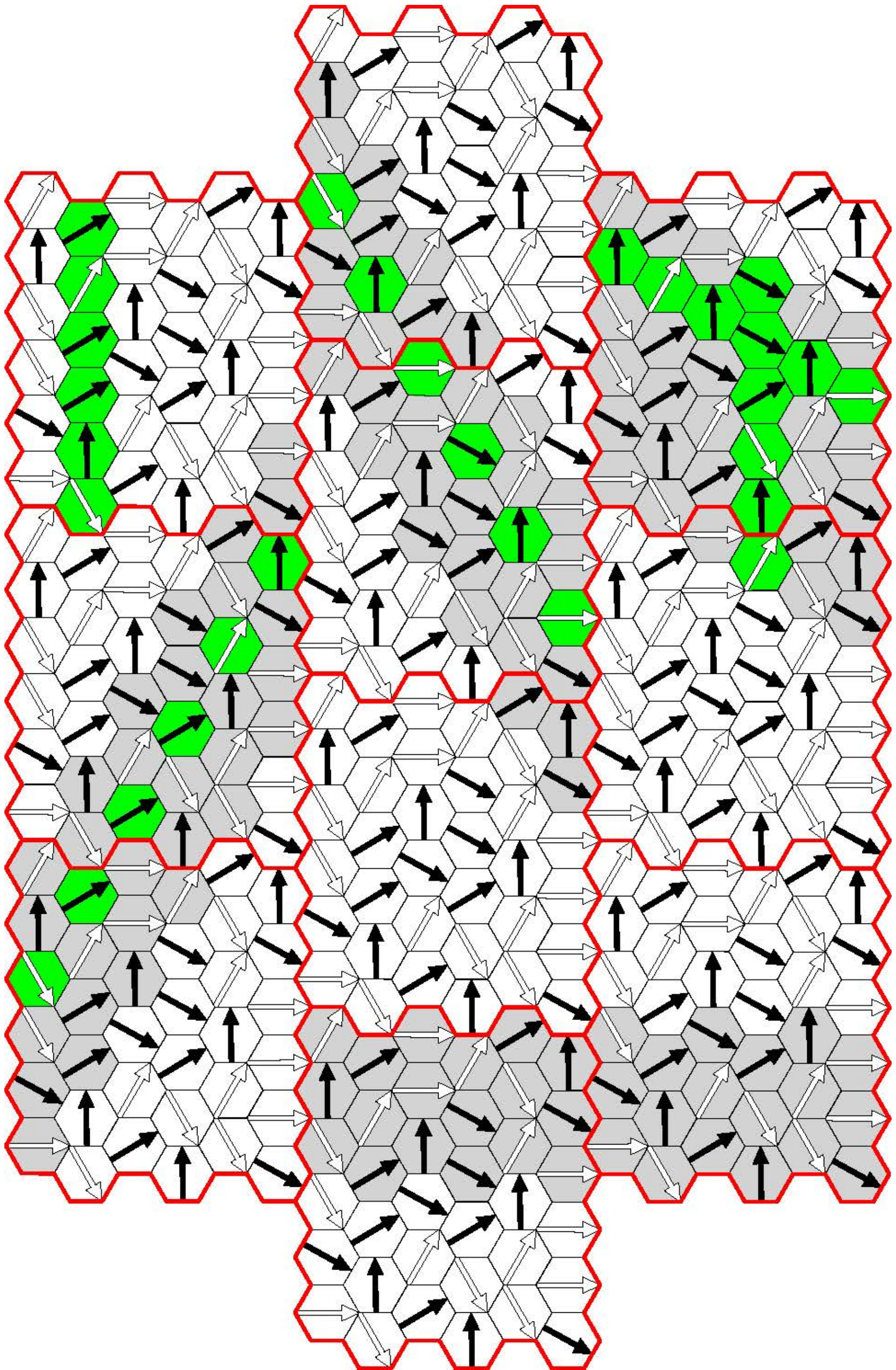
# n - OX game on the hexagonal grid

Players alternate placing thir own symbols pieces (tipically O and X) on the hexagonal grid. The first player to get n consecutive symbols in one of the 6 symmetry line directions is the winner.



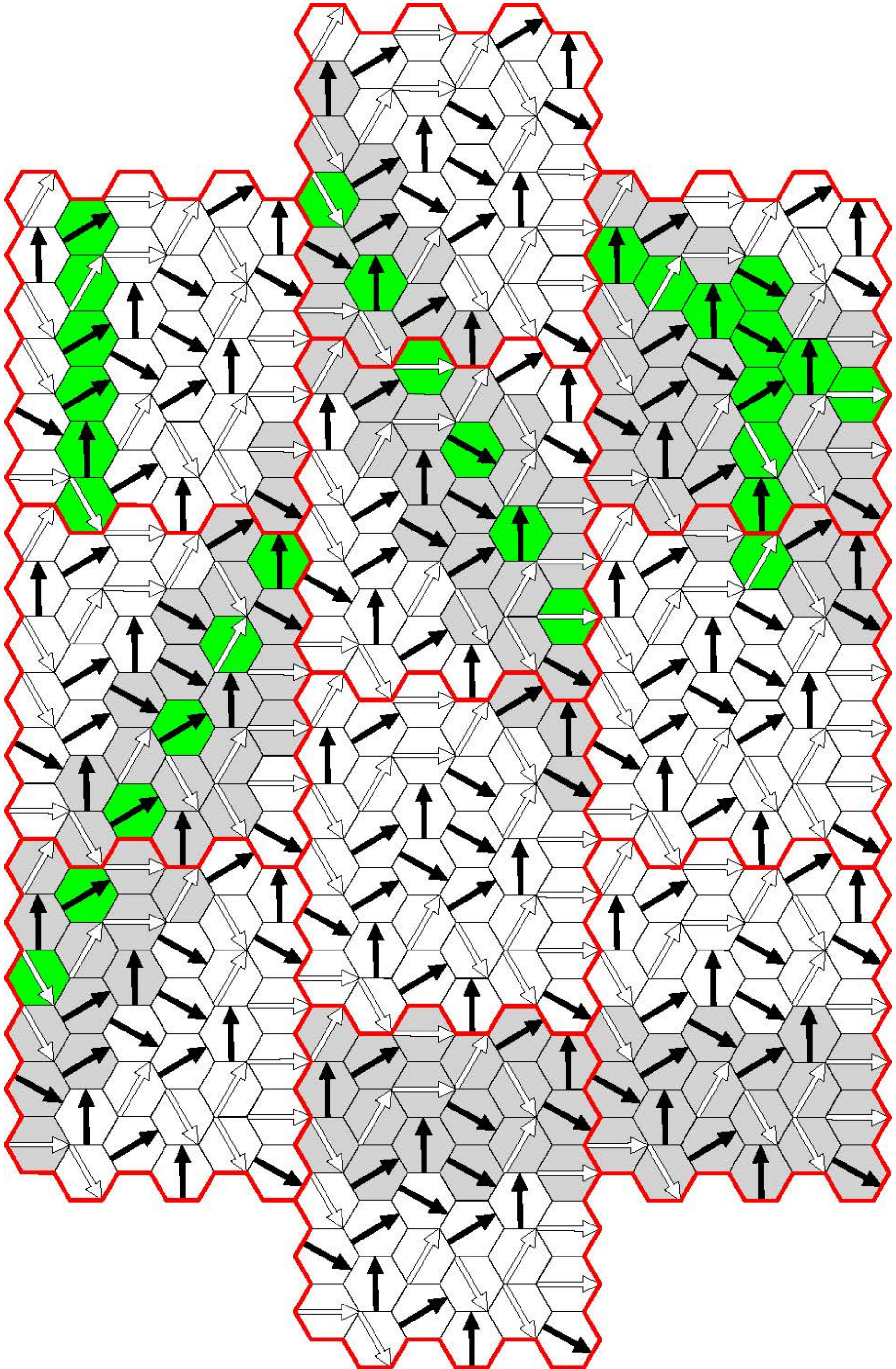
# 18- OX game.

2nd player can be prevented from winning the 18 - OX game.

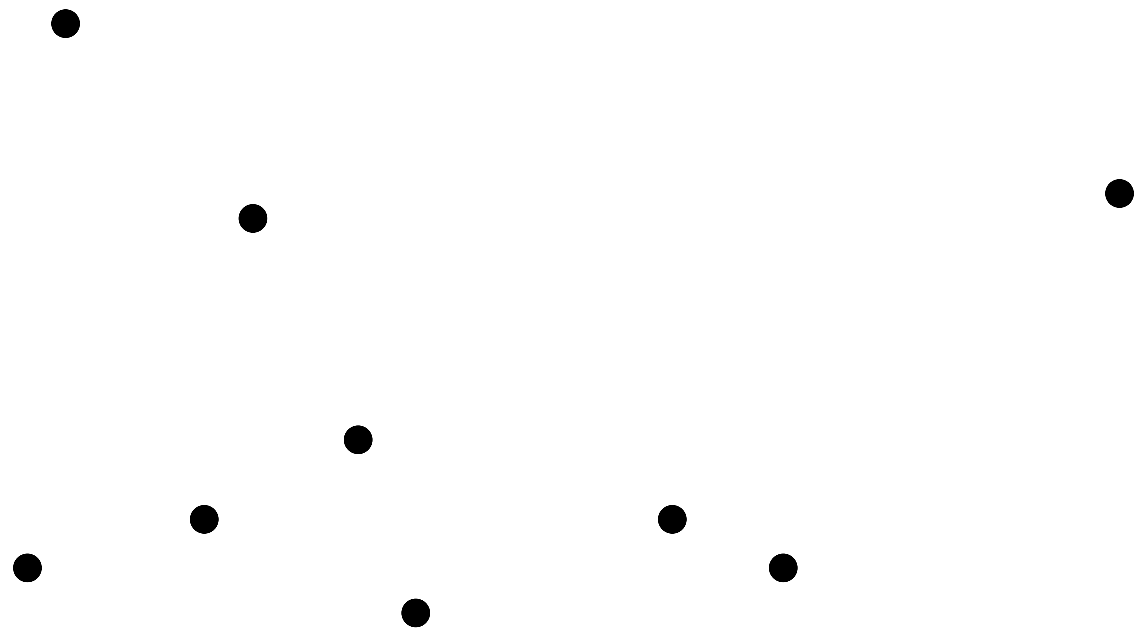


# 18- OX game.

1st player can be prevented from winning the 18 - OX game.

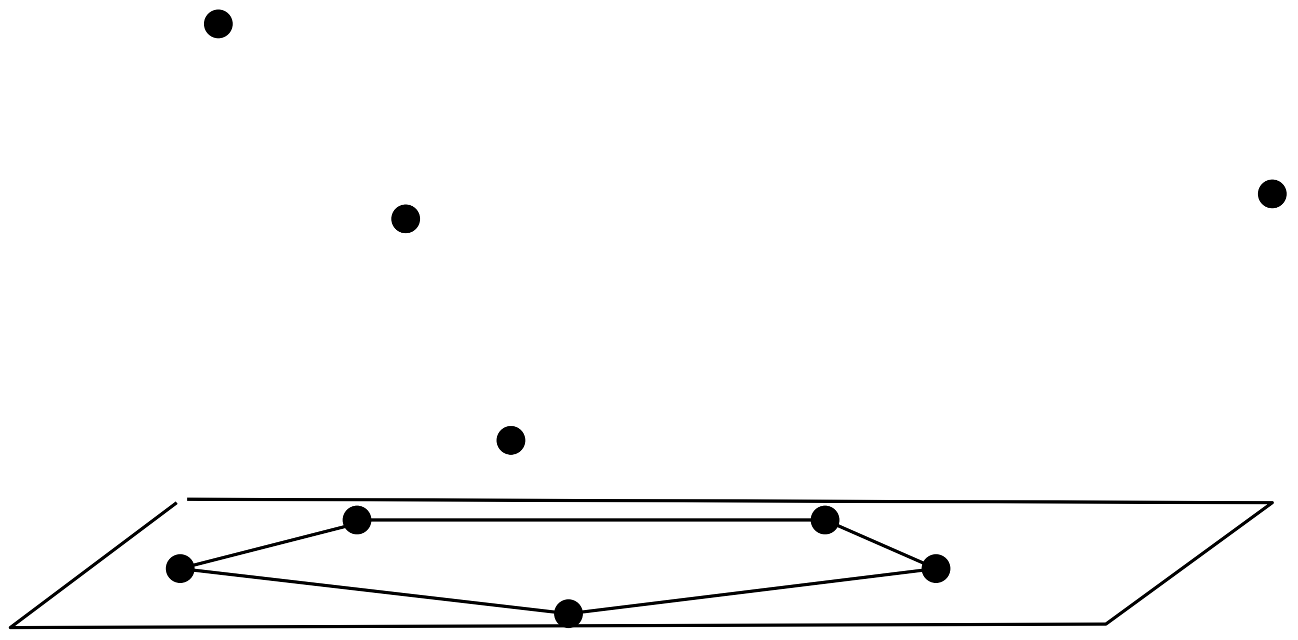


$n = 3$

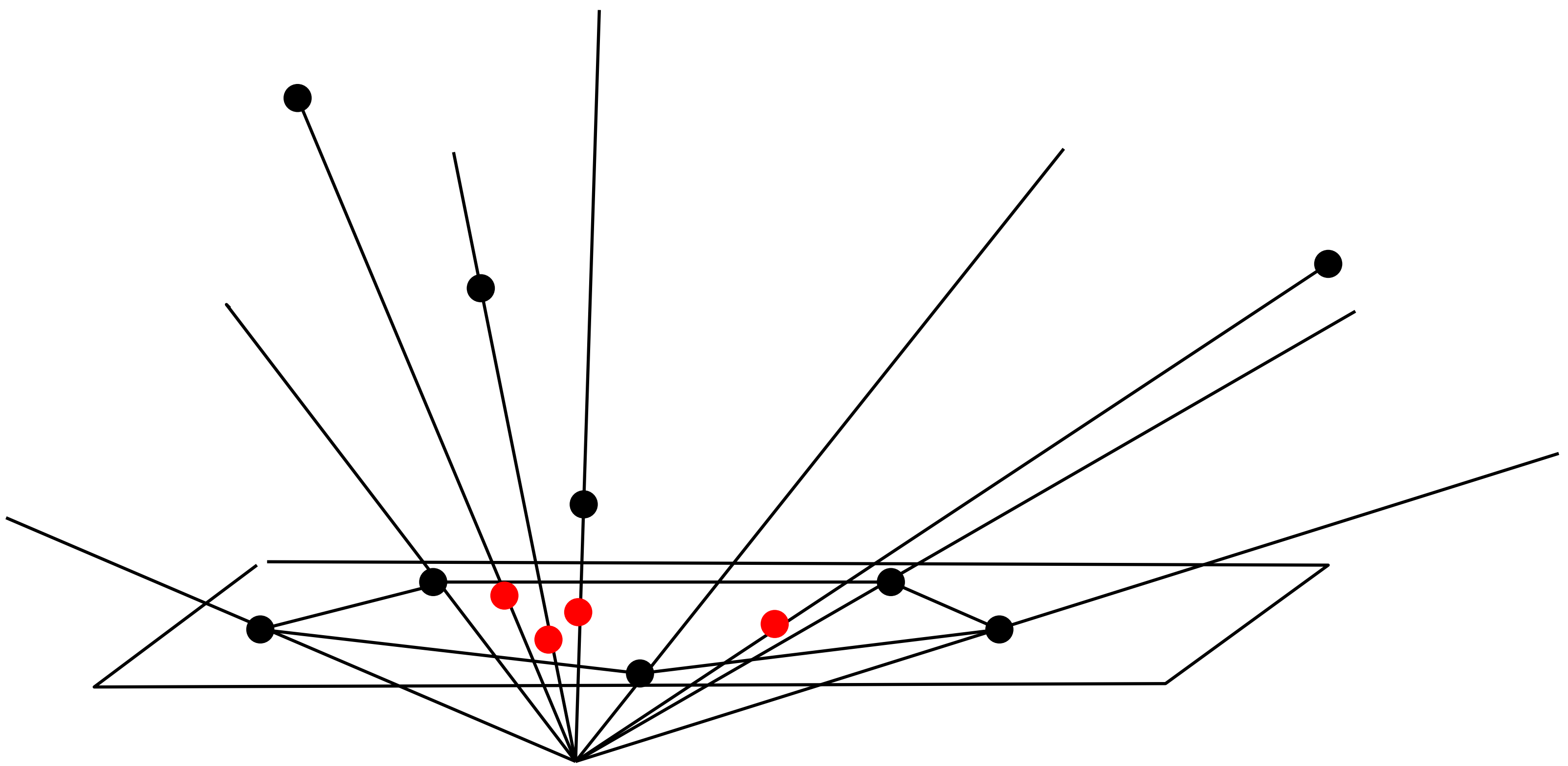


2nd problem

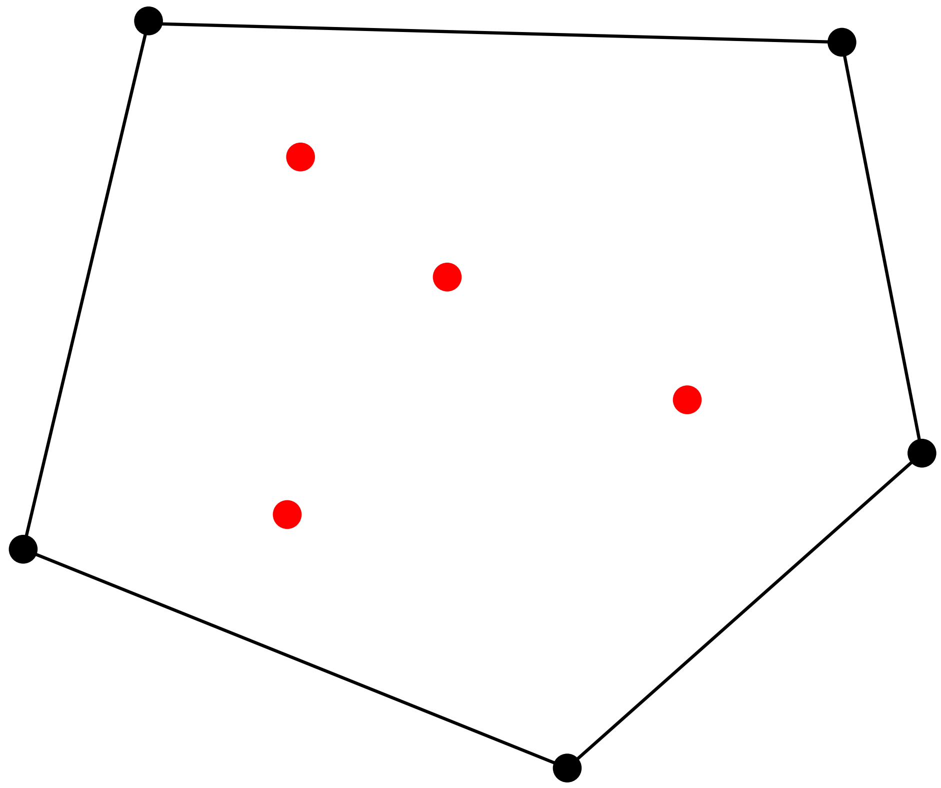
Given n points in the space (not all on a plane) find a polyhedron whose vertices are exactly the given points.



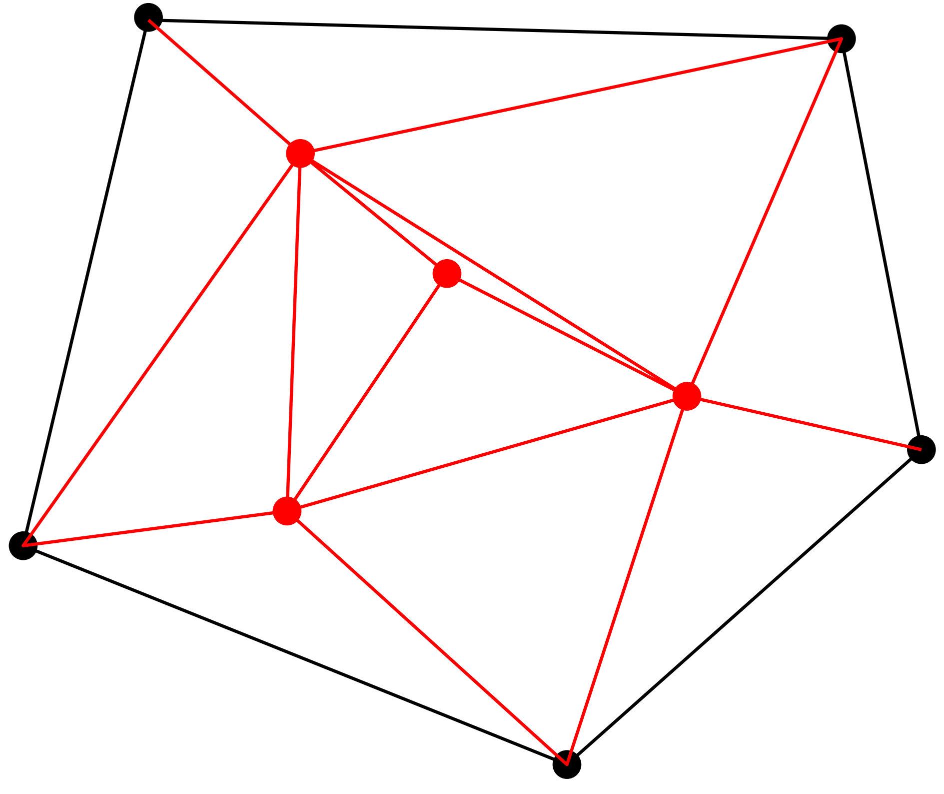
Choose a face of the convex hull.

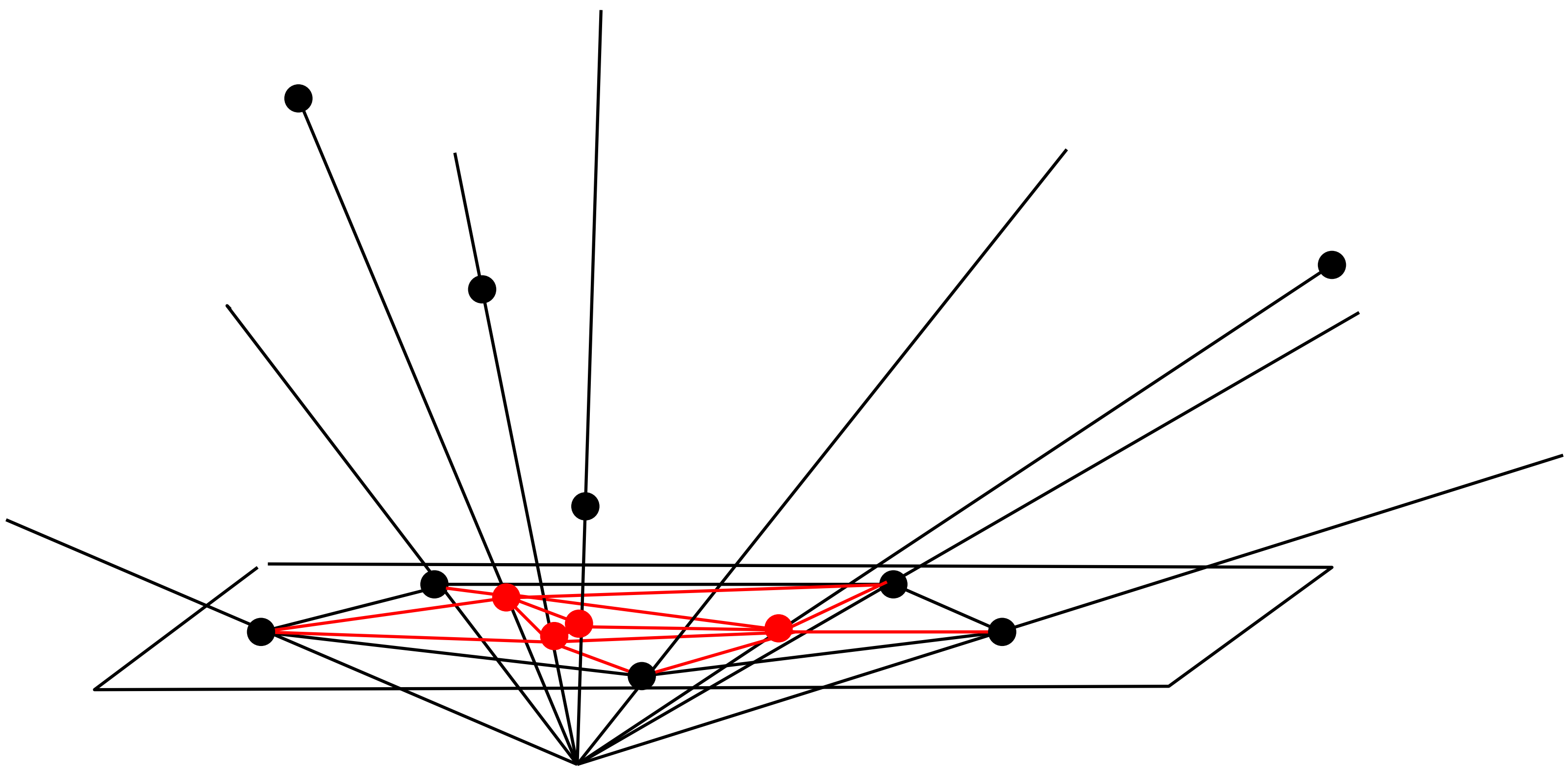


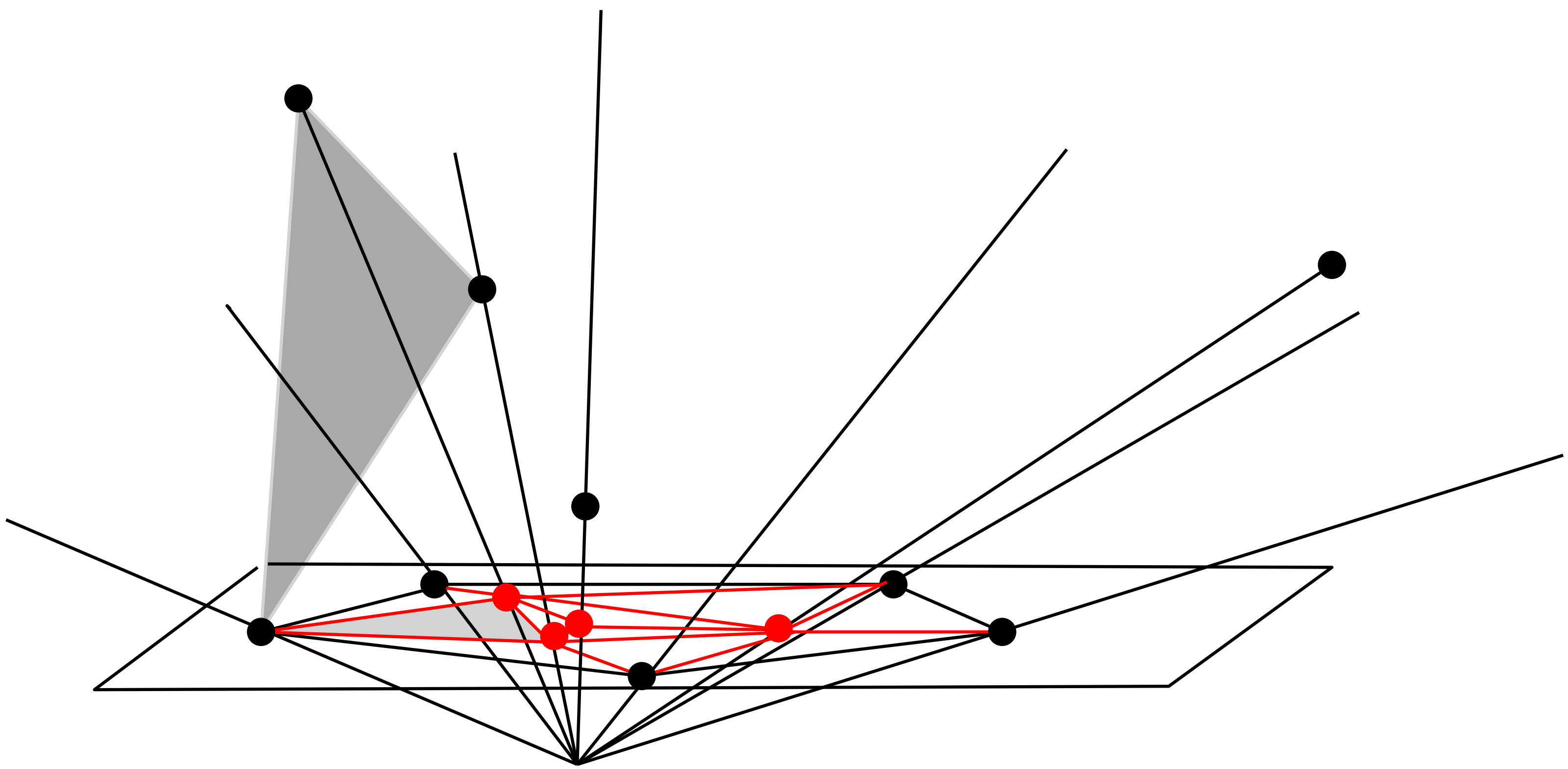
Choose a center and project radially the rest of the points so that...



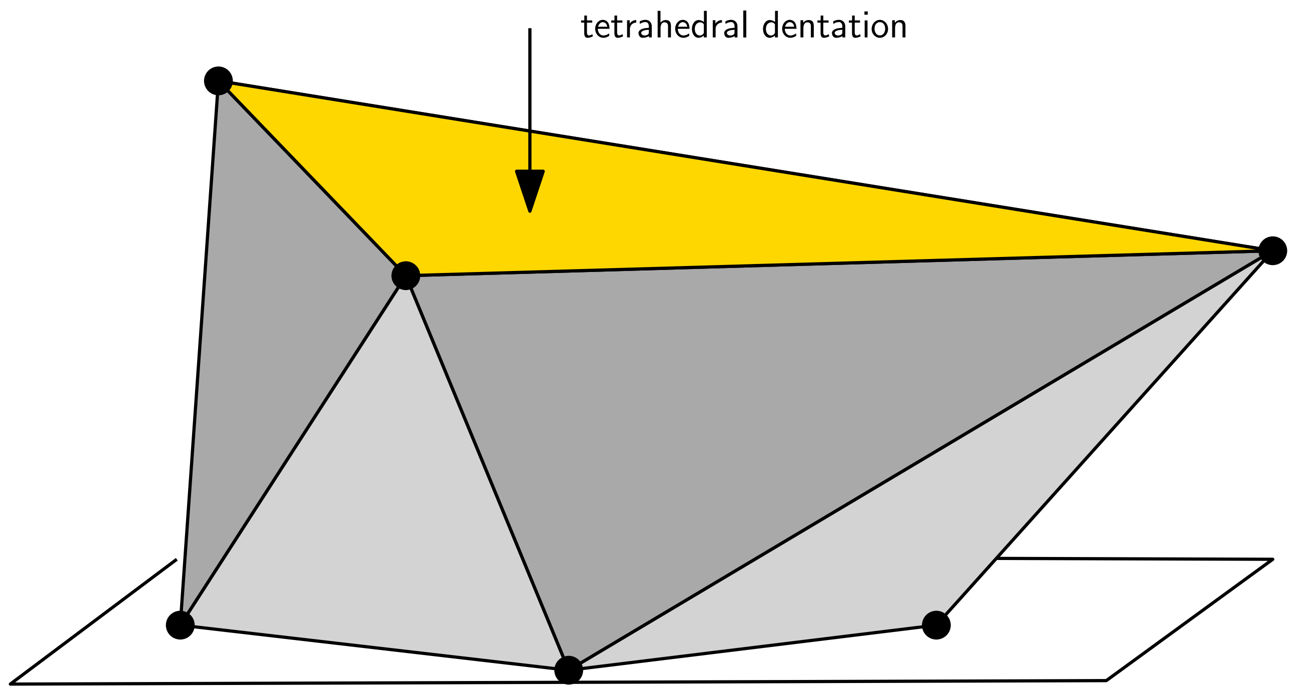
Lemma: There is a triangualtion without using diagonals....



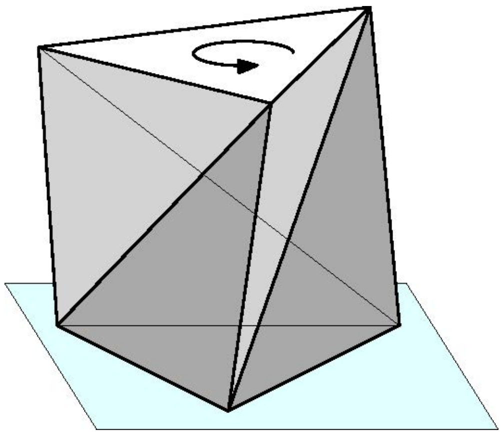




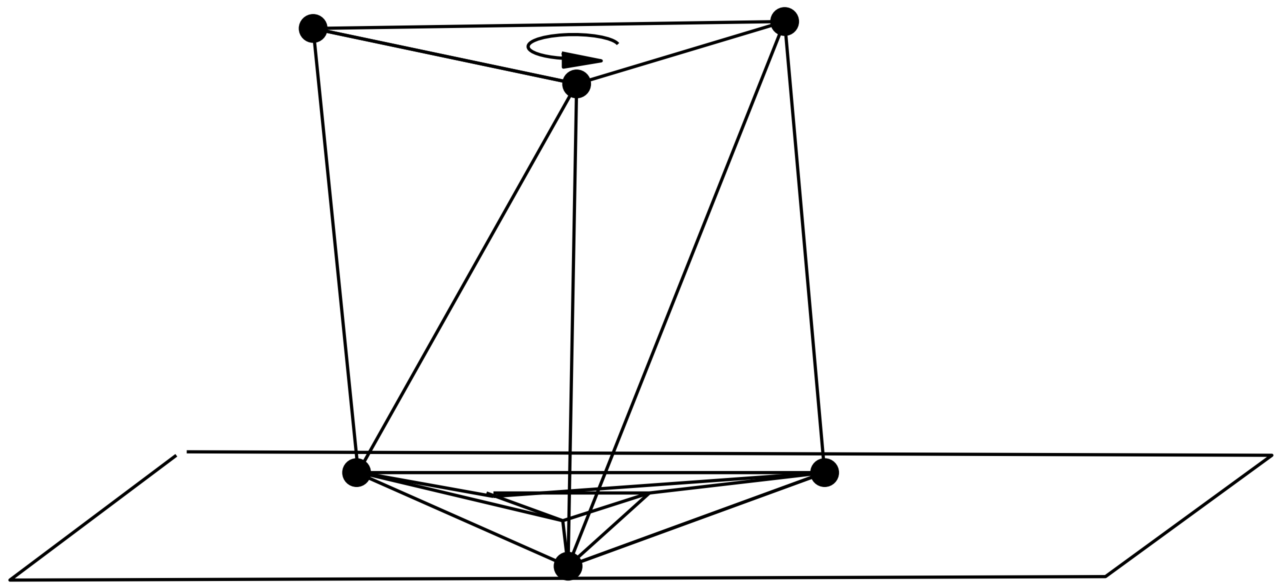
Pull back the triangles.

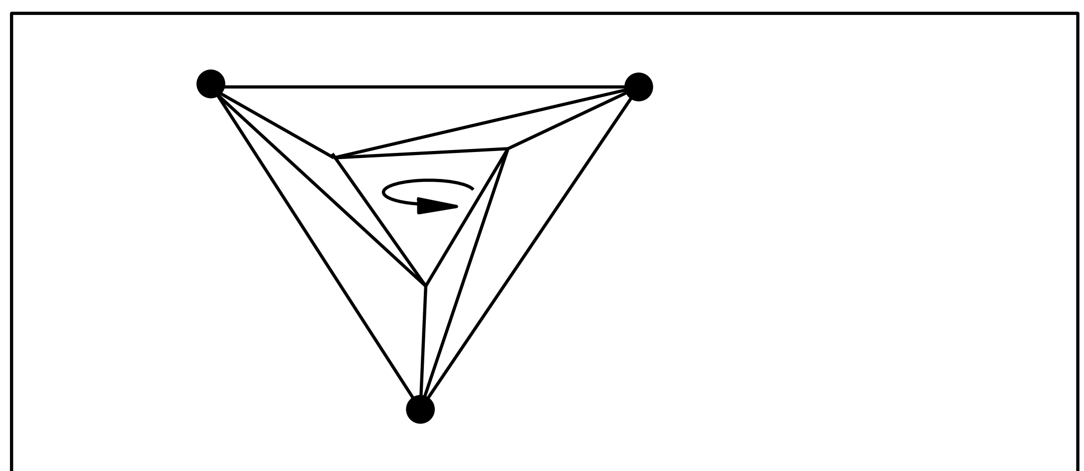

tetrahedral dentation

# Joint work with Osman Yardimci



Schonhardt





$n = 3$

Polygonalization: started with Hugo Steinhous (1964),
there are n pints lying in the plane no three on a line.....

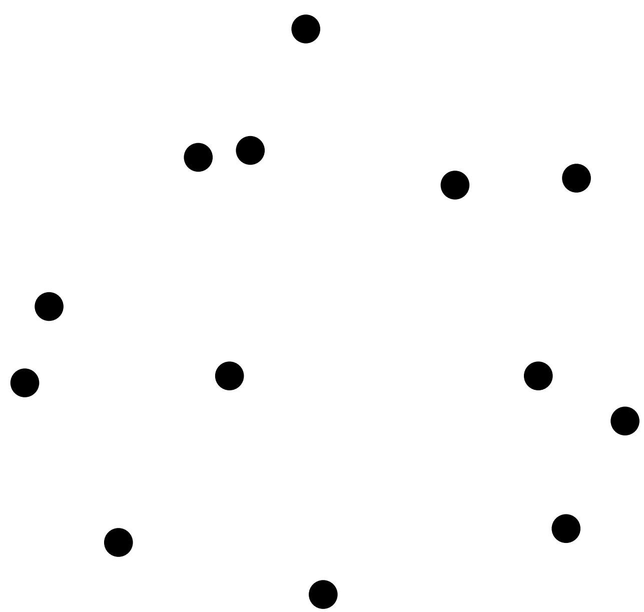
Polyhedralization: started with B. Grunbaum (1994) and
continued by many others, see
Hurtado, Toussaint Trias, Agarwal, Demaine, Mitchel, Sharir **etc.**
there are n points in the space, no for coplanar.....

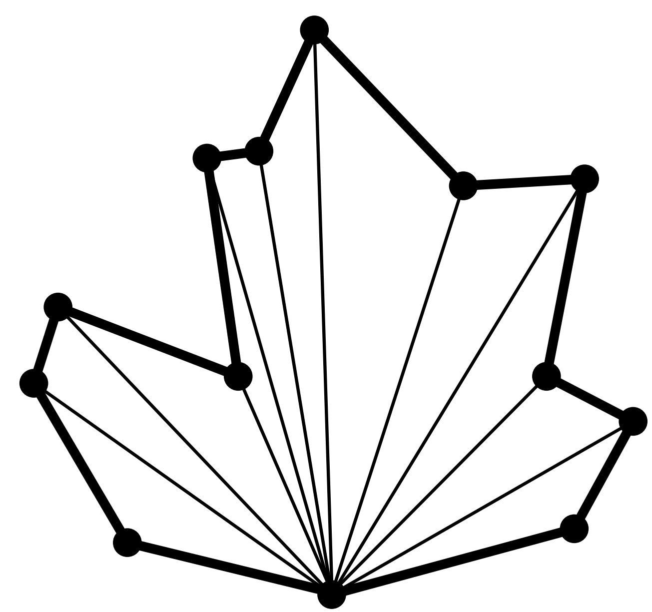
Several methods are known:

hinge polyhedralization
orange polyhedralization
cone polyhedralization
monotonic polyhedralization

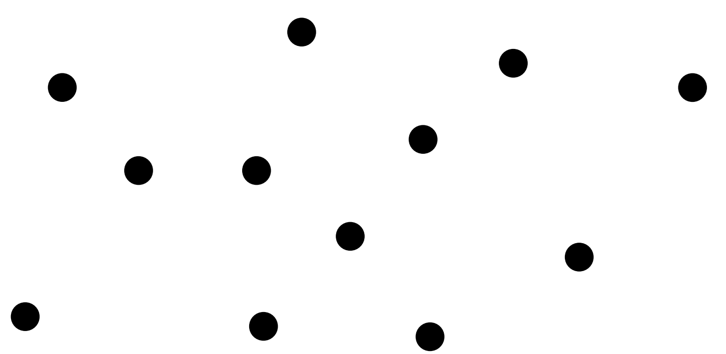
$n = 2$

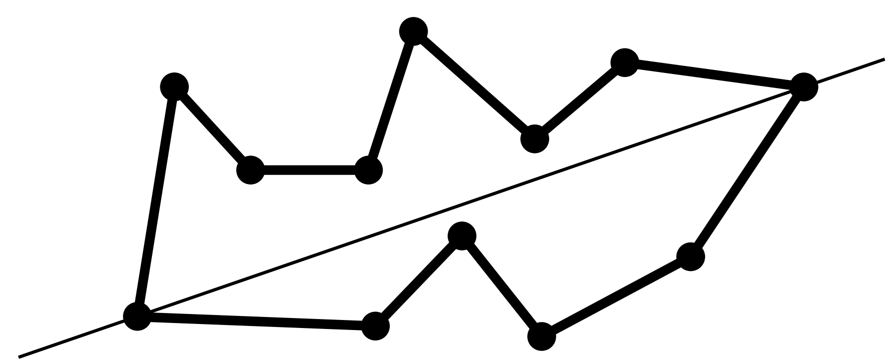
Fan polygonalization Ron Graham 1994.



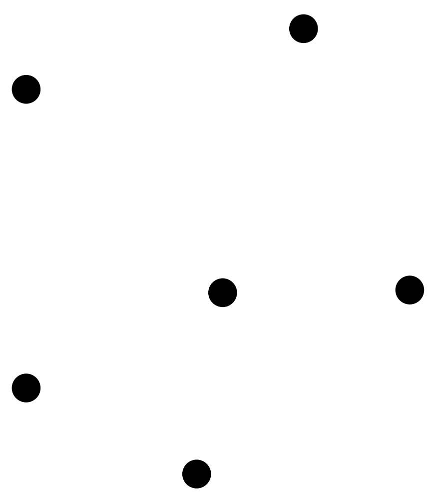


Monotonic polygonalization. Branko Grunbaum, 1994

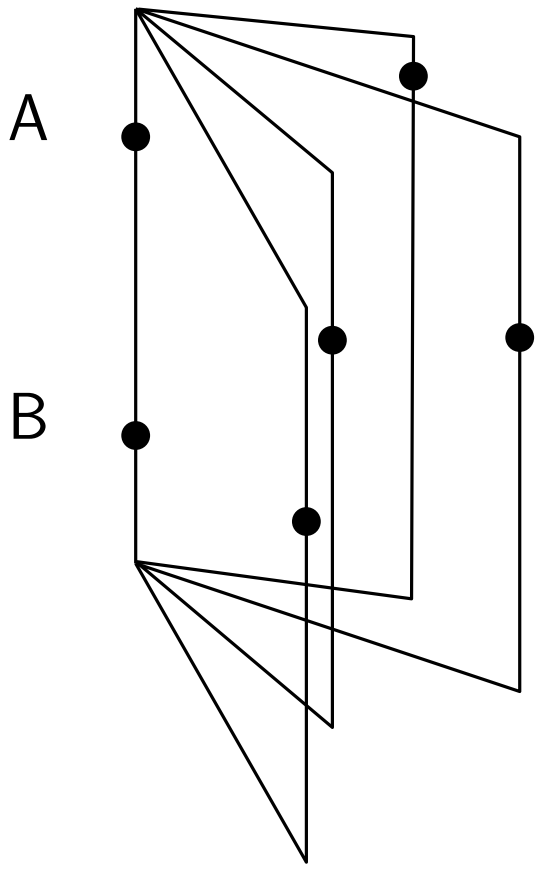


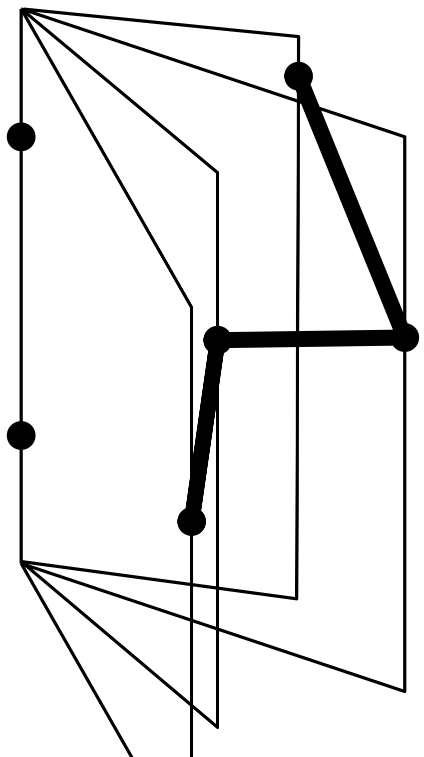


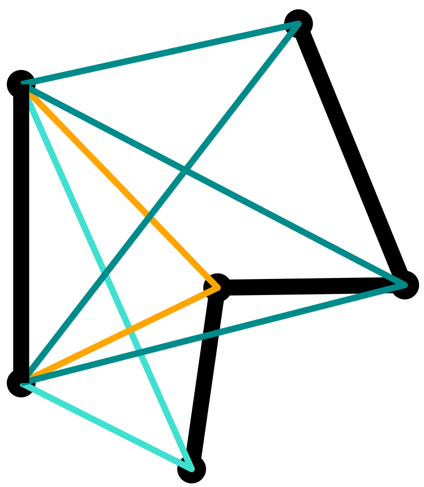
3D: n points are given, no 4 coplanar.

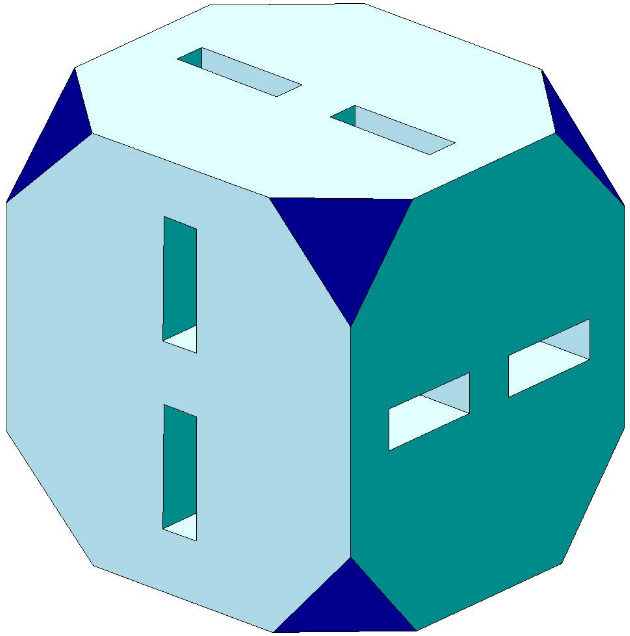


Choose an edgae of the convex hull, and order the remaining points.

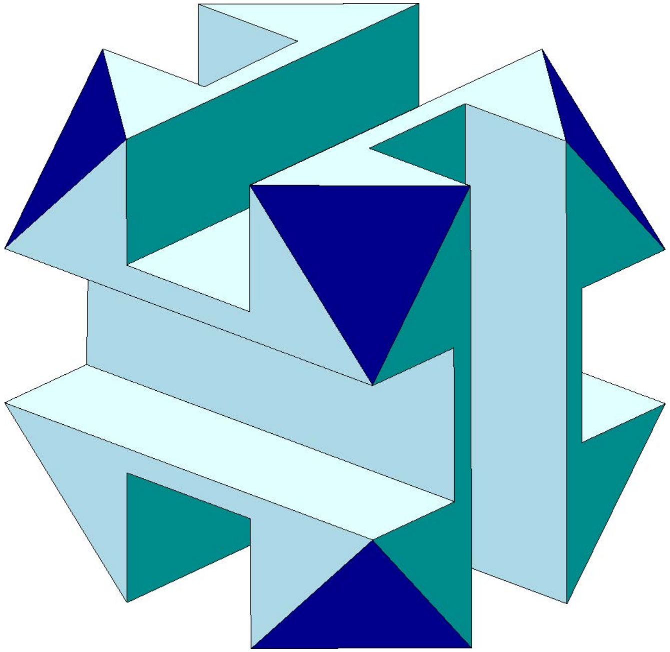




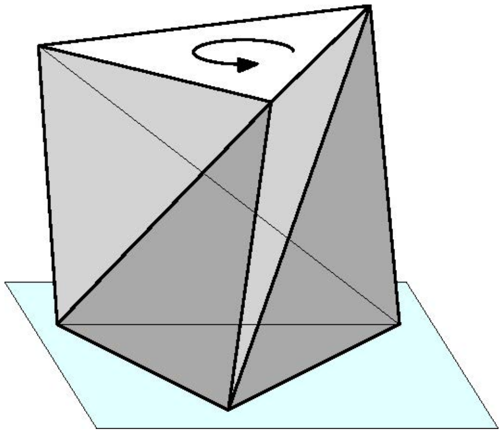




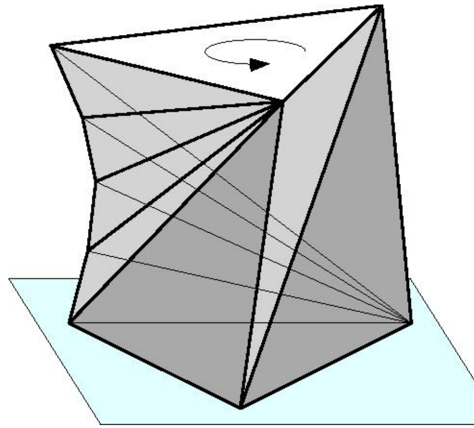
W. Kuperberg's example (2011) of a polyhedron with a point inside which does not see any of the vertices



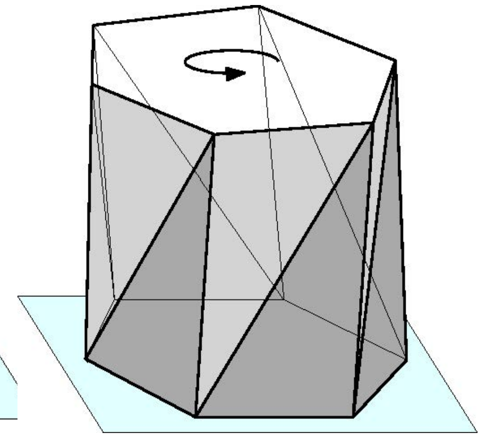
Modified example of Kuperberg.



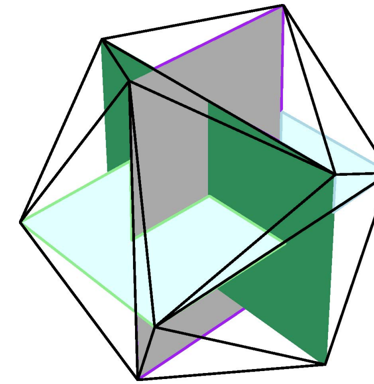
Schonhardt



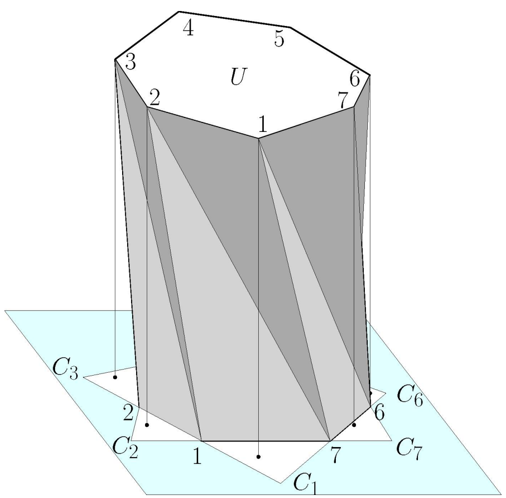
Bagemihl

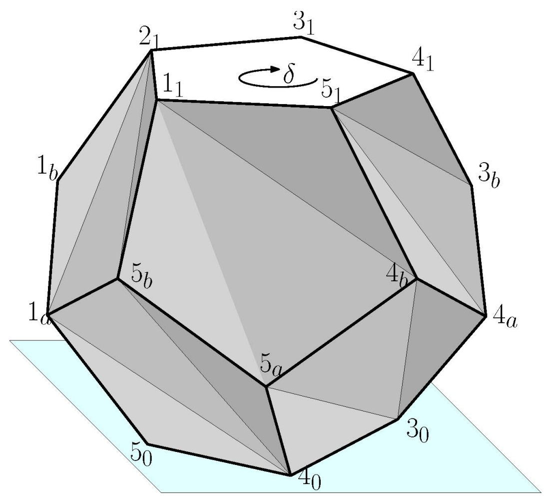


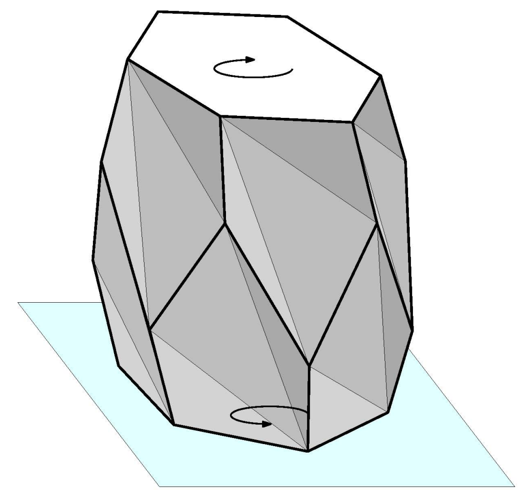
Rambau (2005)



Jessen

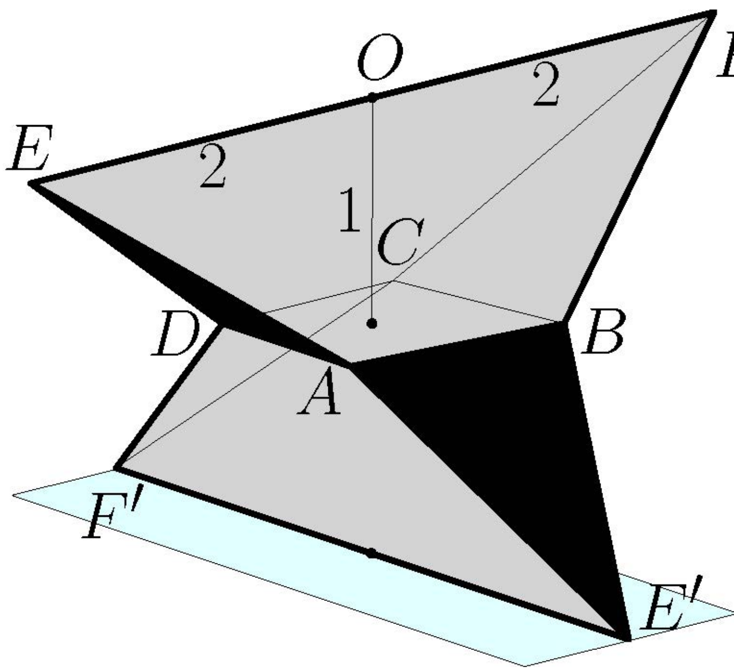


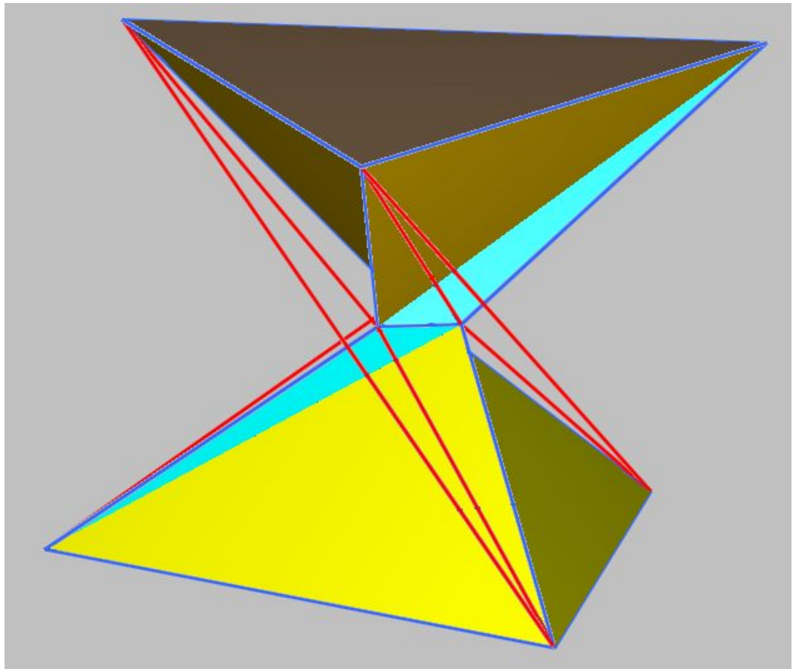



A.B. , B. Carrigan (2014)

Key idea: Instead of triangulations tilings were studied (more general theorem was proved)



Lajos Szilassi:



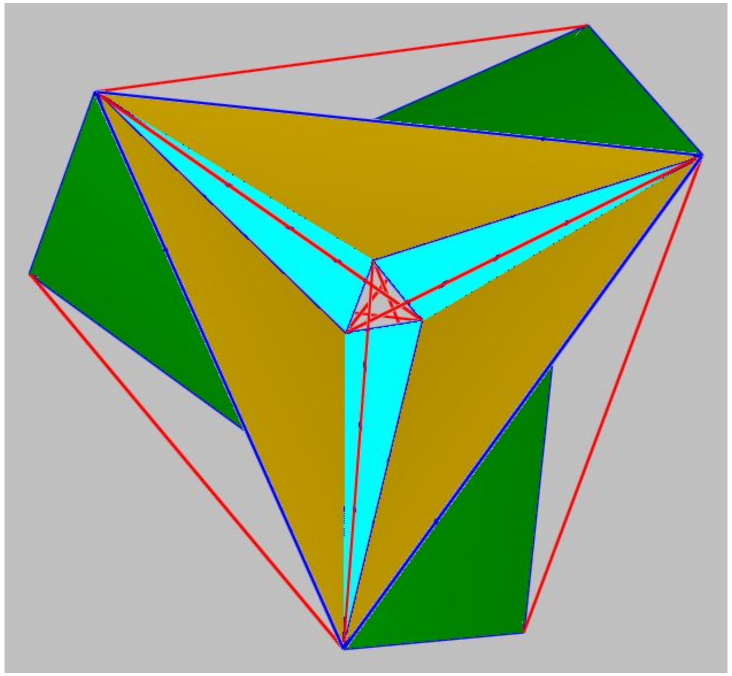
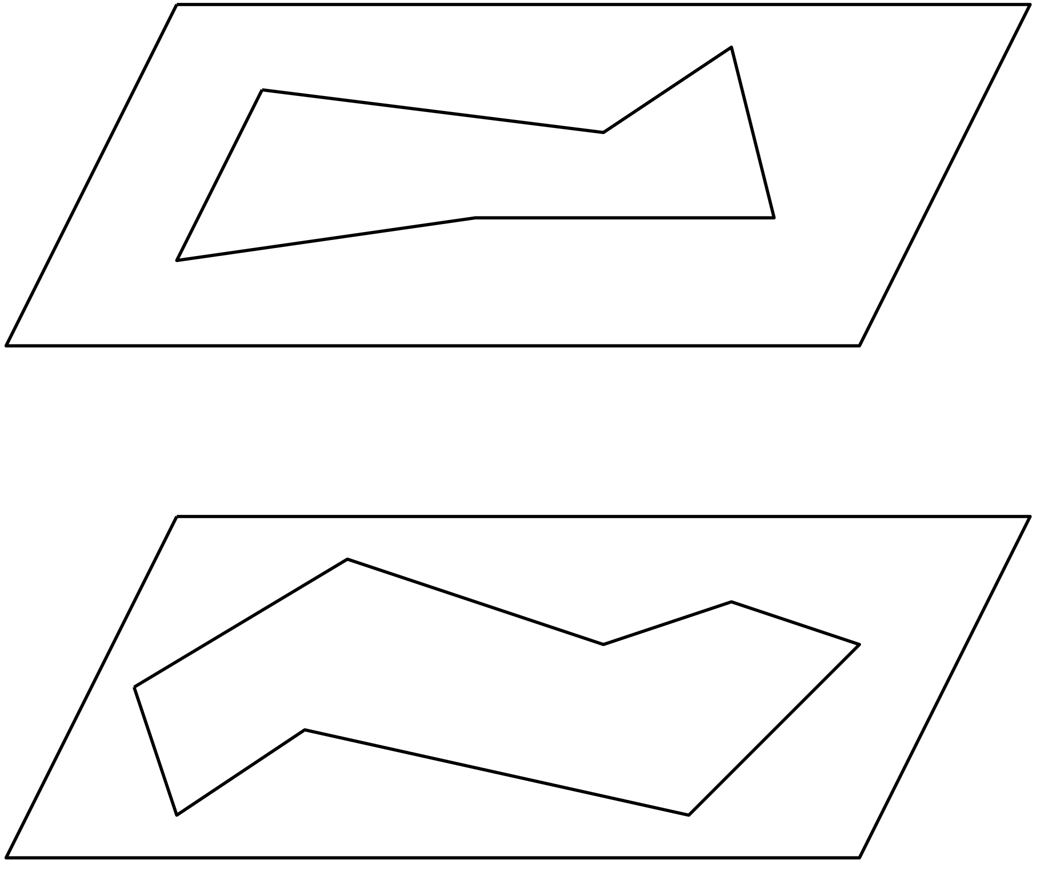
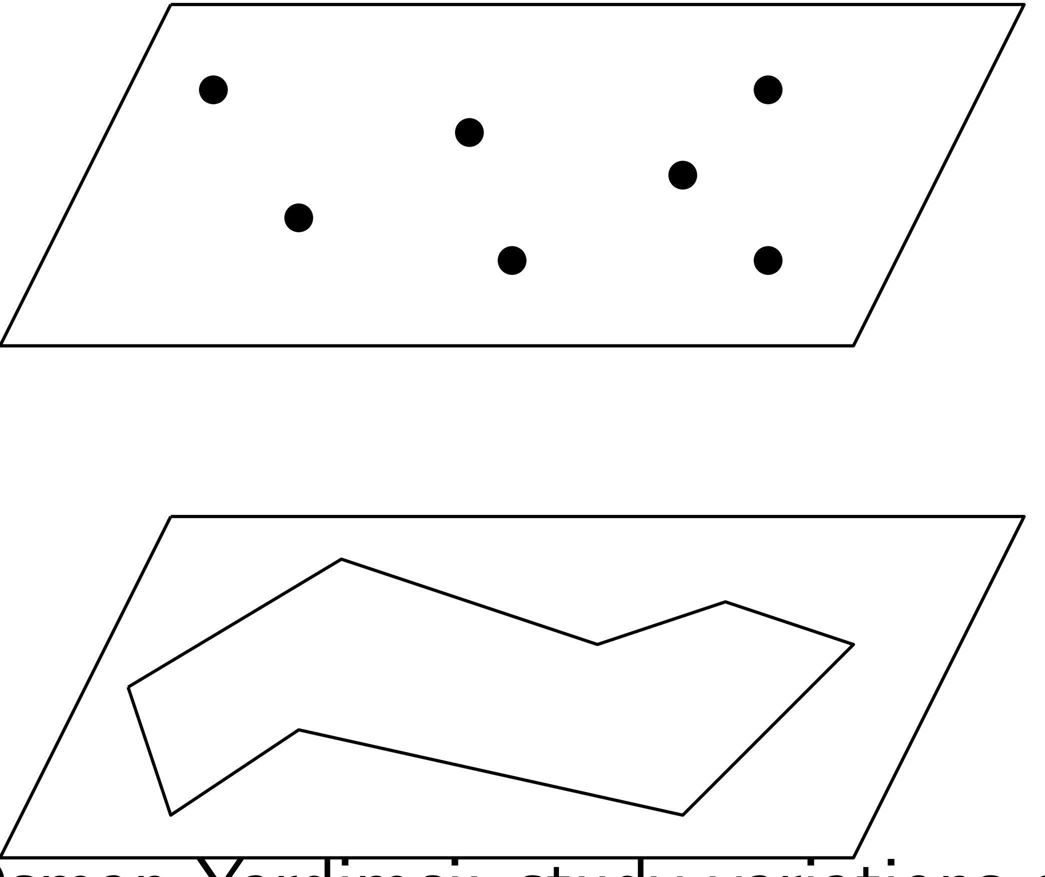


Image from Lajos Szilasi

Connection to surface reconstruction: Polyhedronize two parallel simple simple polygons



Problem Toussaint at al:
Polyhedronize given set of n points
and a simple polygon



Joint work with Osman Yardimci: study variations of
this problem.

O'Rourke at all (2002)

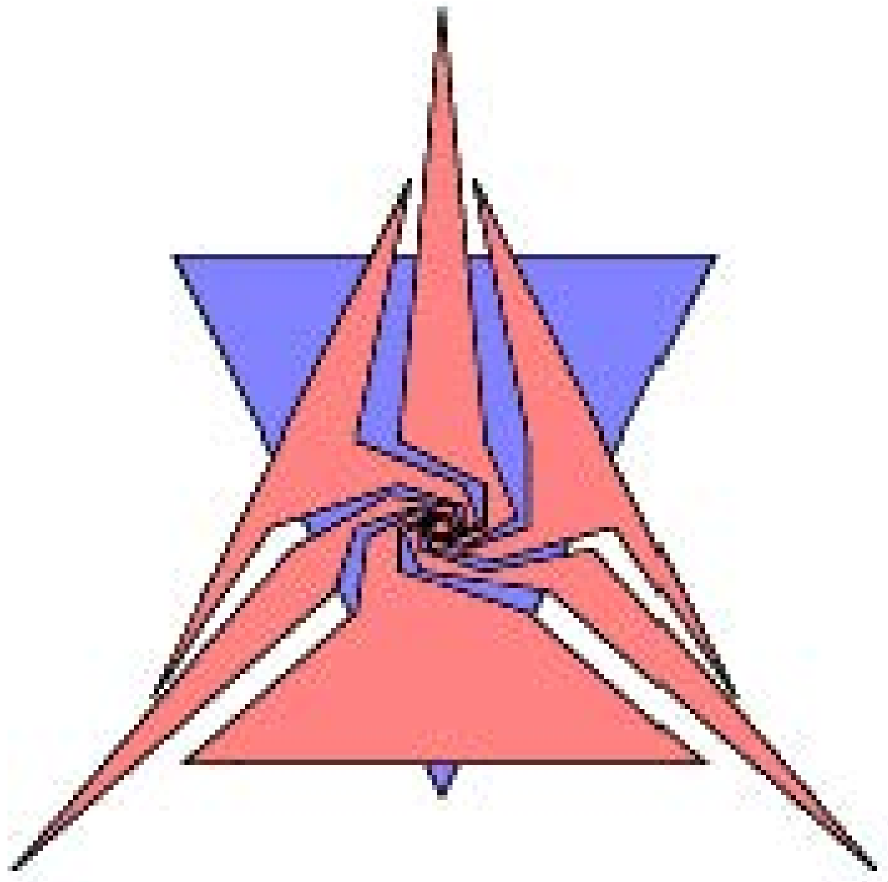
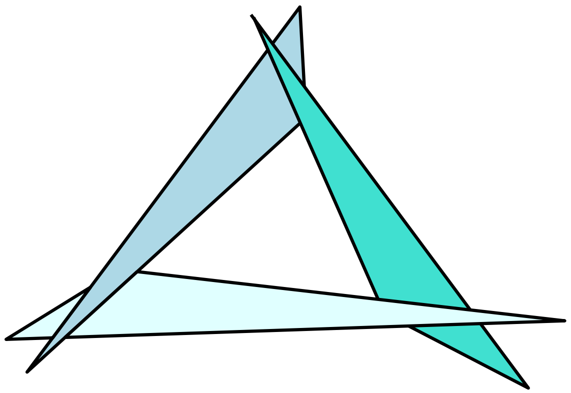
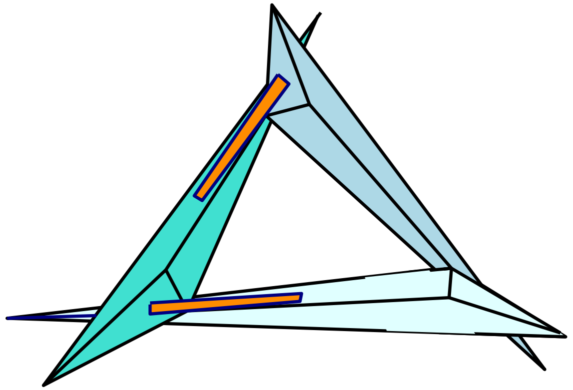


Image: Jeff Ericson

It is possible that a point outside of the convex hull of the polyhedron does not see any complete edge or any coplete face.

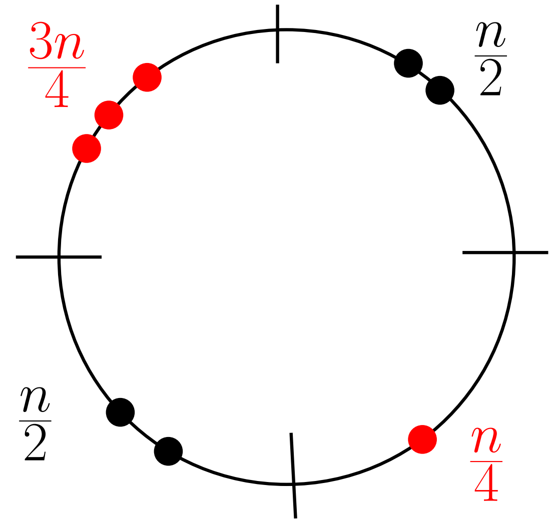


Front view



Back view

Coloured polygonalization/ polyhedralization
Long alternating path problem

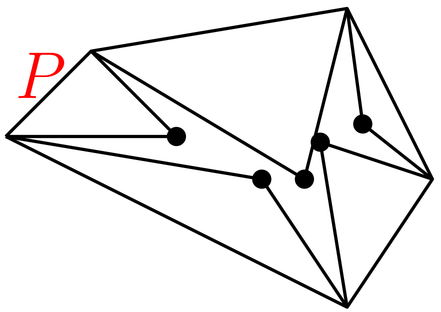
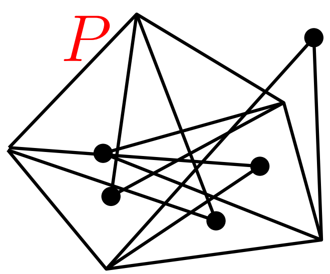



Erdos conjecture:
$maxvertexnumber \leq \frac{3n}{2} + 2$

Kynel,Pach,Toth (2006):
Thm: $maxvertexno \geq n + c\sqrt{\frac{n}{\log n}}$
Constr: $maxvertexno \leq \frac{4n}{3} + c\sqrt{n}$

Merino, Saazar, Urrutia (2006):
Gave an algorithm to decide if alternating path exist in case points are in convex position.
If the red and bue points are separated by a line such path exists.

# Pair given points (n) to sides of a given n-gon.

| Authors: | Dim: | Δ's cannot overlap   | Δ's must cover P   |
|---|---|---|---|
| Bogomolnaya, Nazarov, Rukshin (1988) | 2 | | **Points are in P:**<br>**Thm:** covering exists |
| A.Bezdek (2001) | 2,3 | **Points are in P:**<br>**Thm:** n Δ's exist<br>**Points-general pos. :**<br>**Thm:** n/3 Δ's exist,<br>**Conj.:** n/2 Δ's exist, | **Points, general pos. :**<br>**Thm:** covering exist |
| N.Karasev (2002) | n | **Thm:** Same as above in any dimension | **Thm:** Same as above in any dimension |