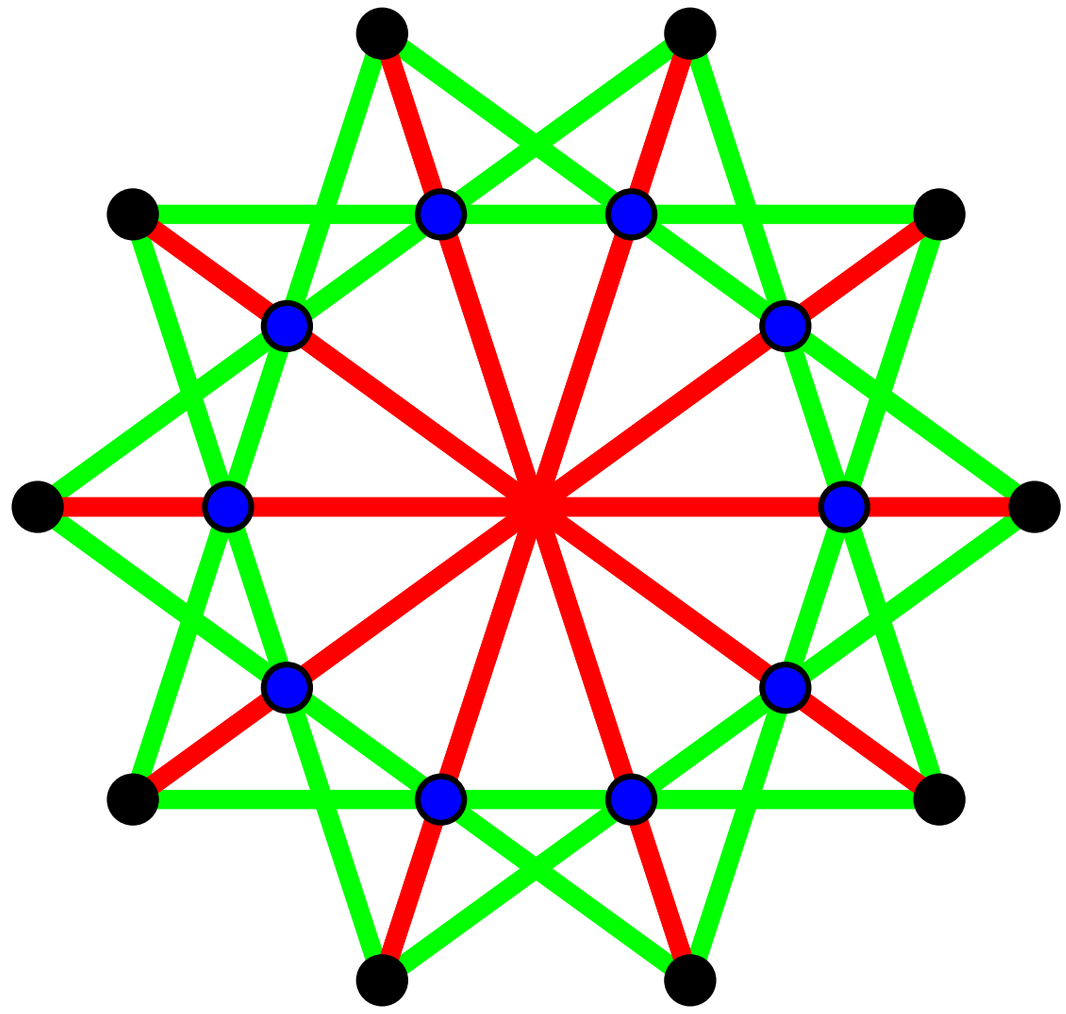
LEAH WRENN BERMAN
UNIVERSITY OF ALASKA FAIRBANKS
GEOSYM 2015—29 JUNE - 3 JULY 2015, VESZPRÉM, HUNGARY

SYMMETRIC MOVABLE CONFIGURATIONS

Supported in part by a grant from the Simons Foundation

GEOMETRIC CONFIGURATION

- *p* points
- *n* straight lines
- *q* lines/point
- *k* points/line
- (p_q, n_k) configuration; (*q*,*k*)-configuration
- (n_k) configuration; *k*-configuration



(20_3, 15_4)
(3,4)-configuration

GEOMETRIC CONFIGURATION

- *p* points
- *n* straight lines
- *q* lines/point
- *k* points/line
- (p_q, n_k) configuration; (*q*,*k*)-configuration
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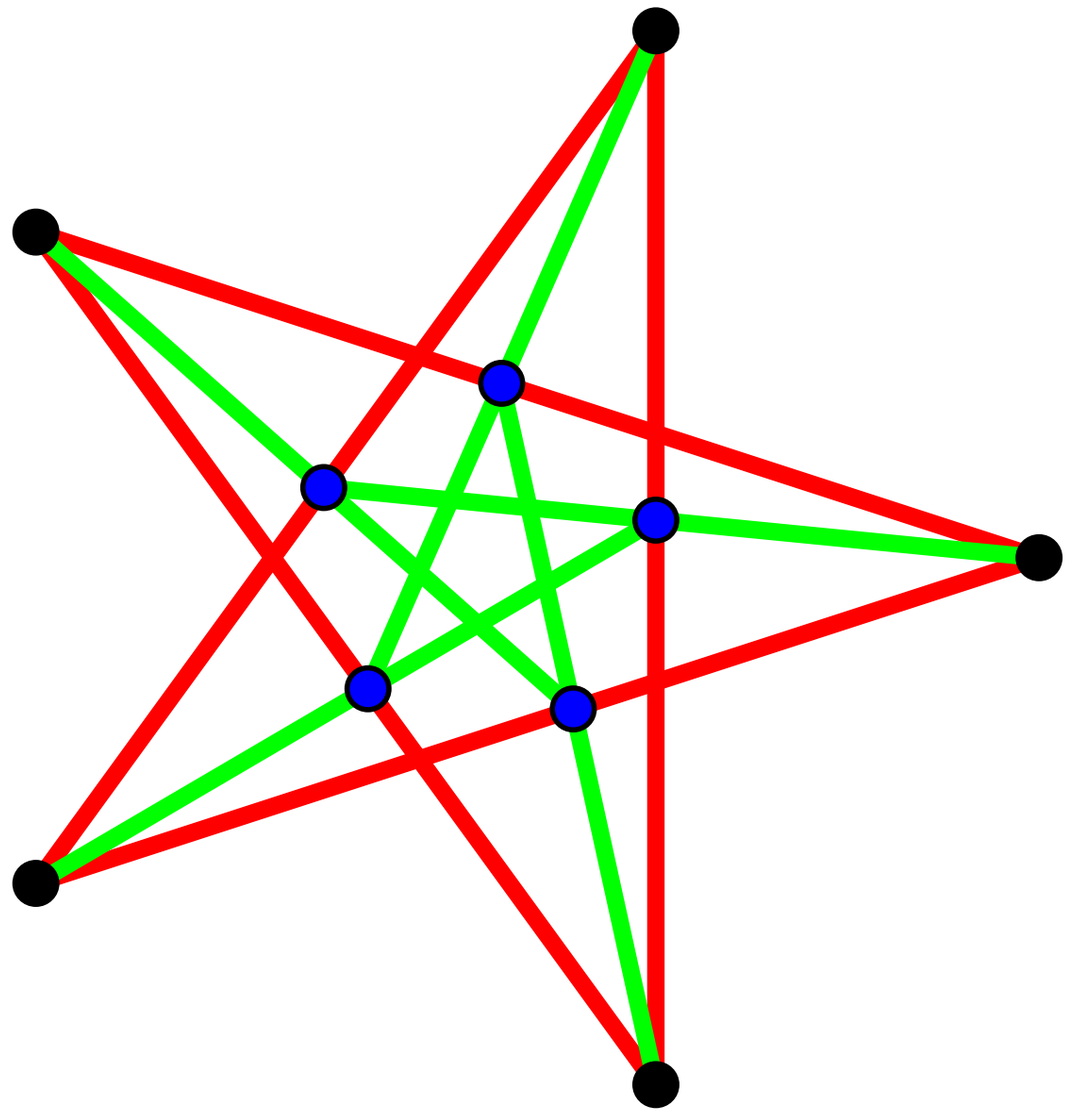
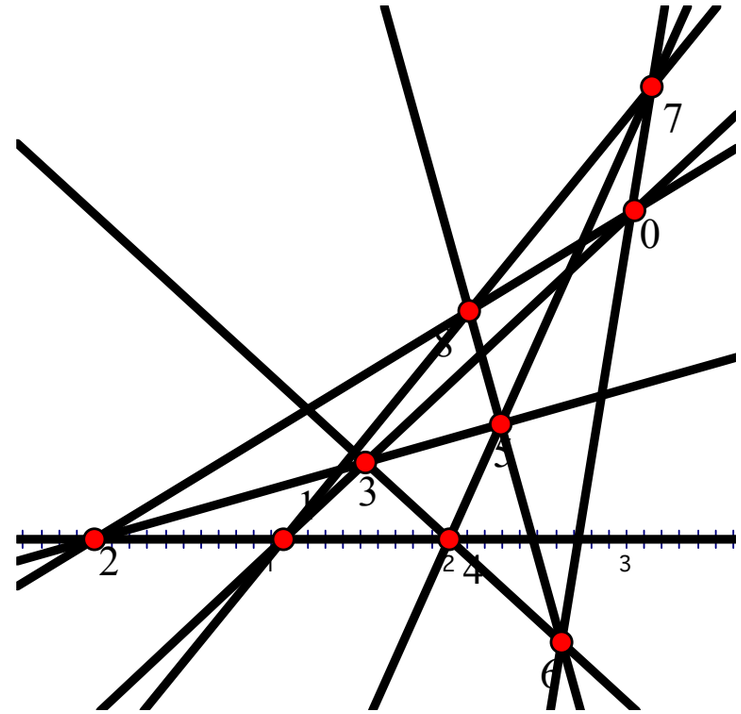
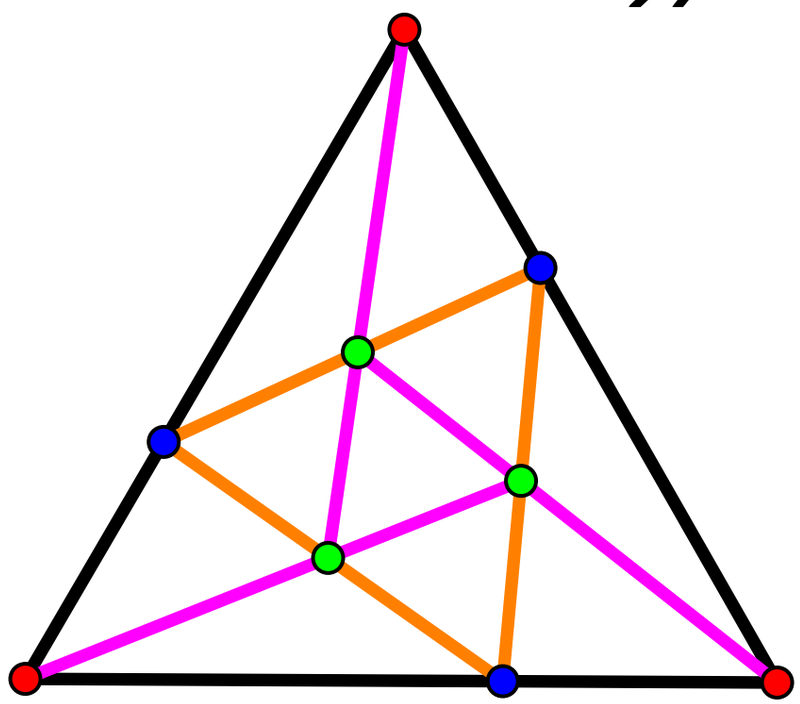


(10_3, 10_3) = (10_3)
3-configuration

SYMMETRIC CONFIGURATION

- isomorphism
- *balanced* vs. *symmetric*
- symmetry classes under isometries of the configuration





(9_3)

SYMMETRIC CONFIGURATION

- isomorphism
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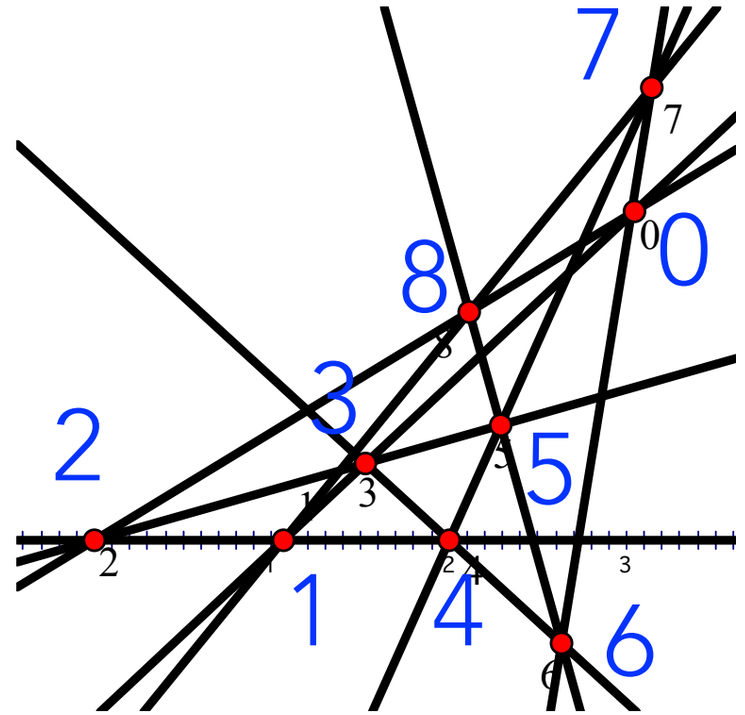

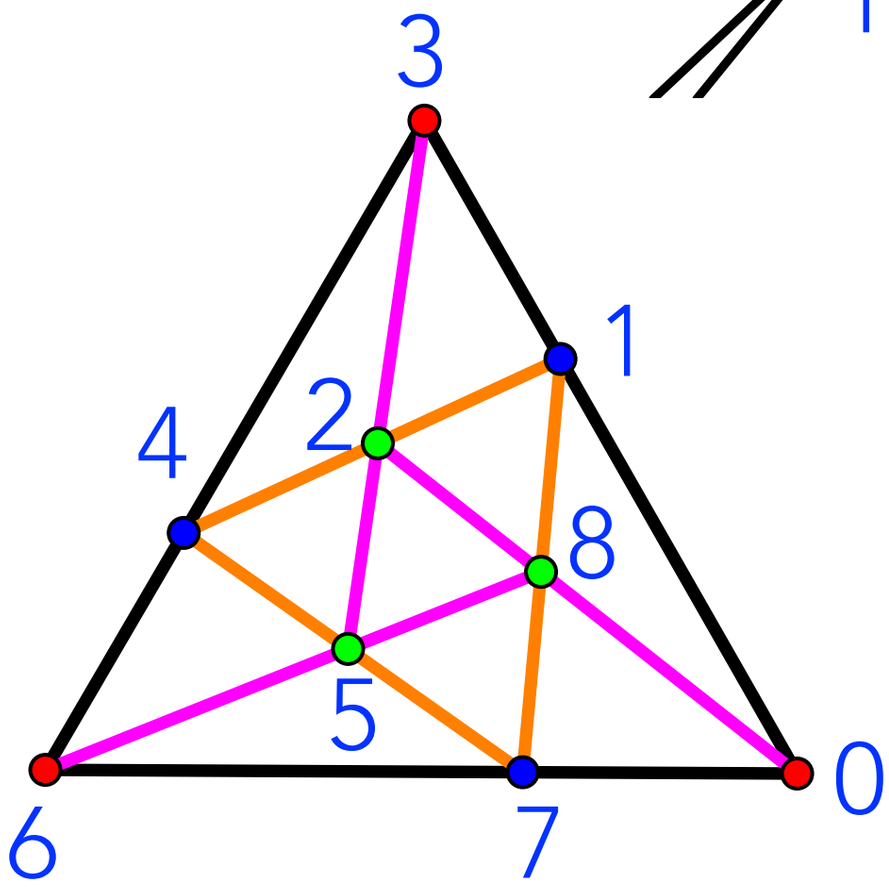

(9_3)

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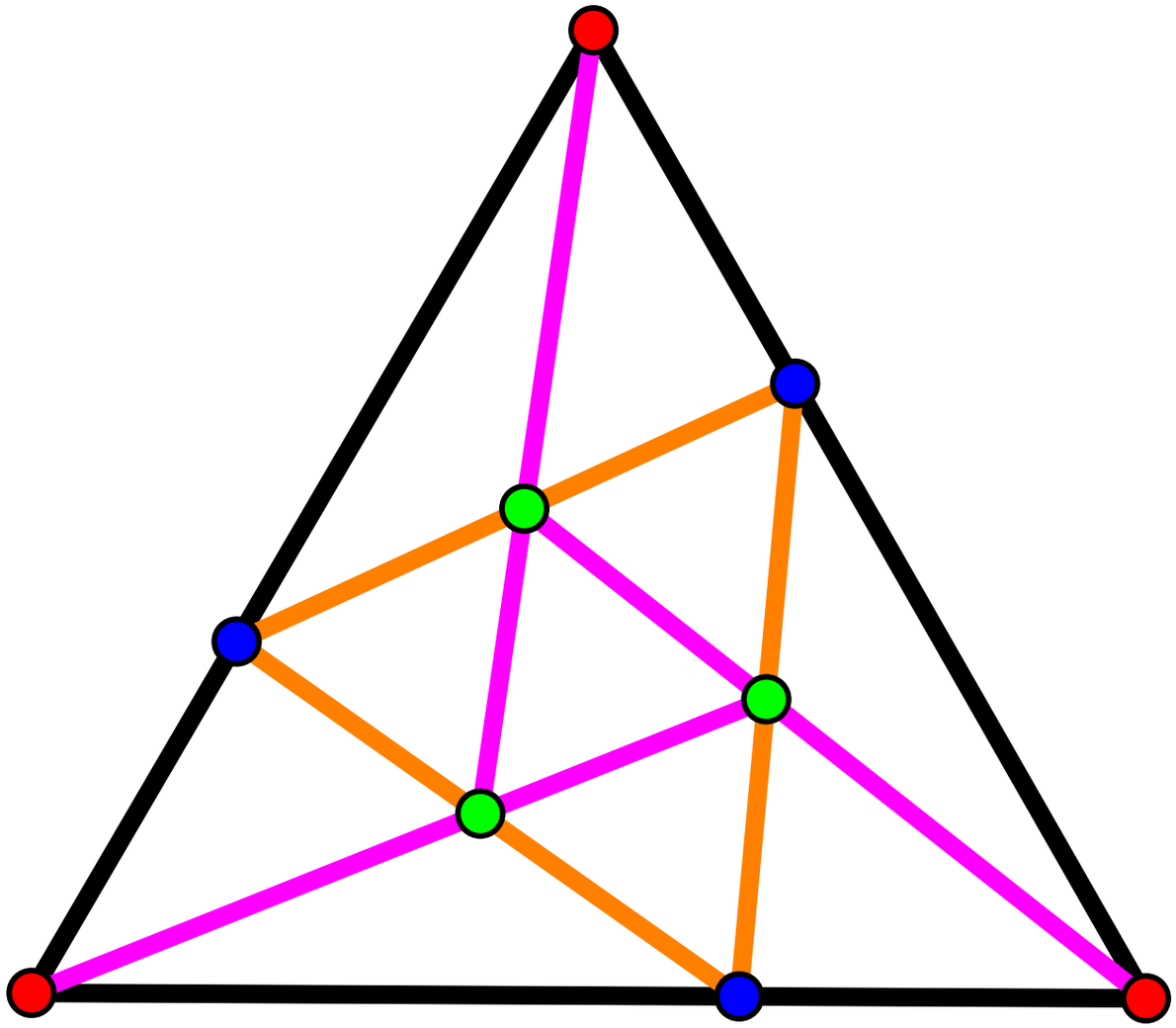

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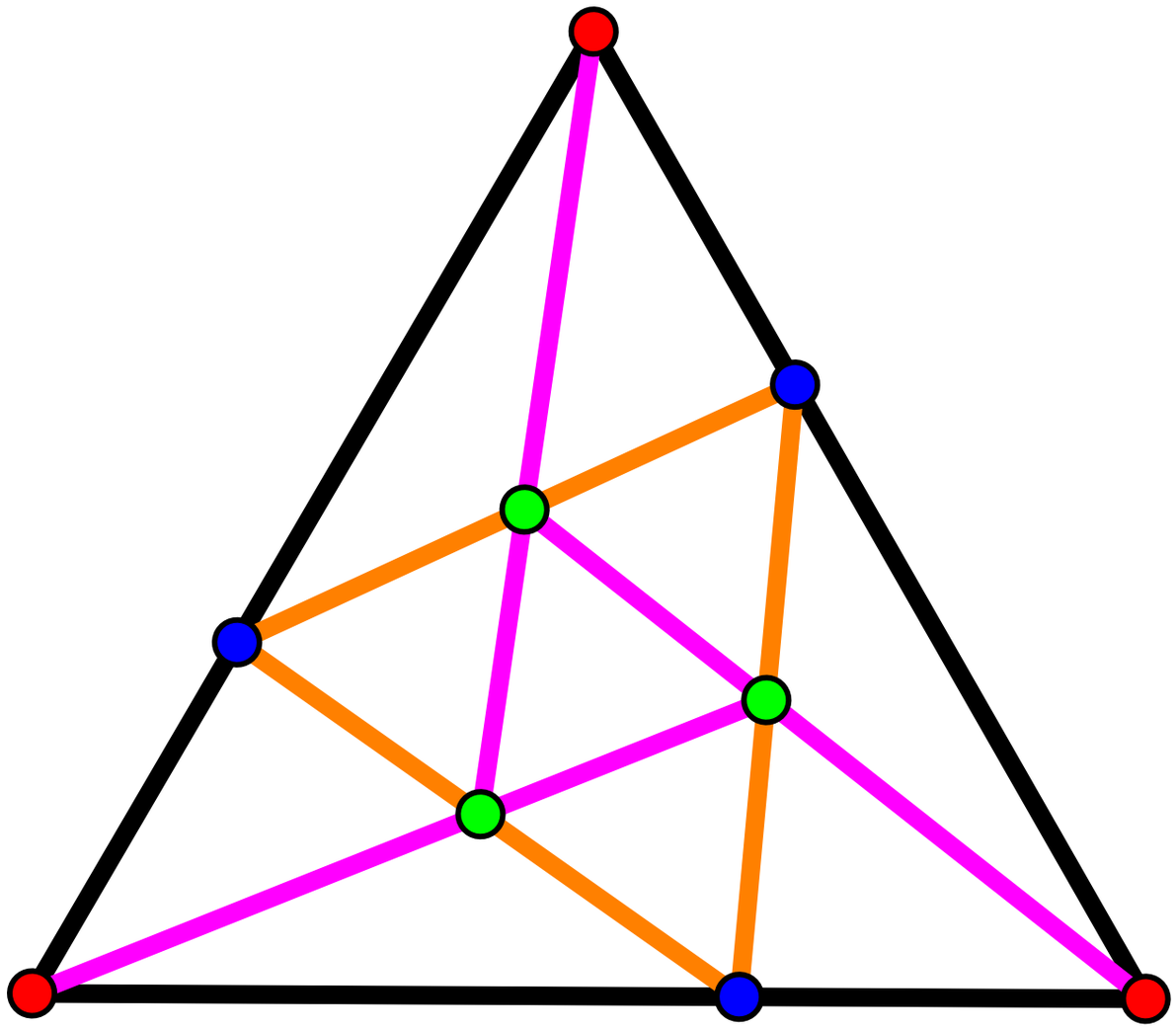

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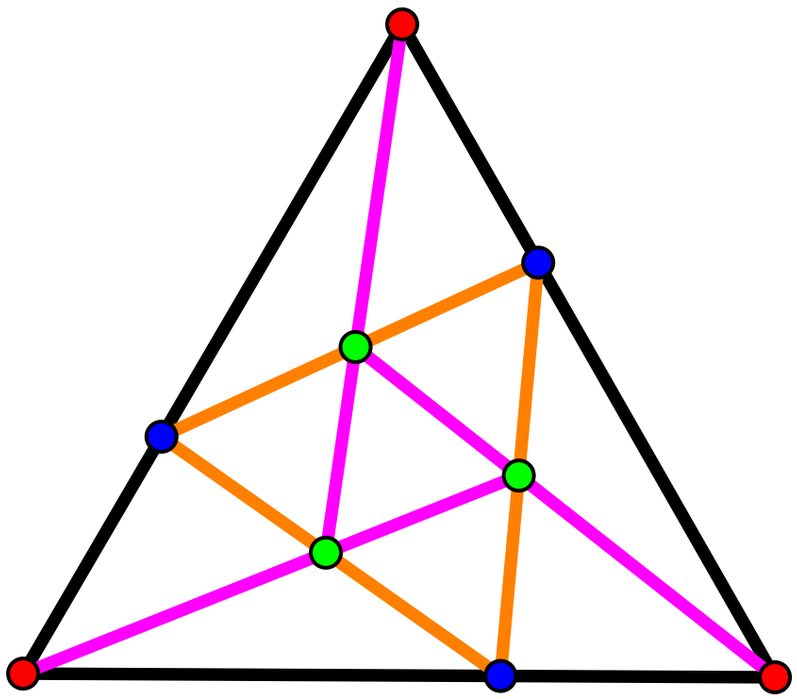
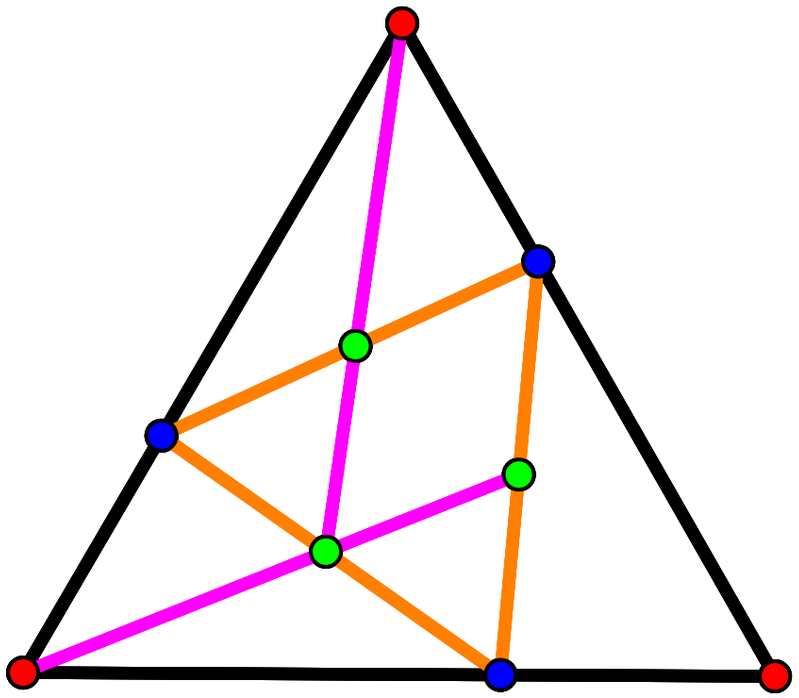


At least 4 points in general position are fixed, but a continuous parameter moves at least one point

MOVABLE CONFIGURATION



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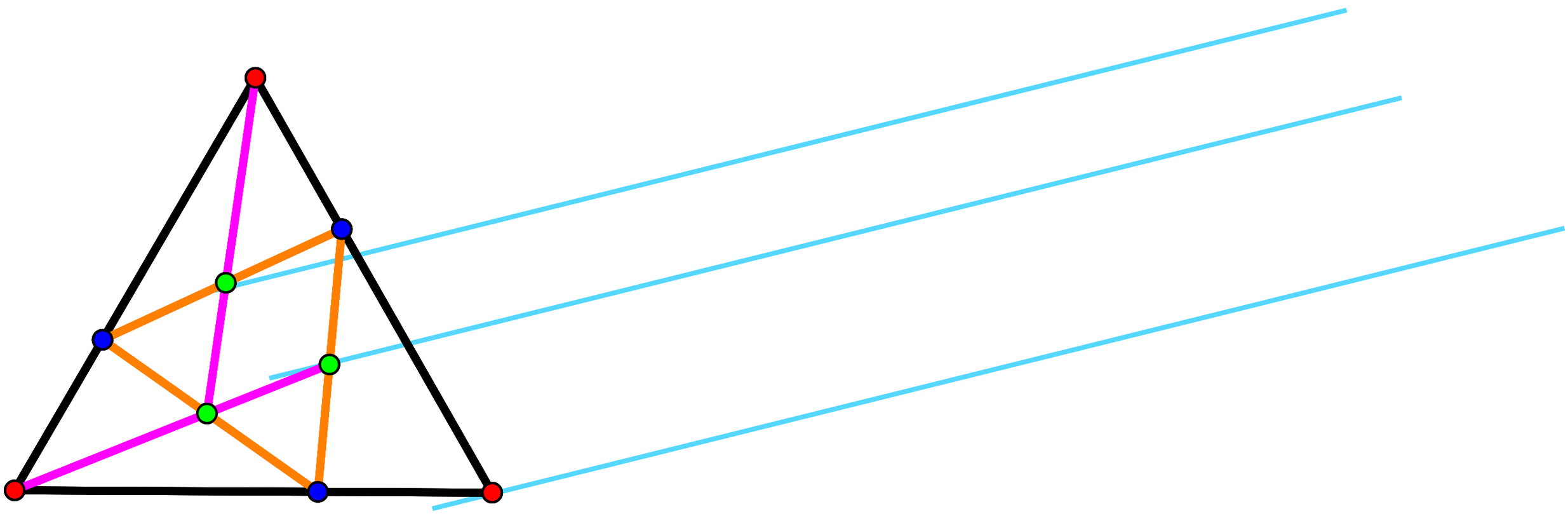

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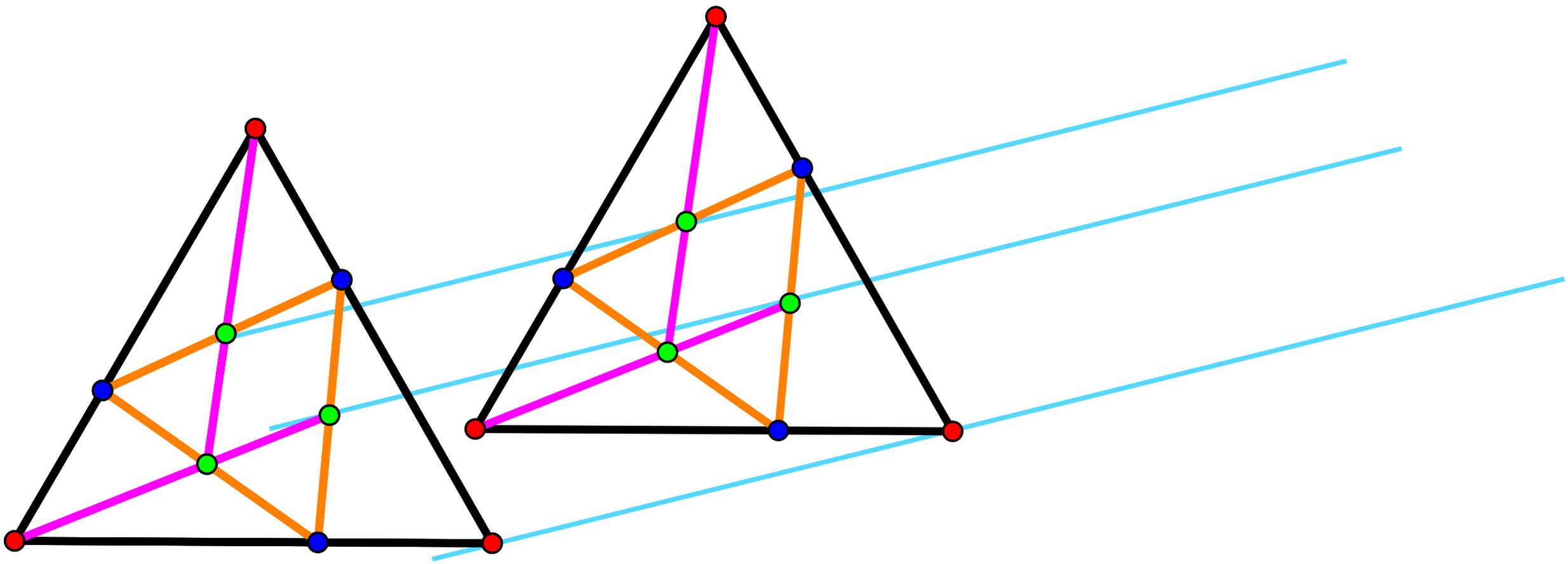

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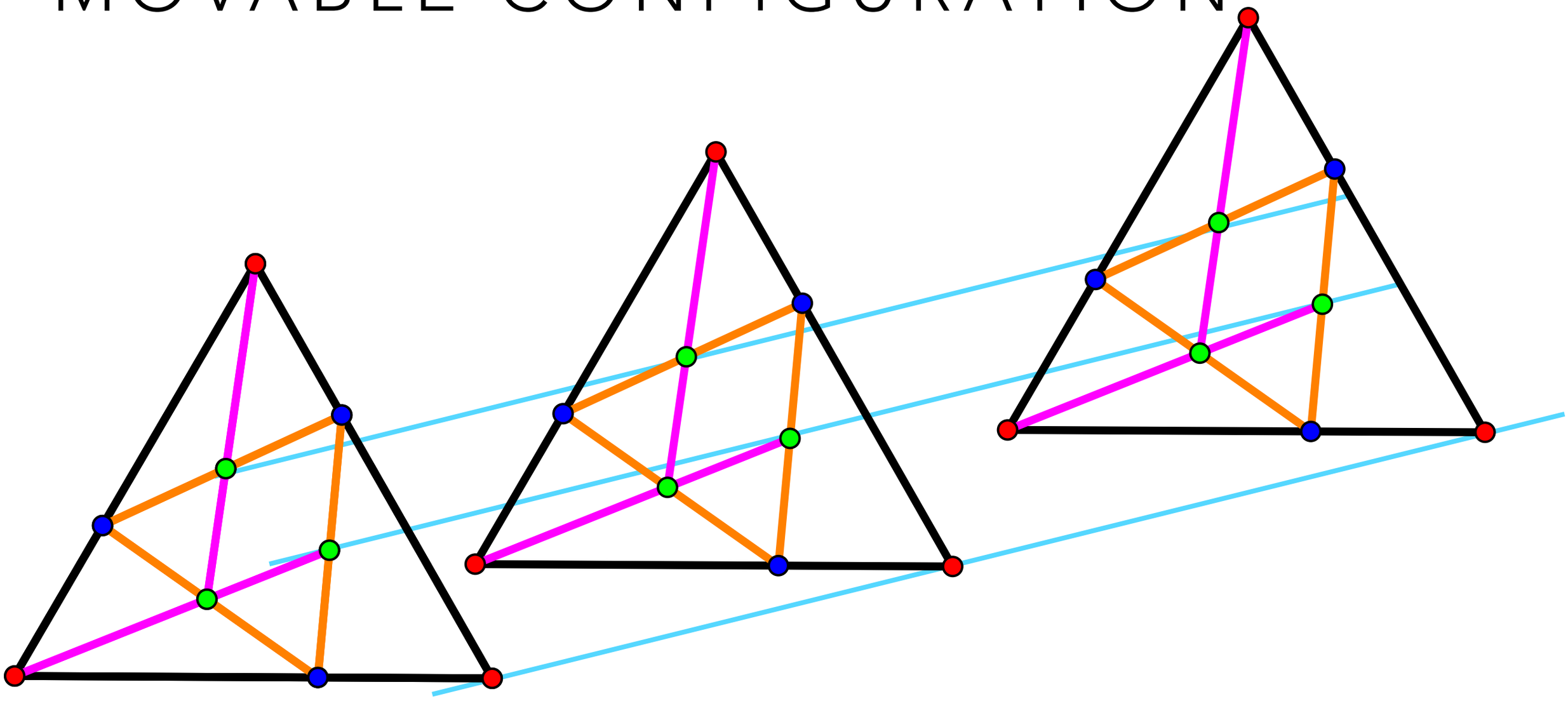

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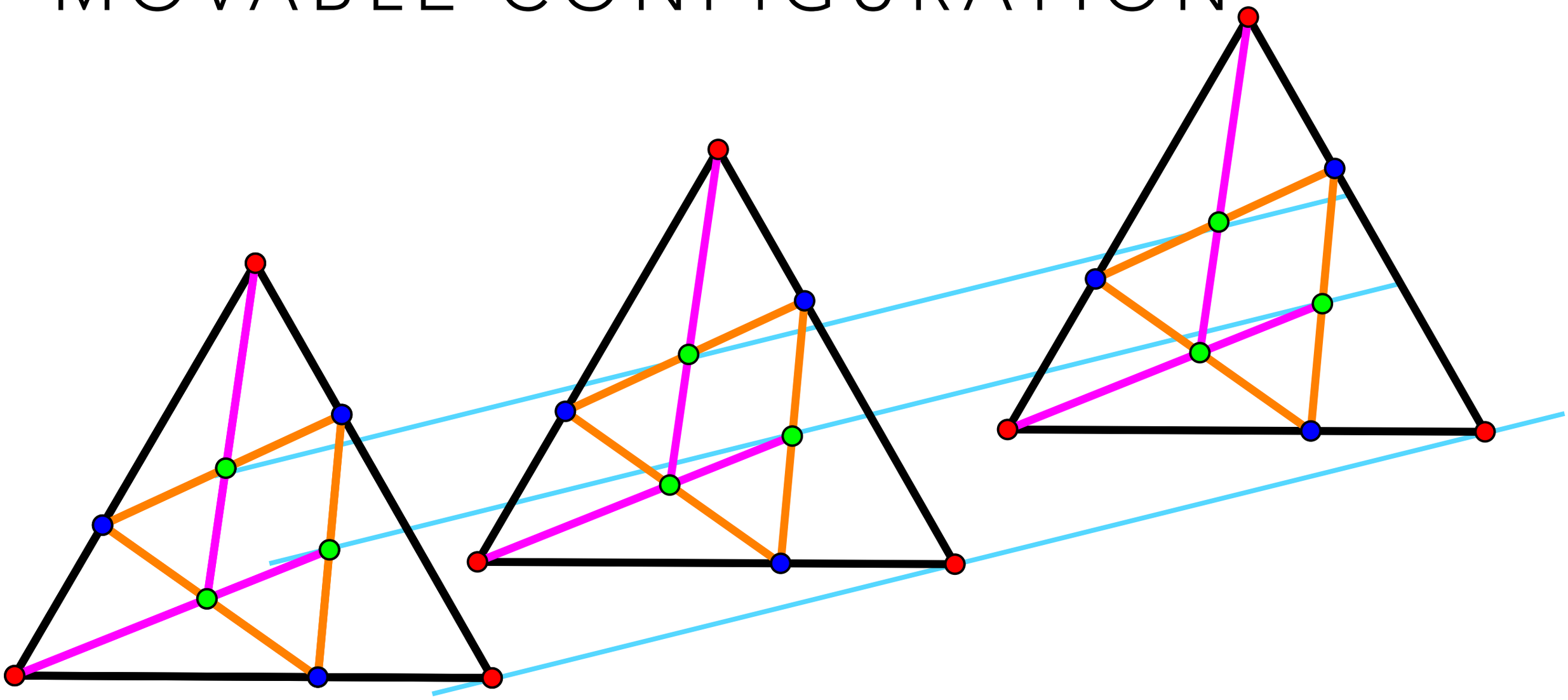

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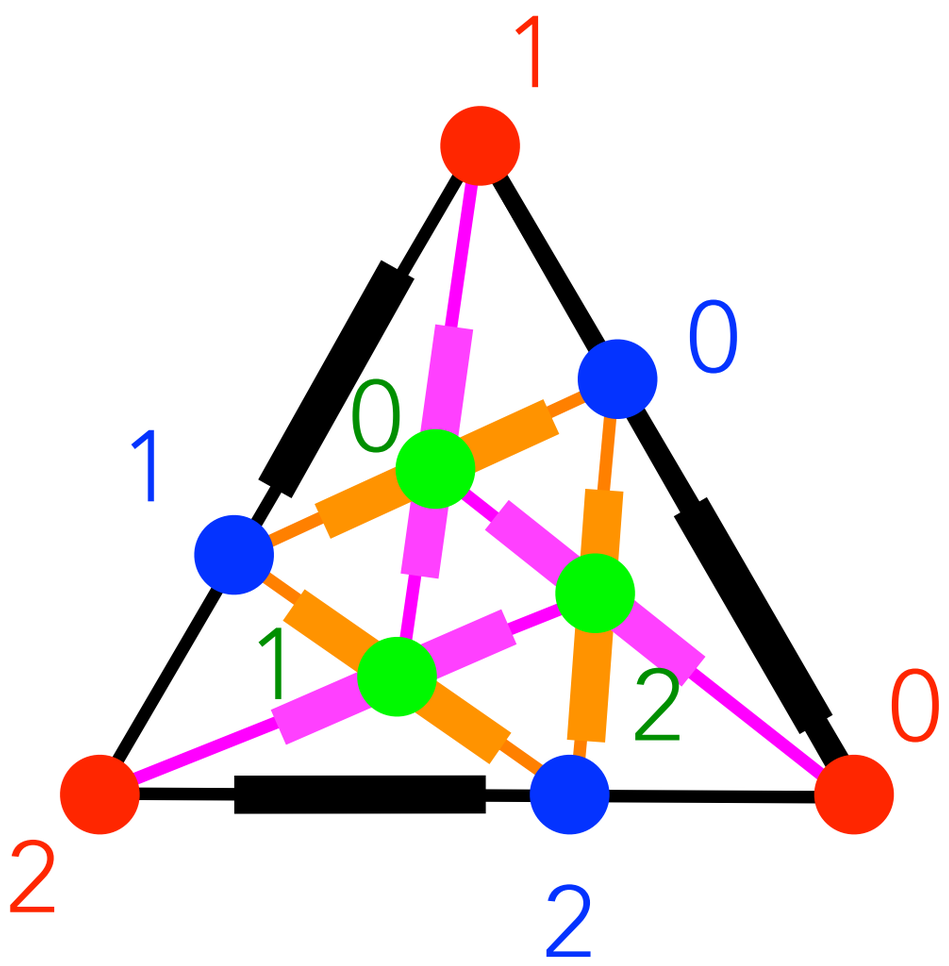
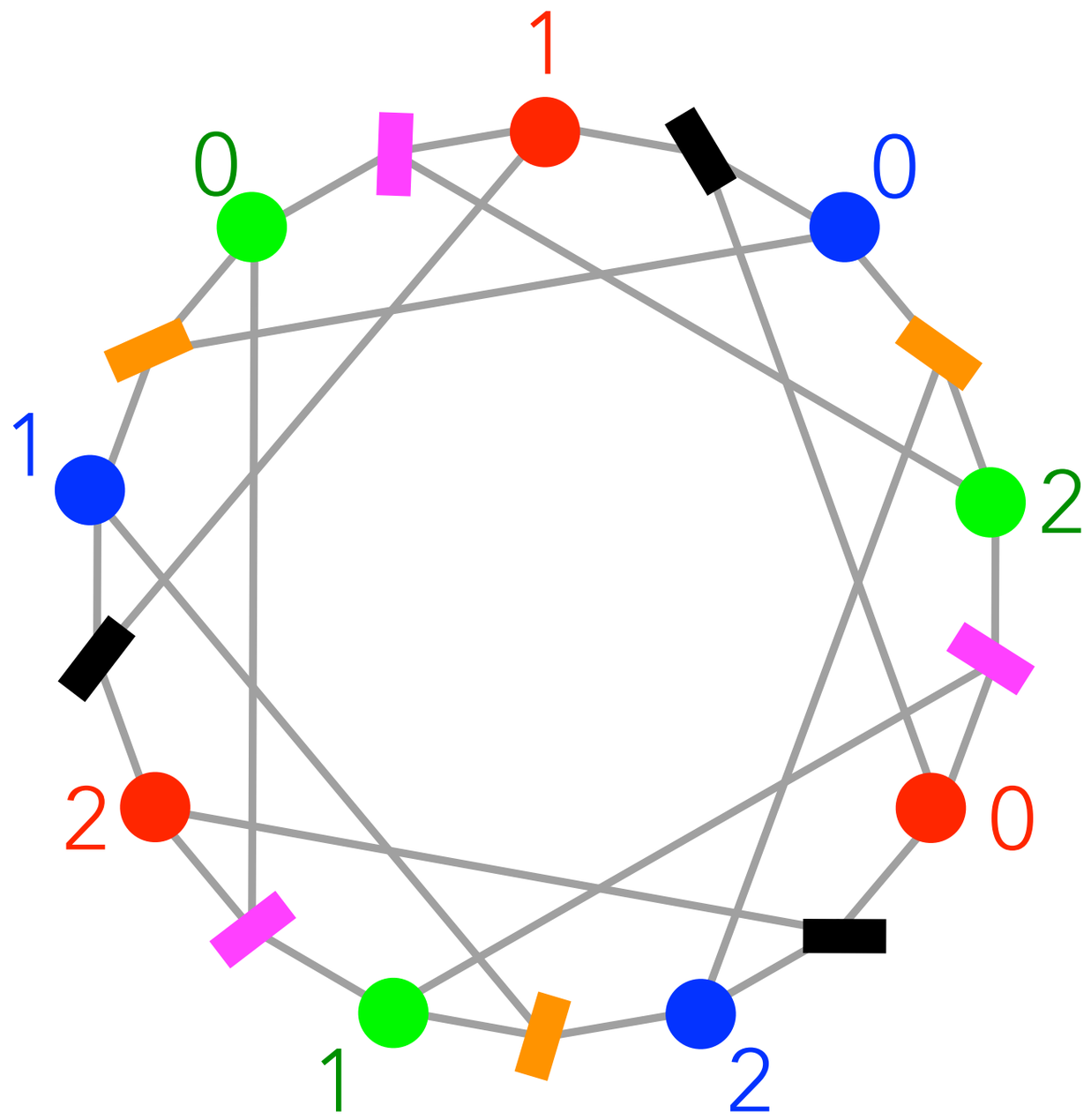
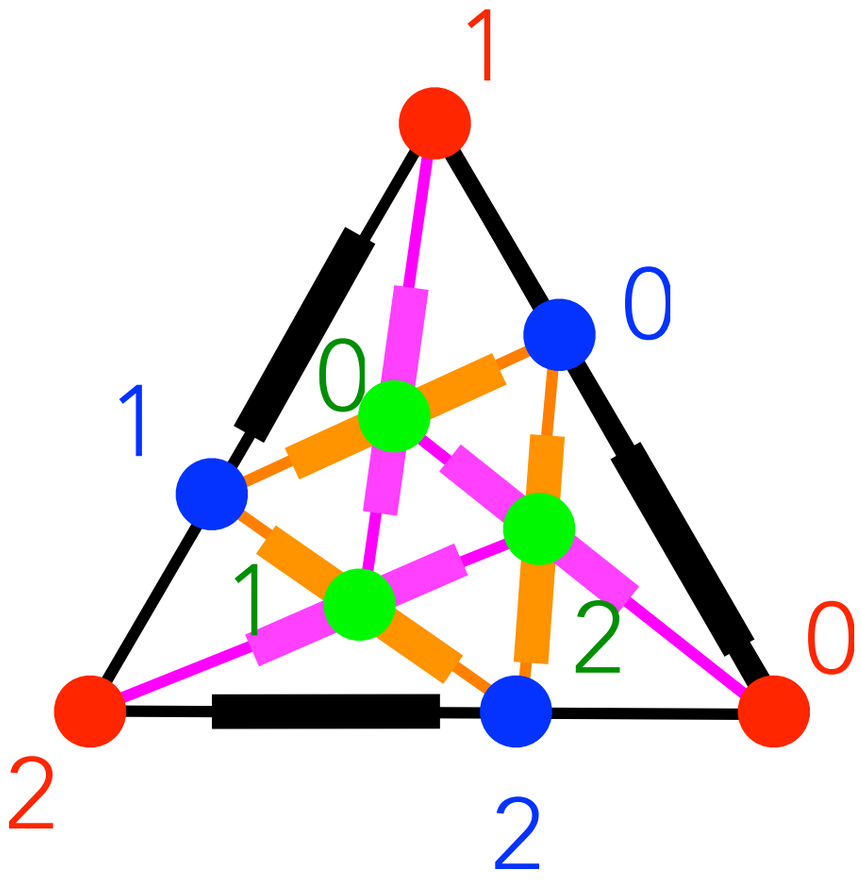
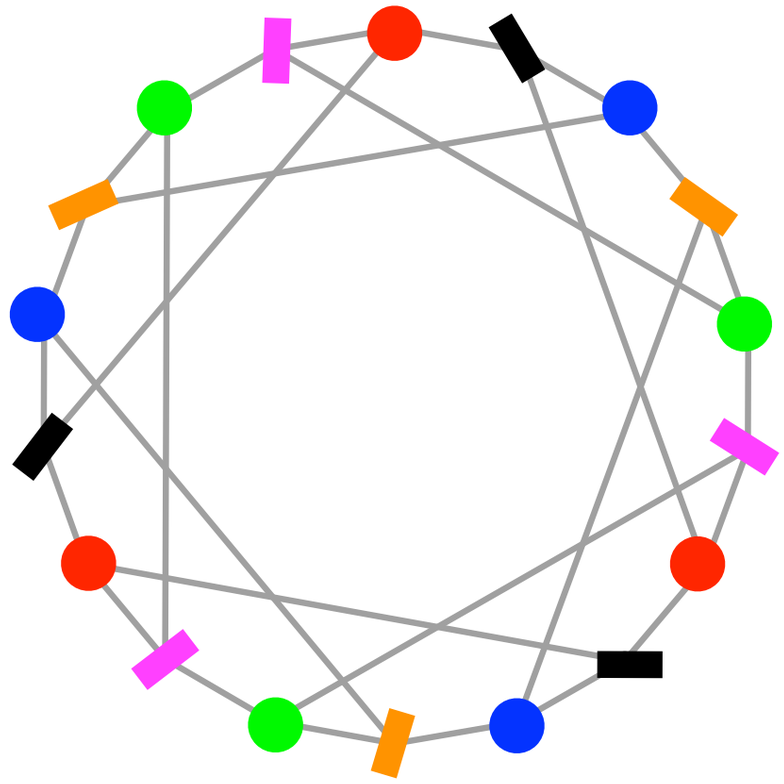
LEVI GRAPH

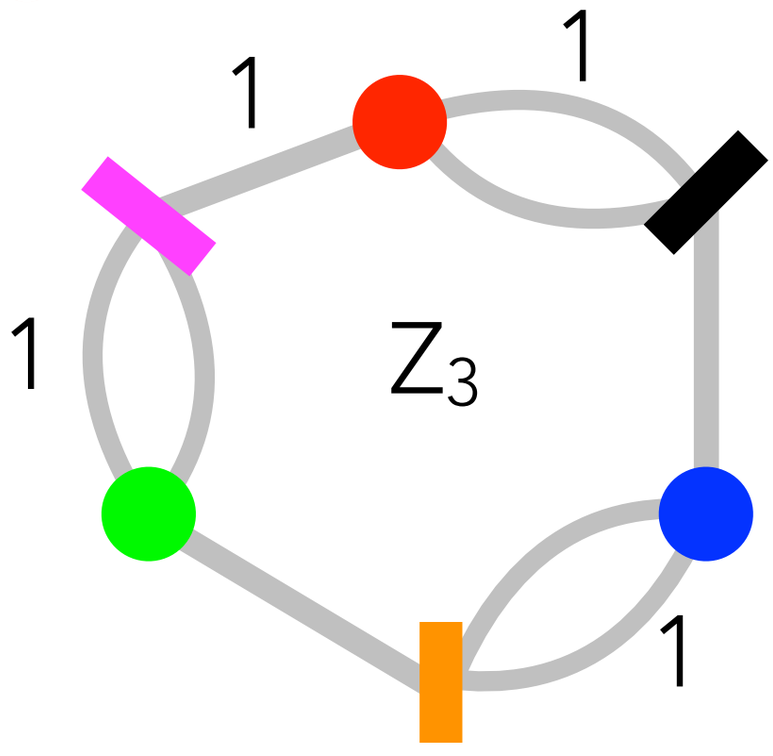


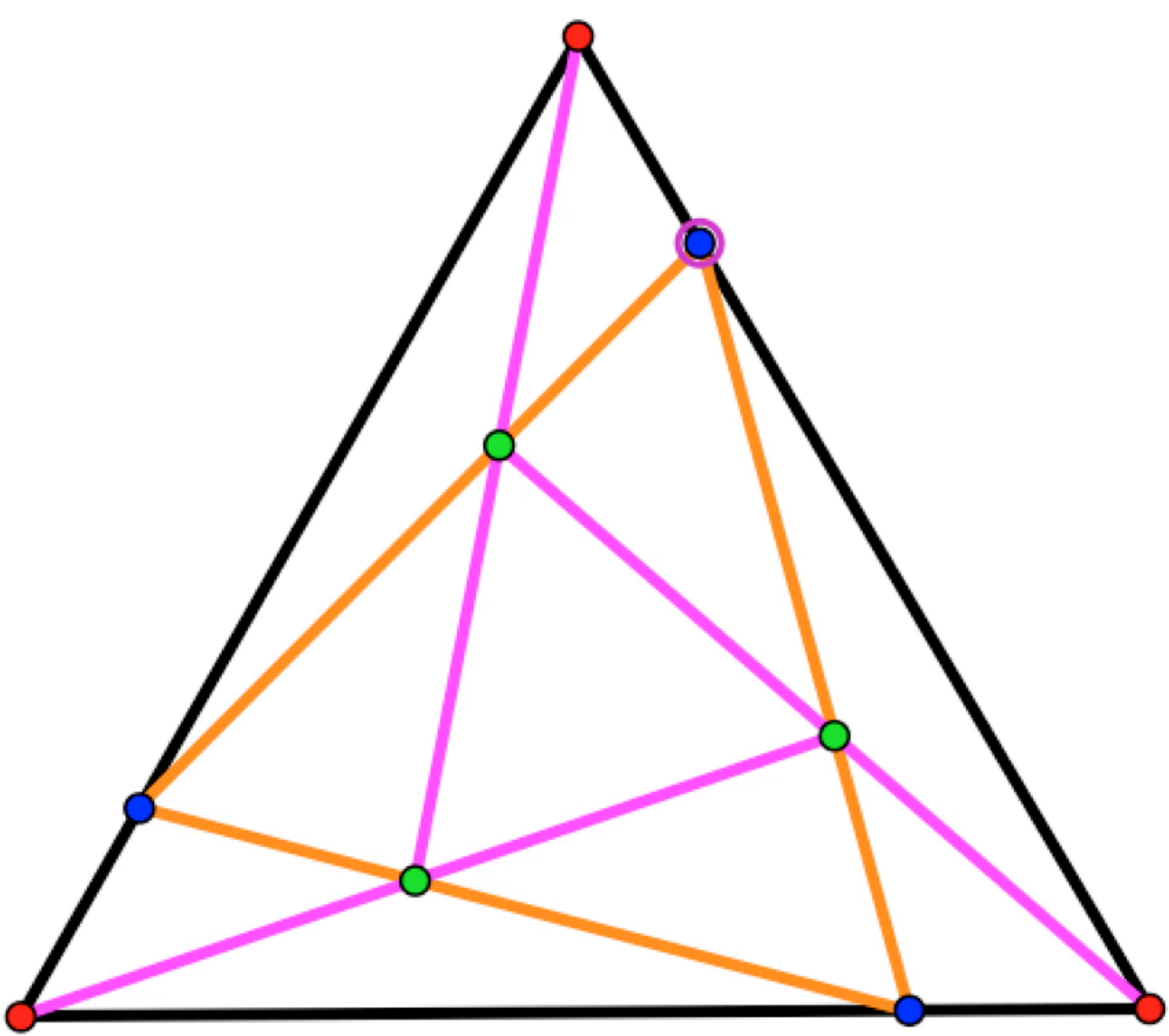


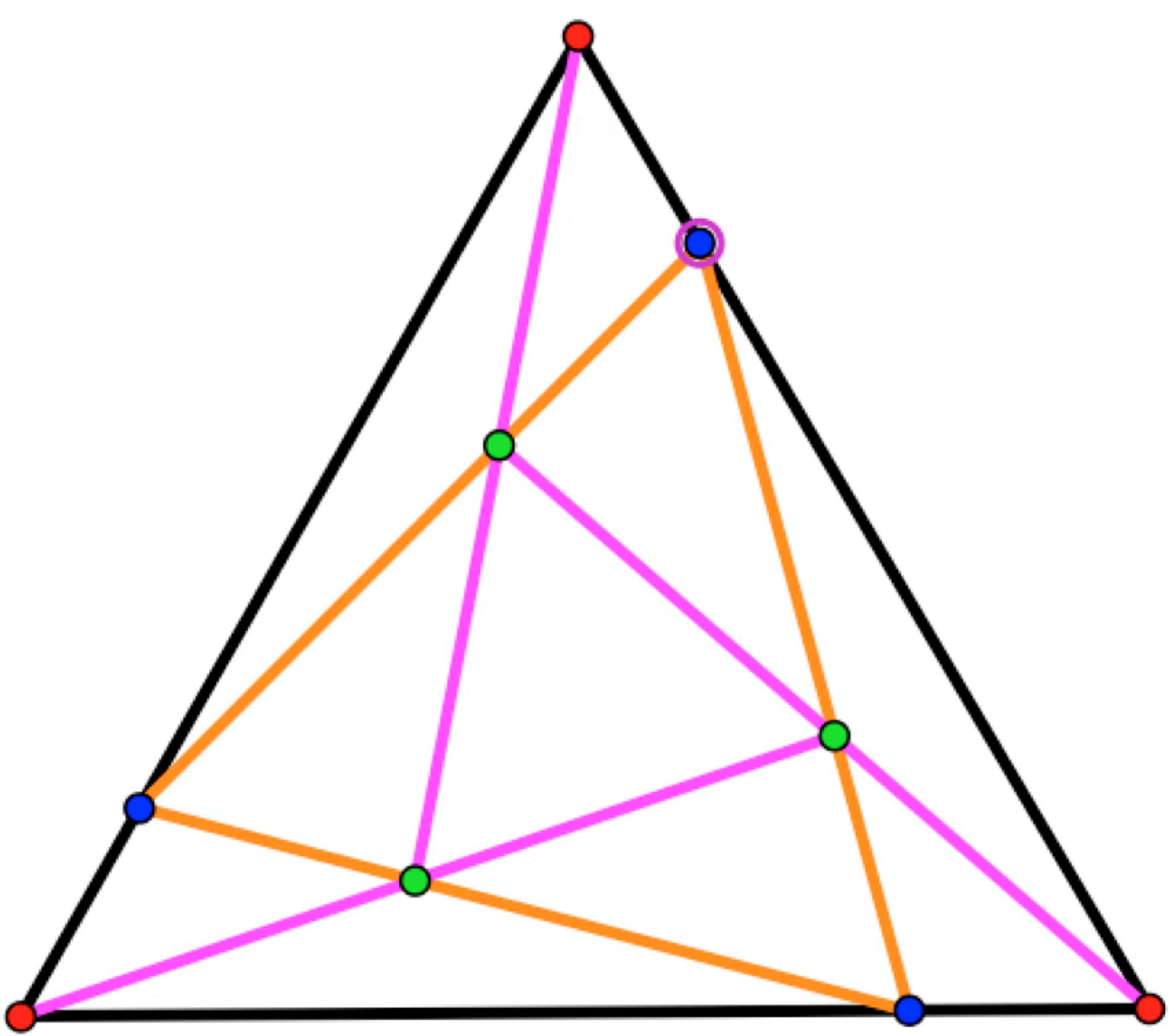
REDUCED LEVI GRAPH

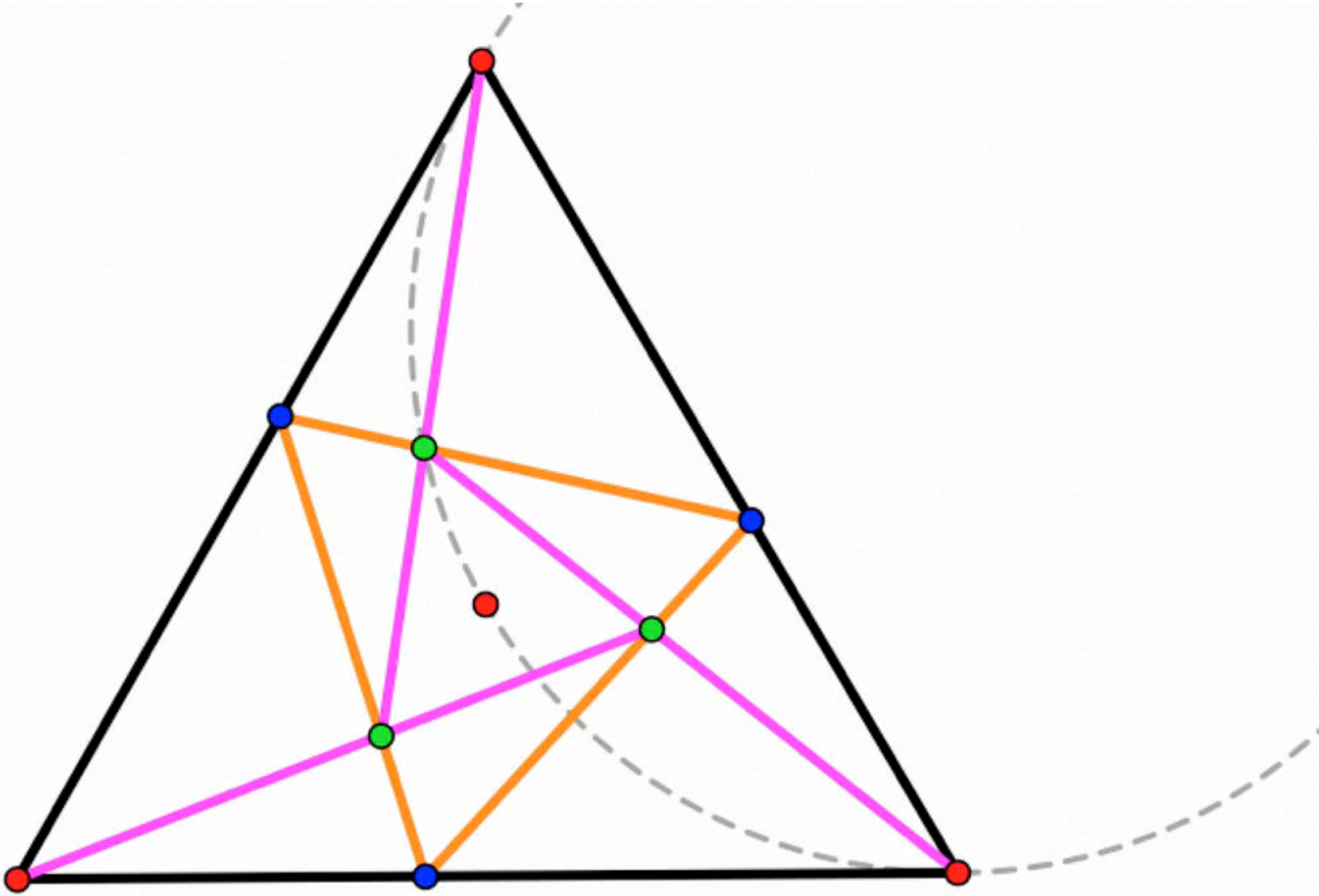


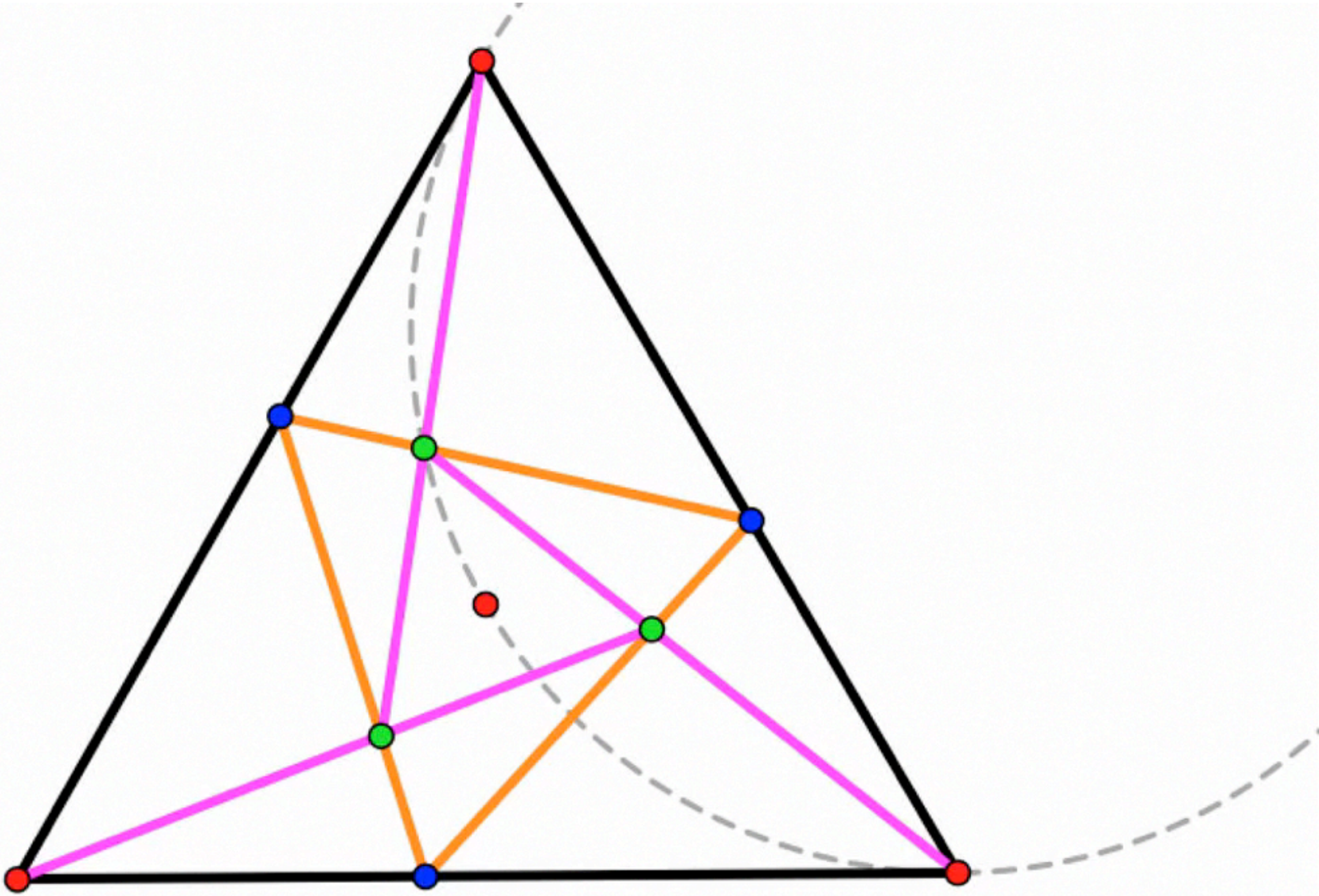




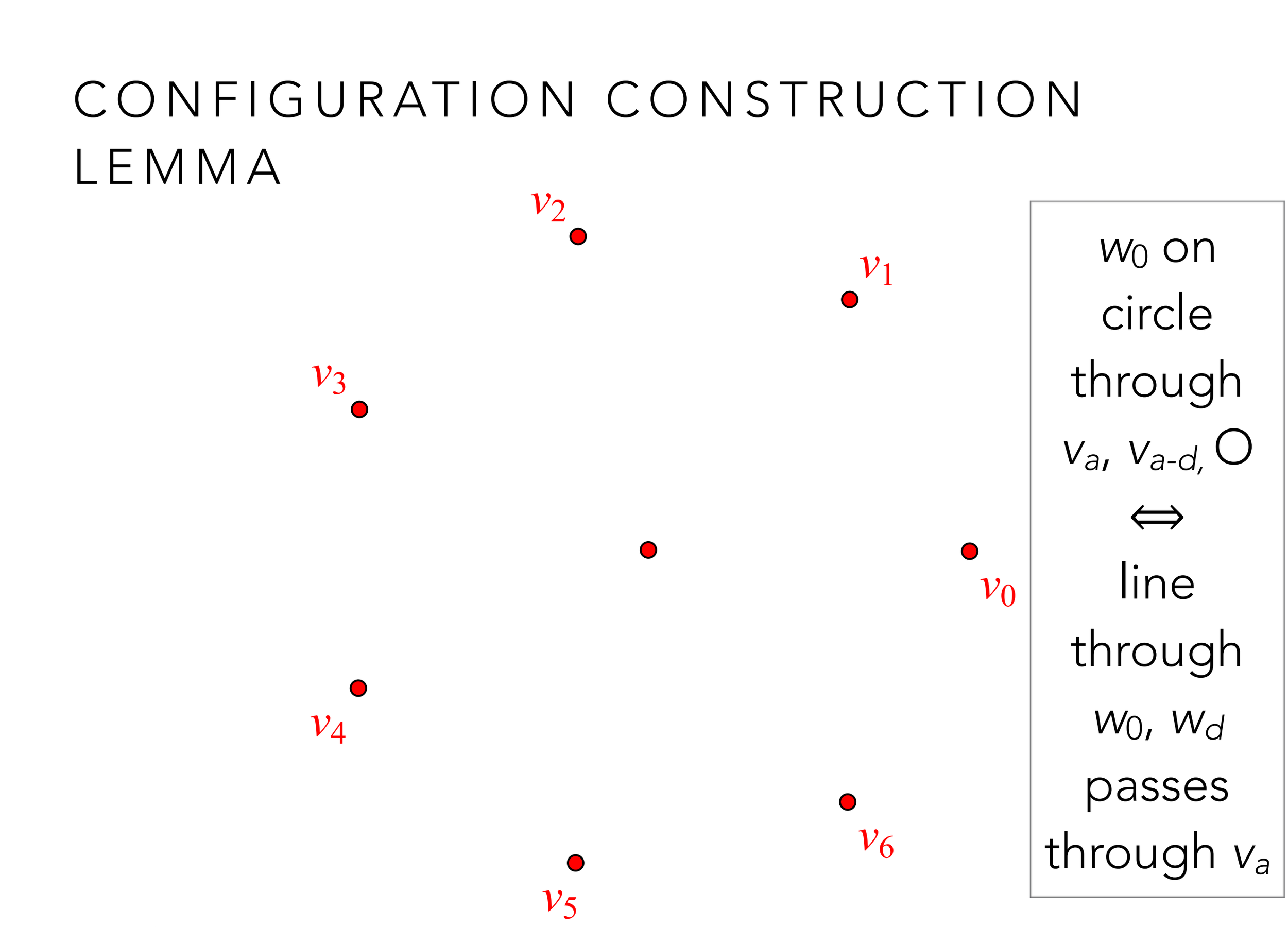








CONFIGURATION CONSTRUCTION LEMMA



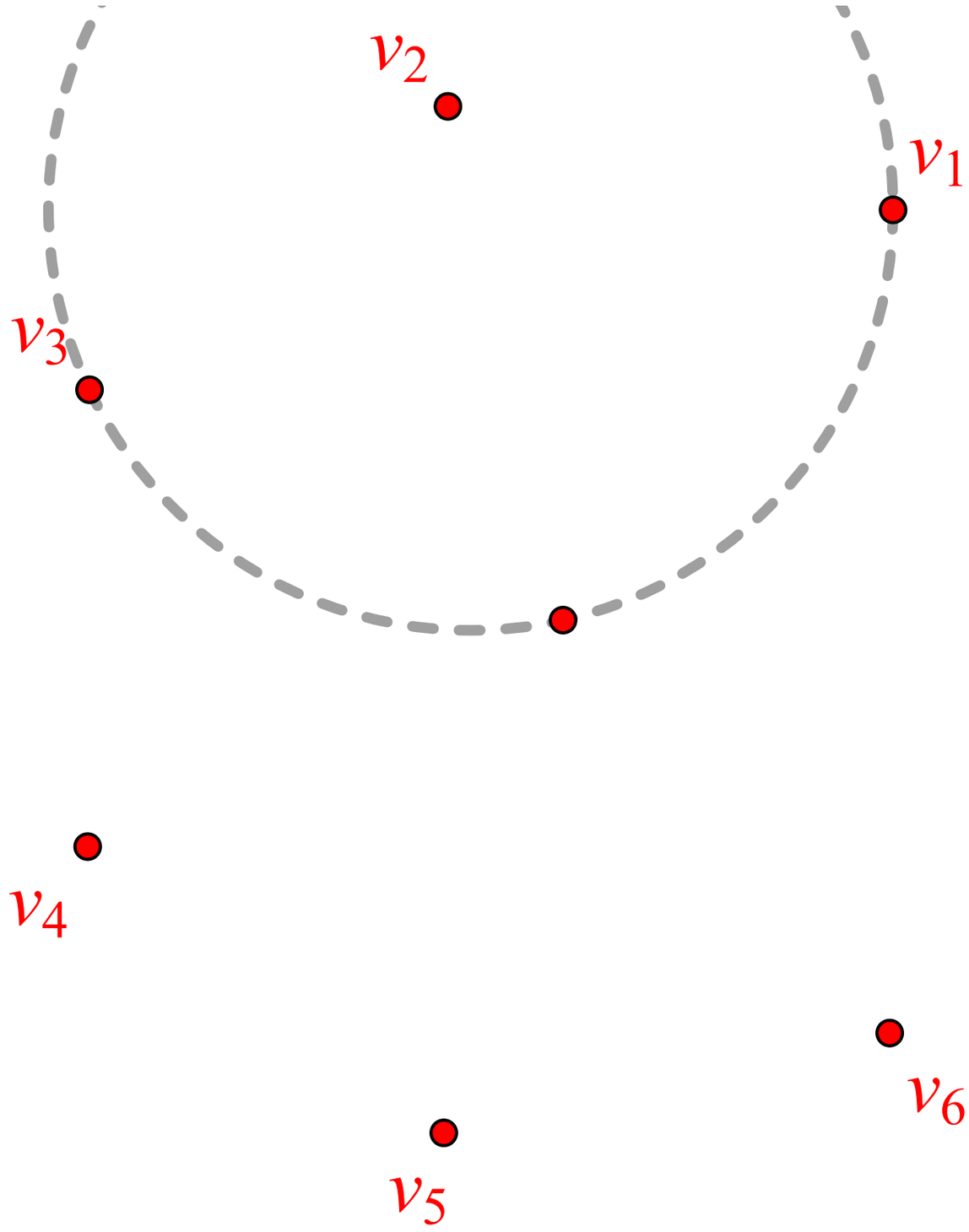
w_0 on circle through v_a, v_{a-d}, O

$\Leftrightarrow$

line through w_0, w_d passes through v_a

Grünbaum 2009, B. 2013

CONFIGURATION CONSTRUCTION LEMMA



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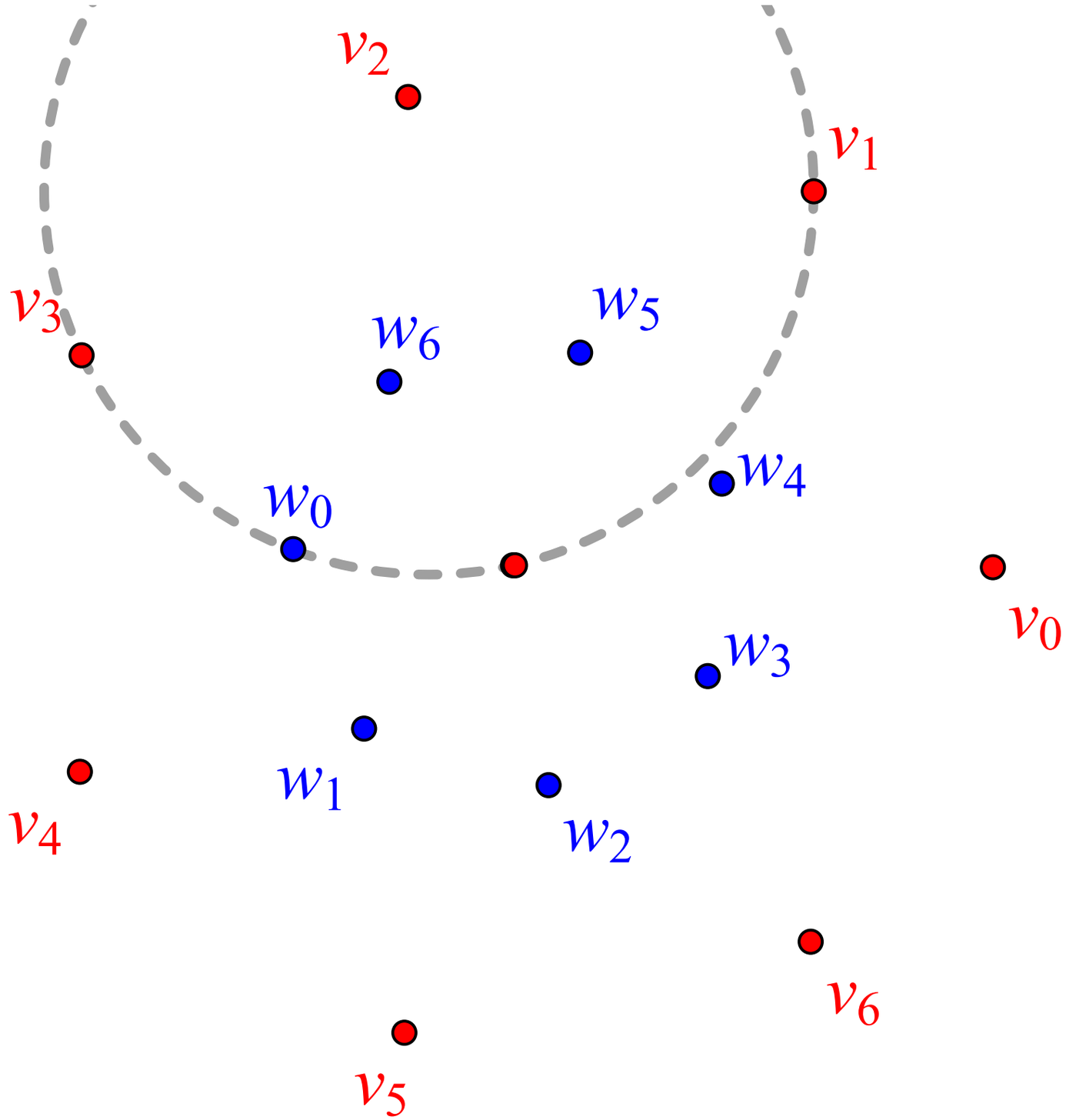

w_0 on circle through v_a, v_{a-d}, ○

⇔

line through w_0, w_d passes through v_a

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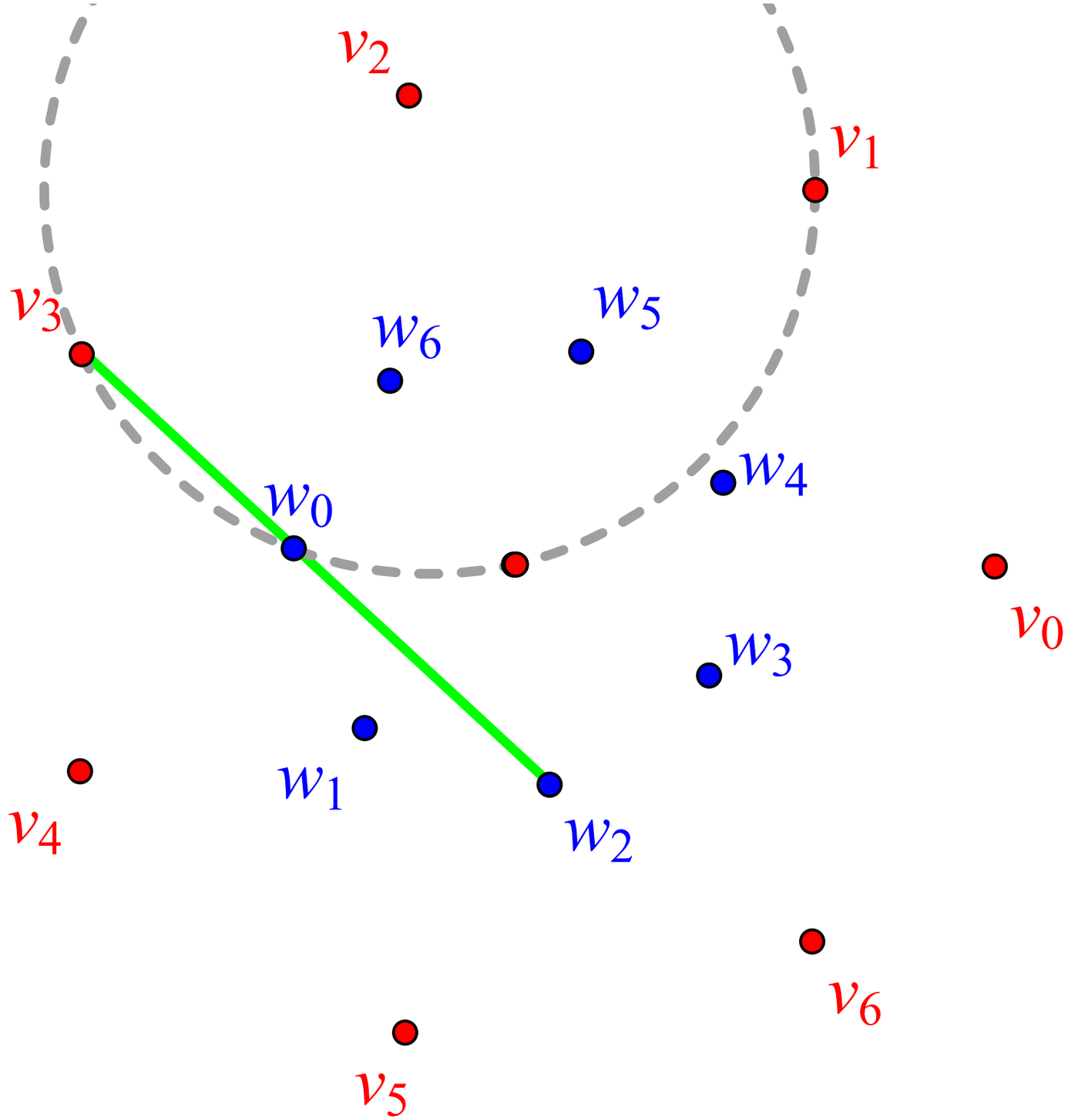

w_0 on circle through v_a, v_{a-d}, O

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Grünbaum 2009, B. 2013

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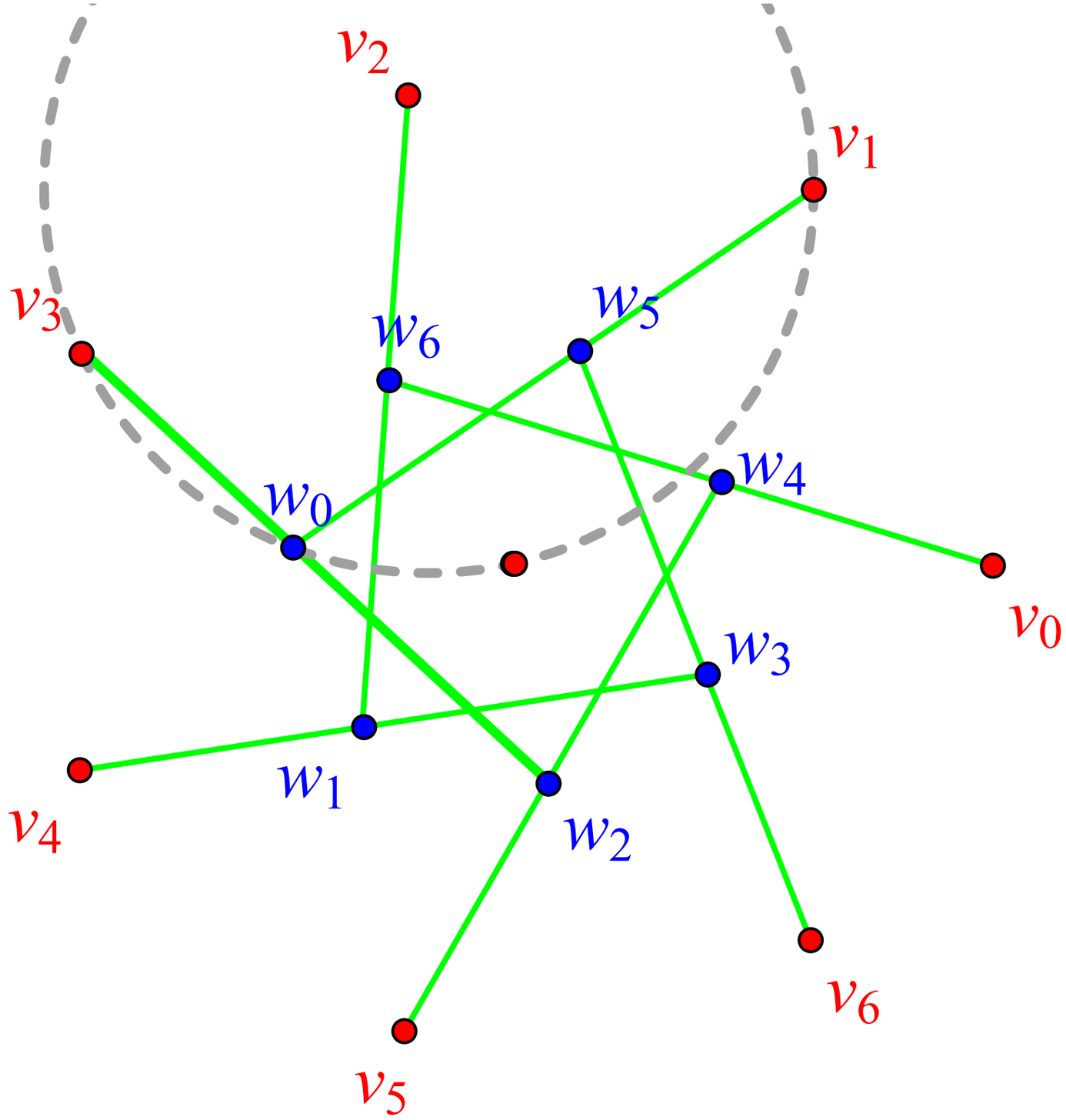

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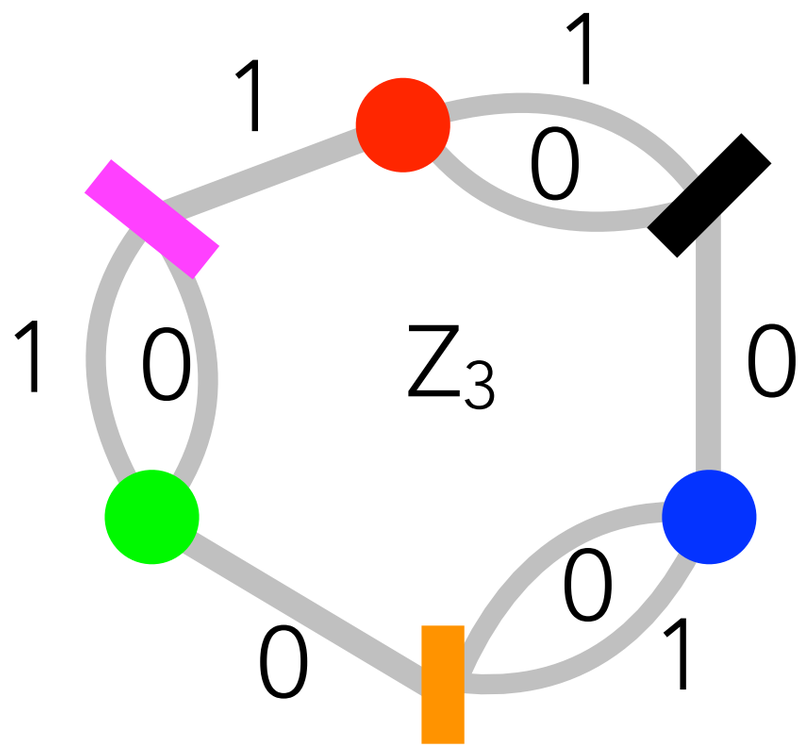
AN EXAMPLE



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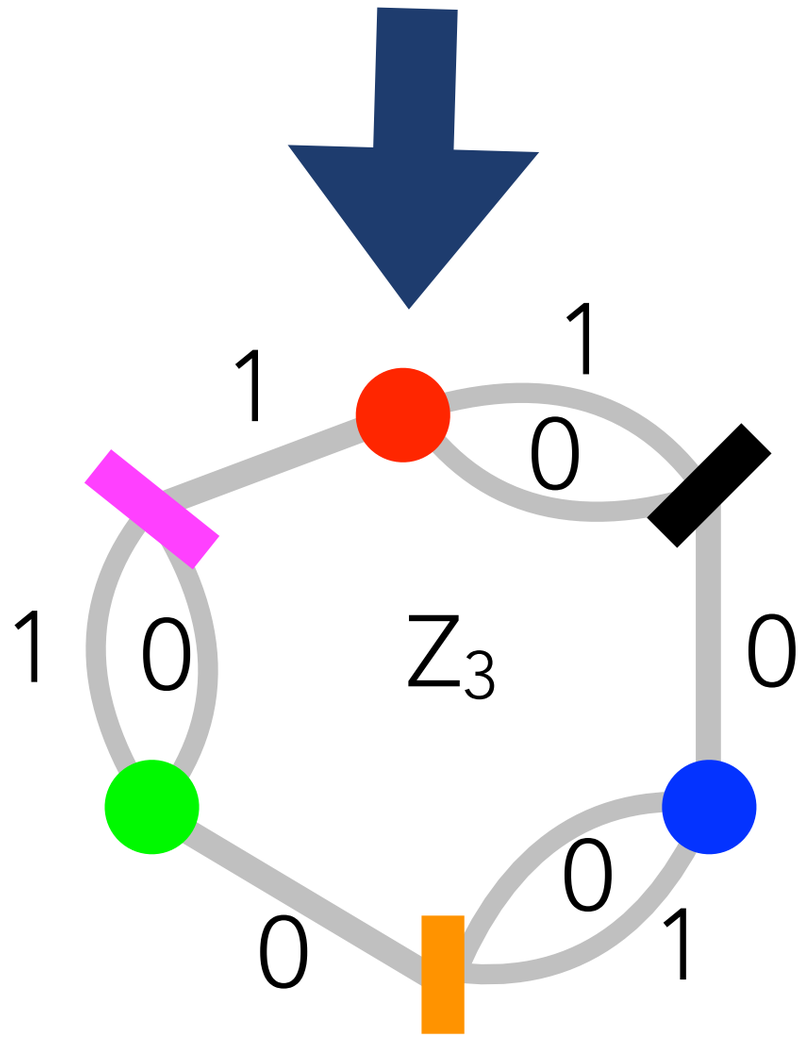

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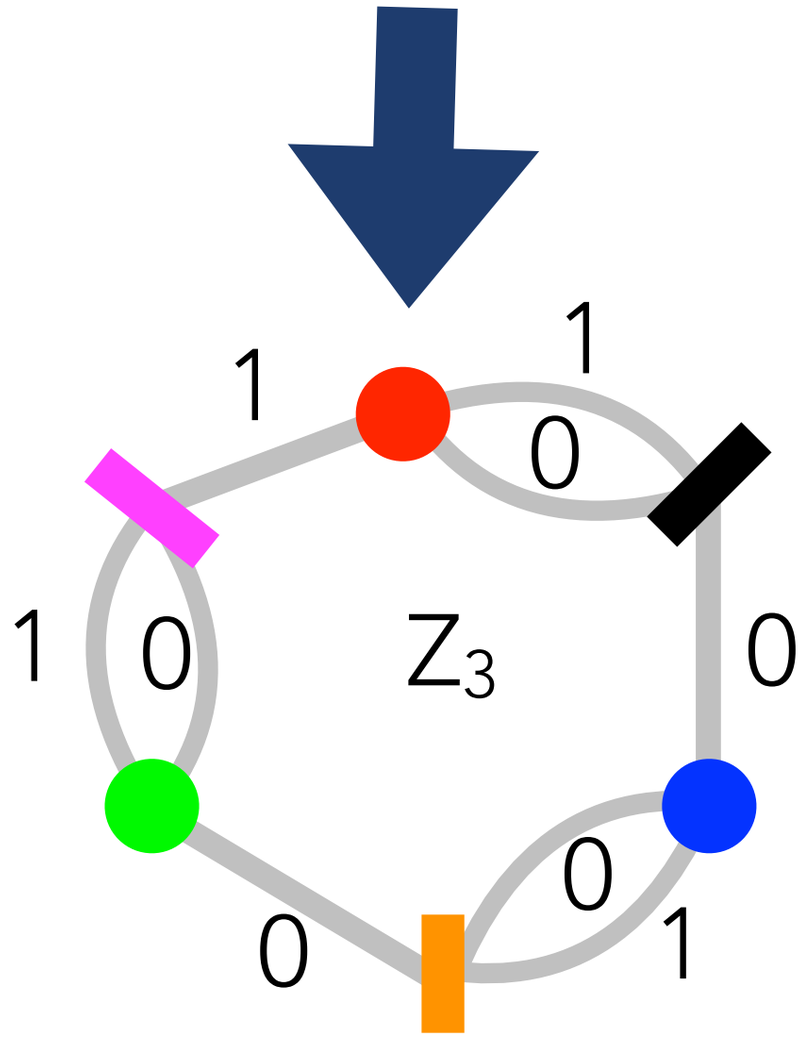

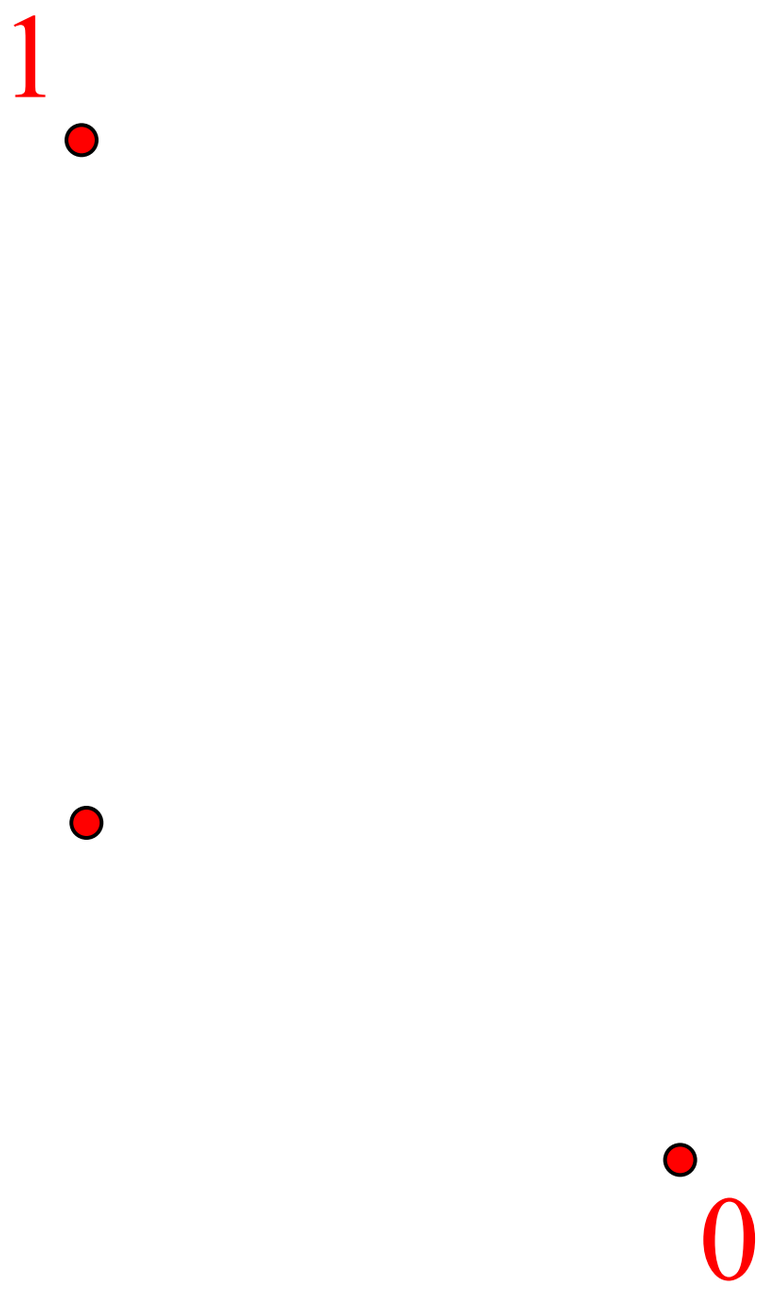

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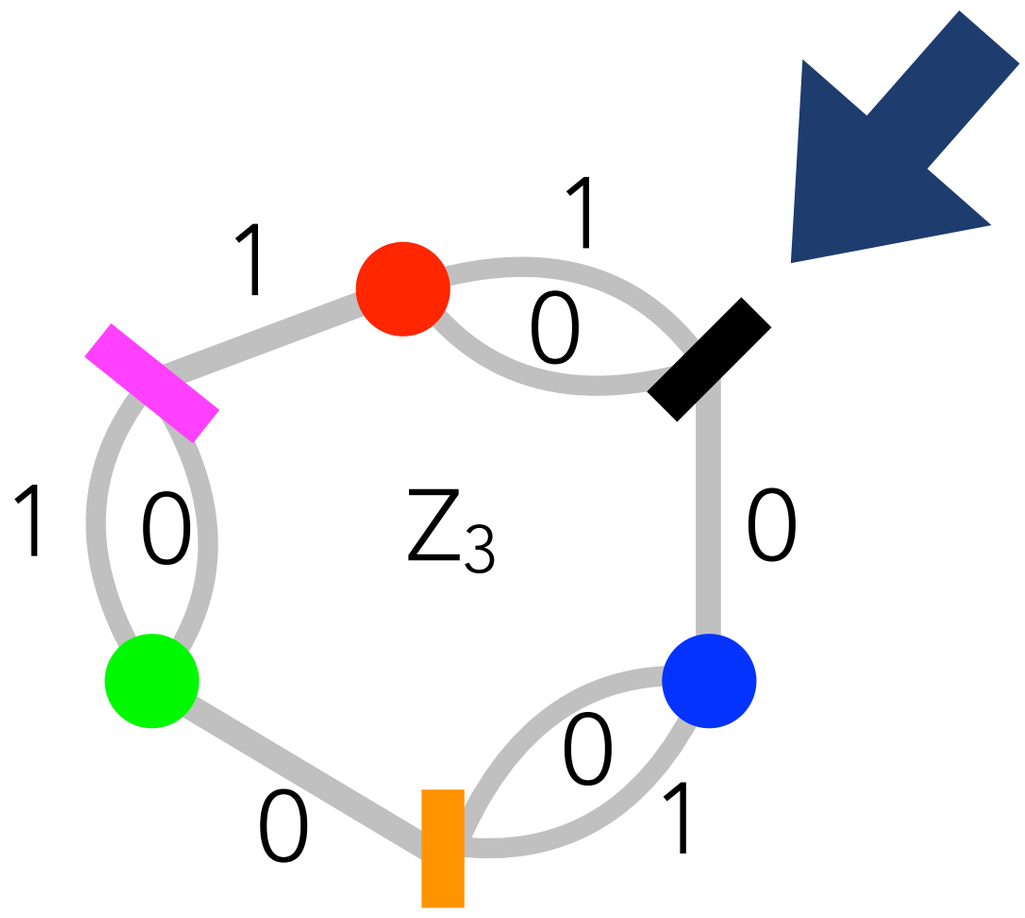

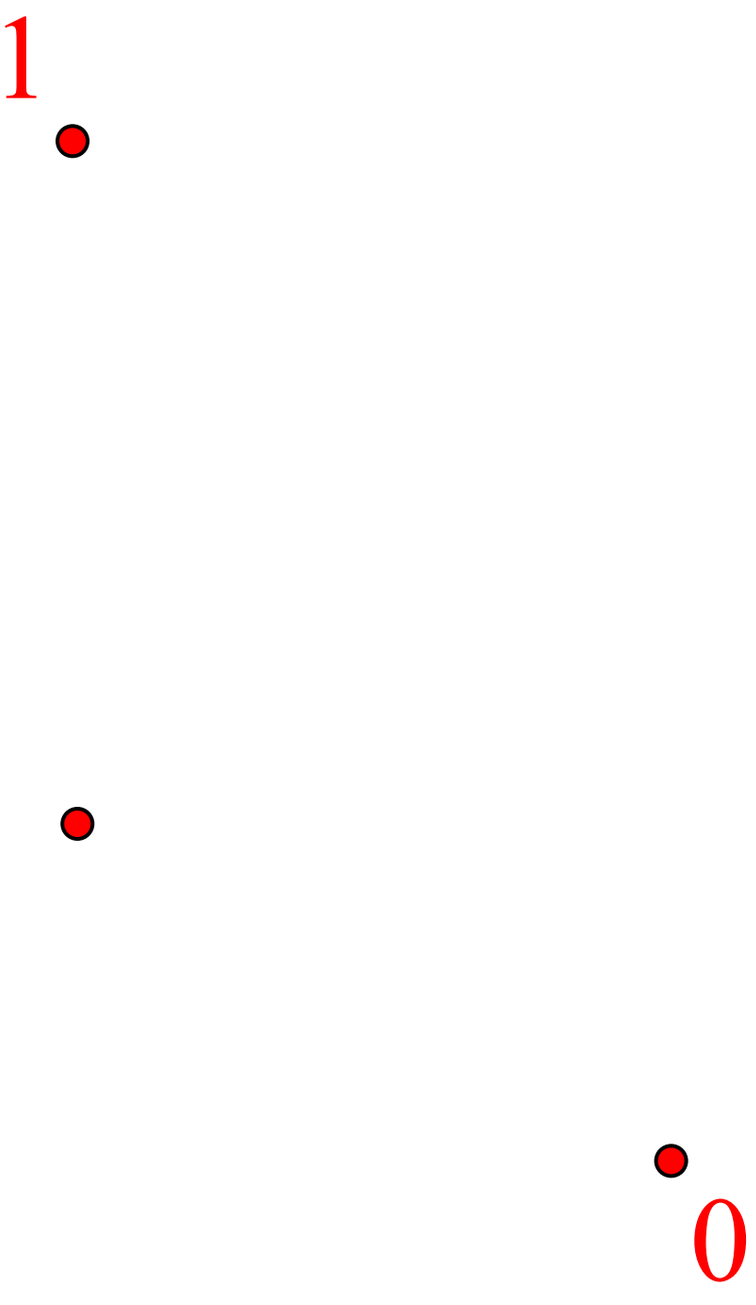

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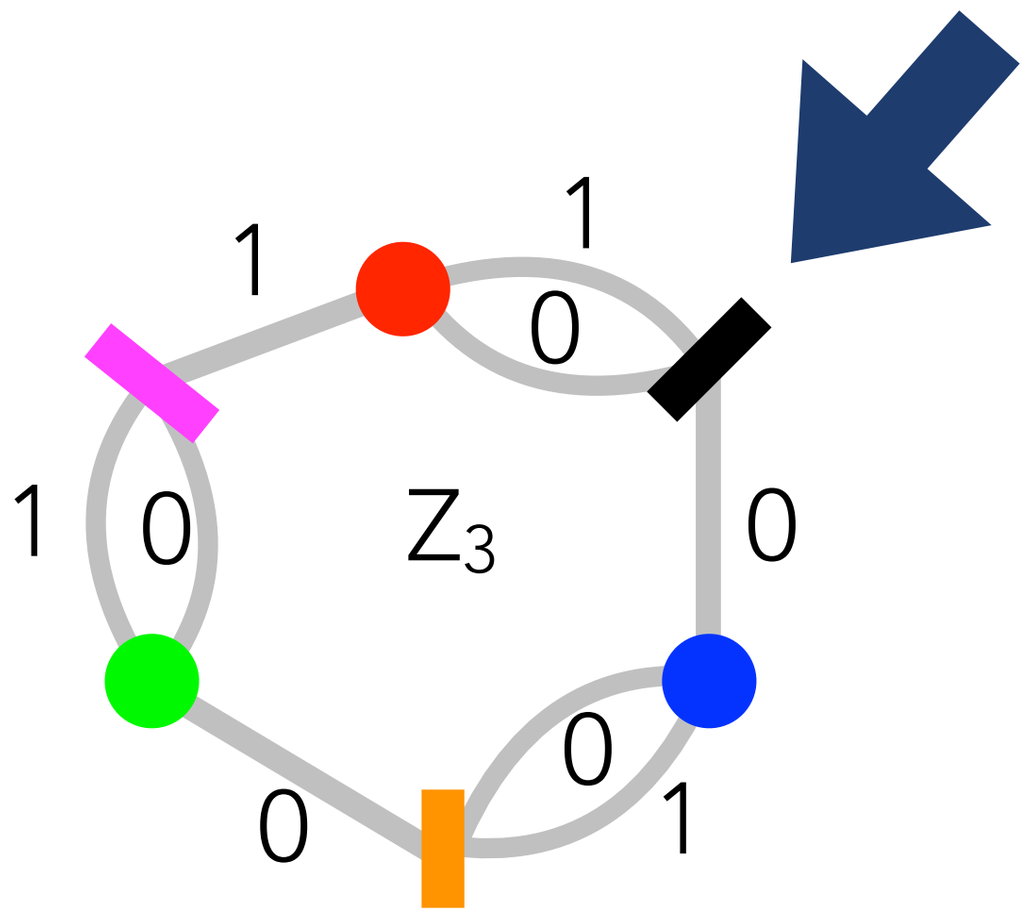

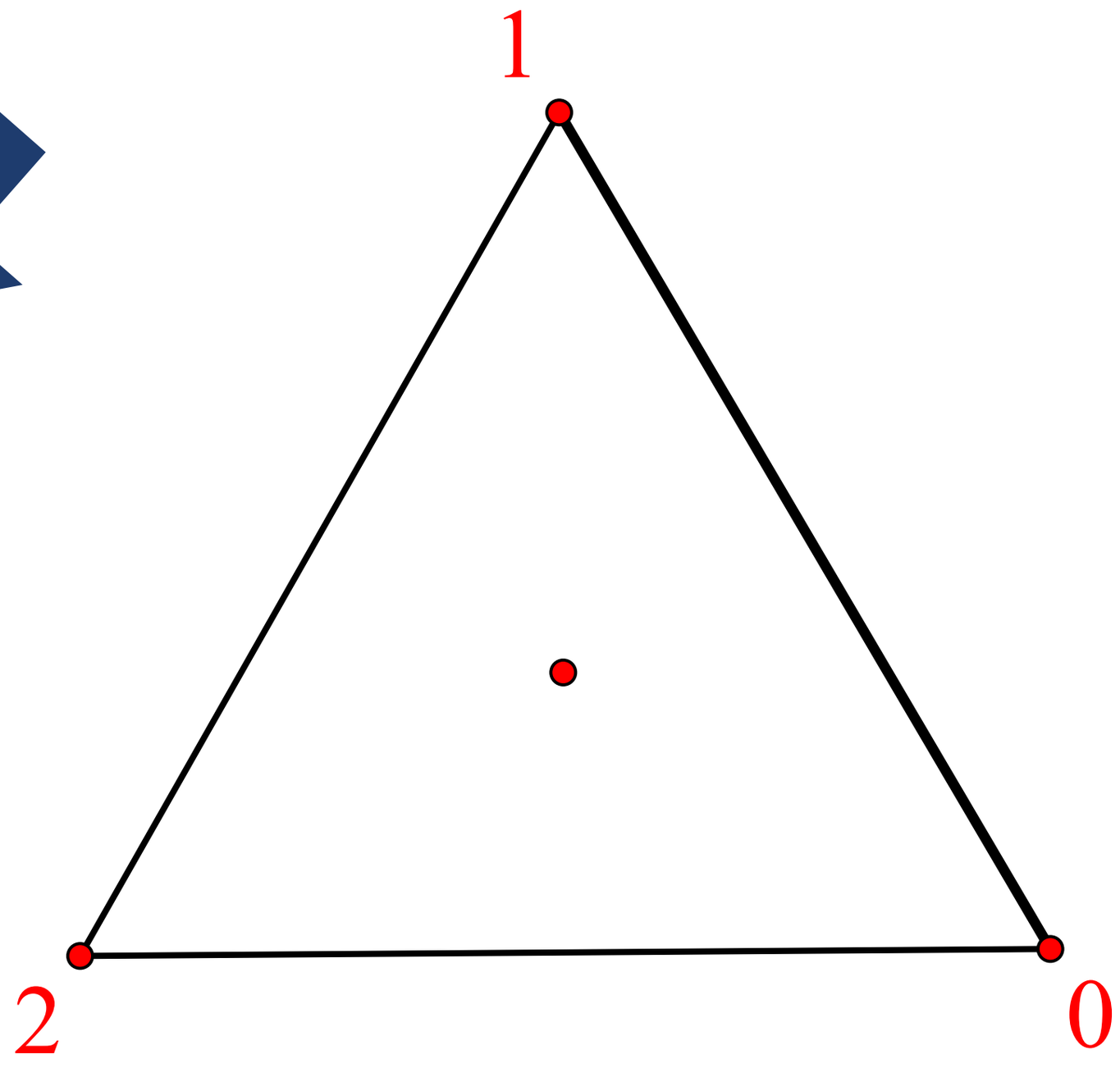

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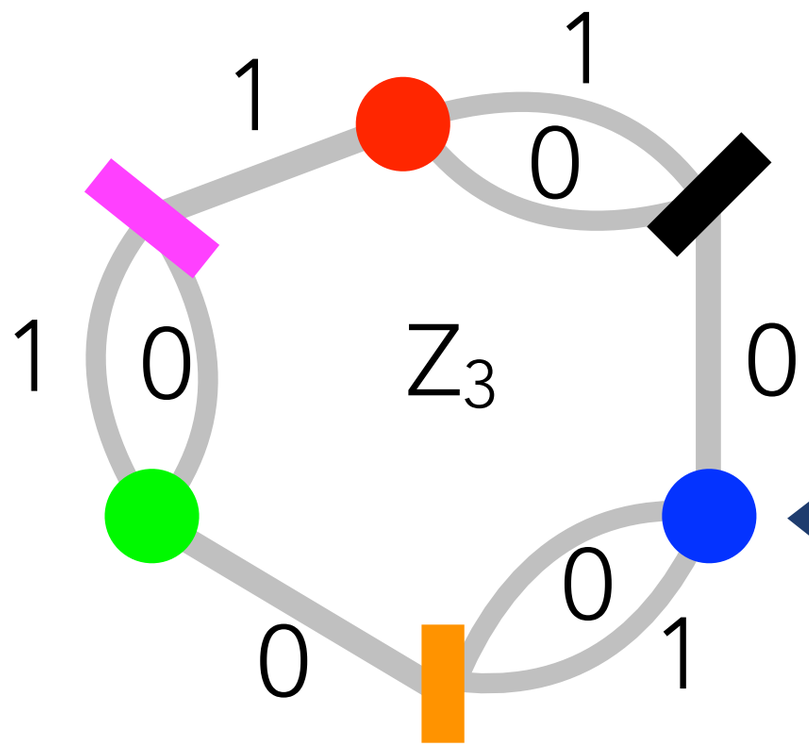

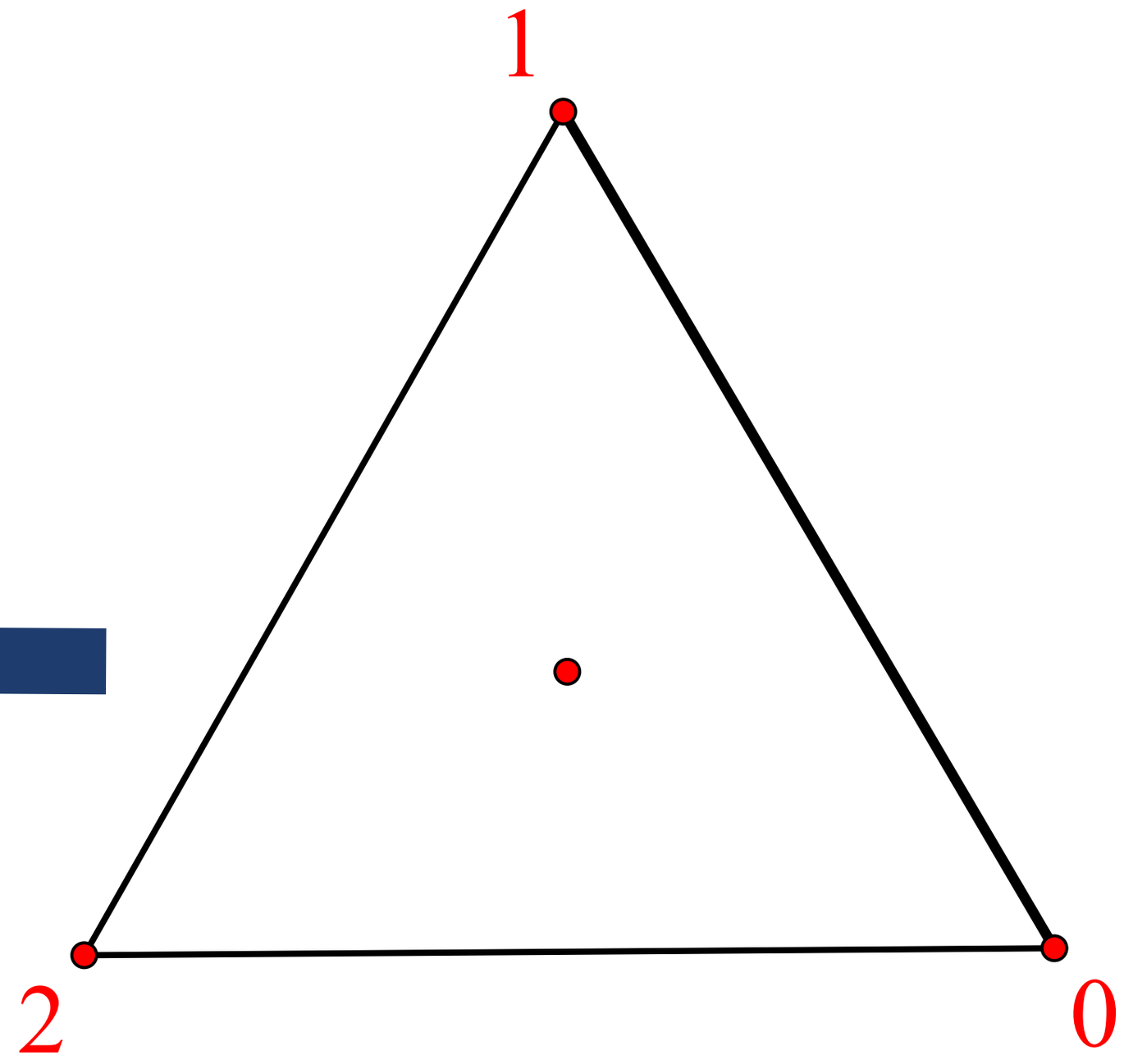

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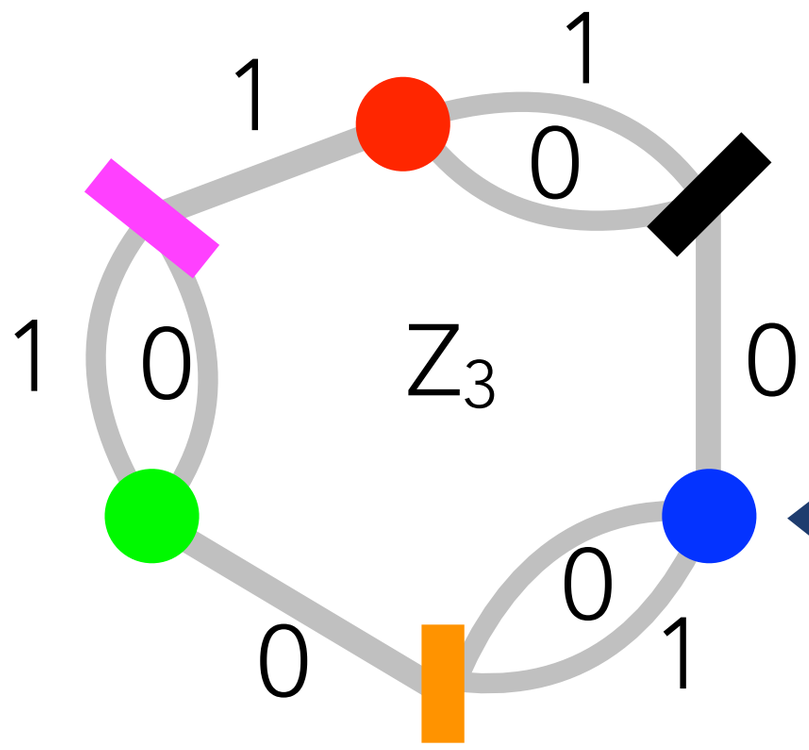

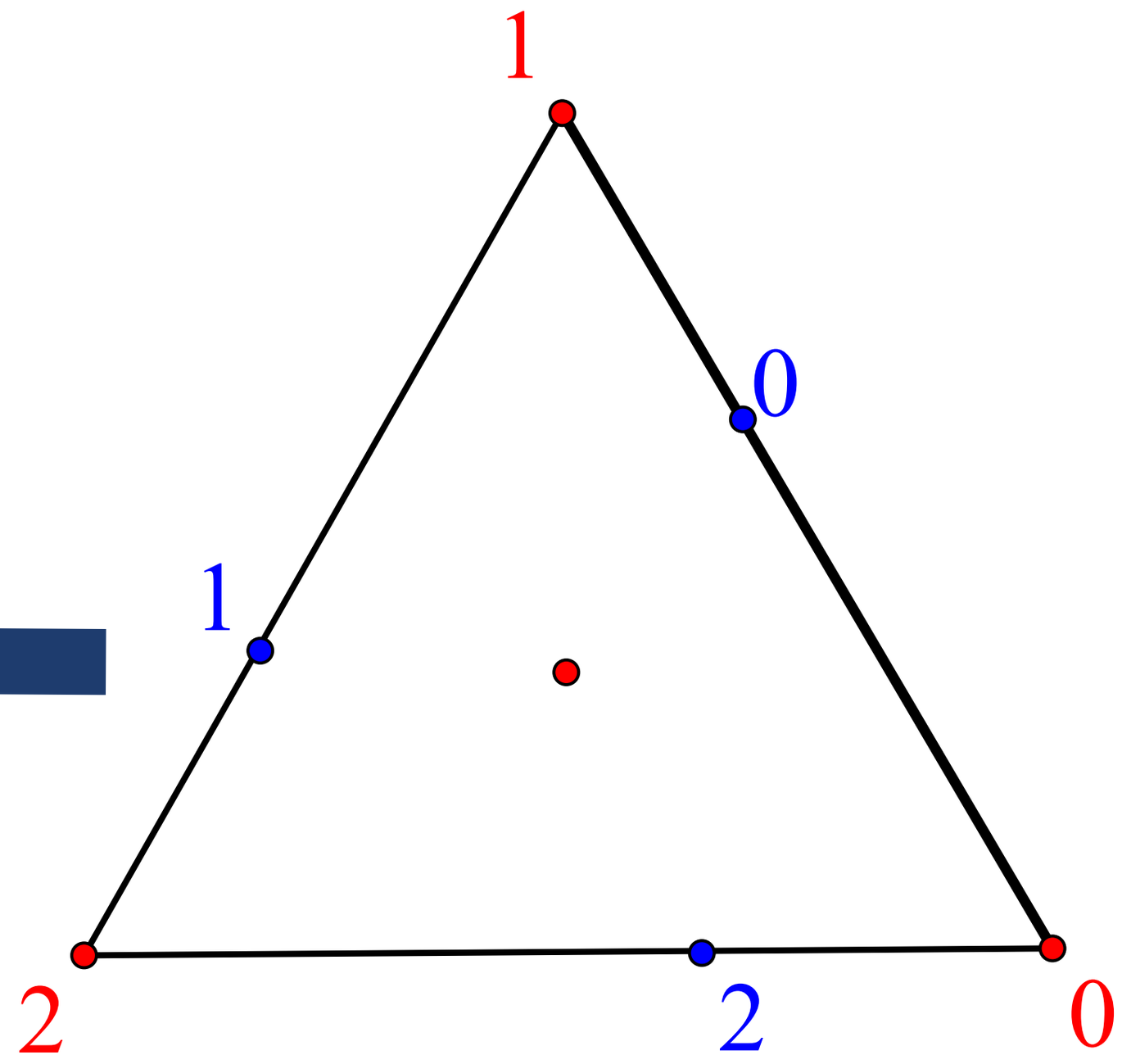

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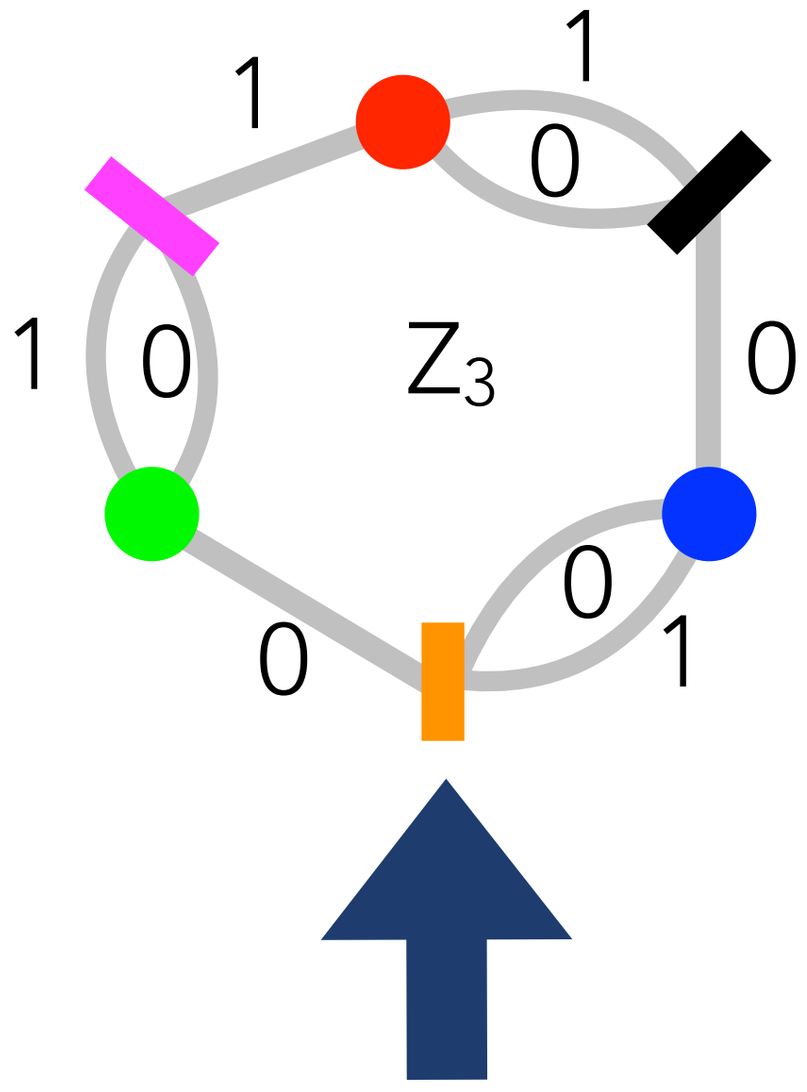

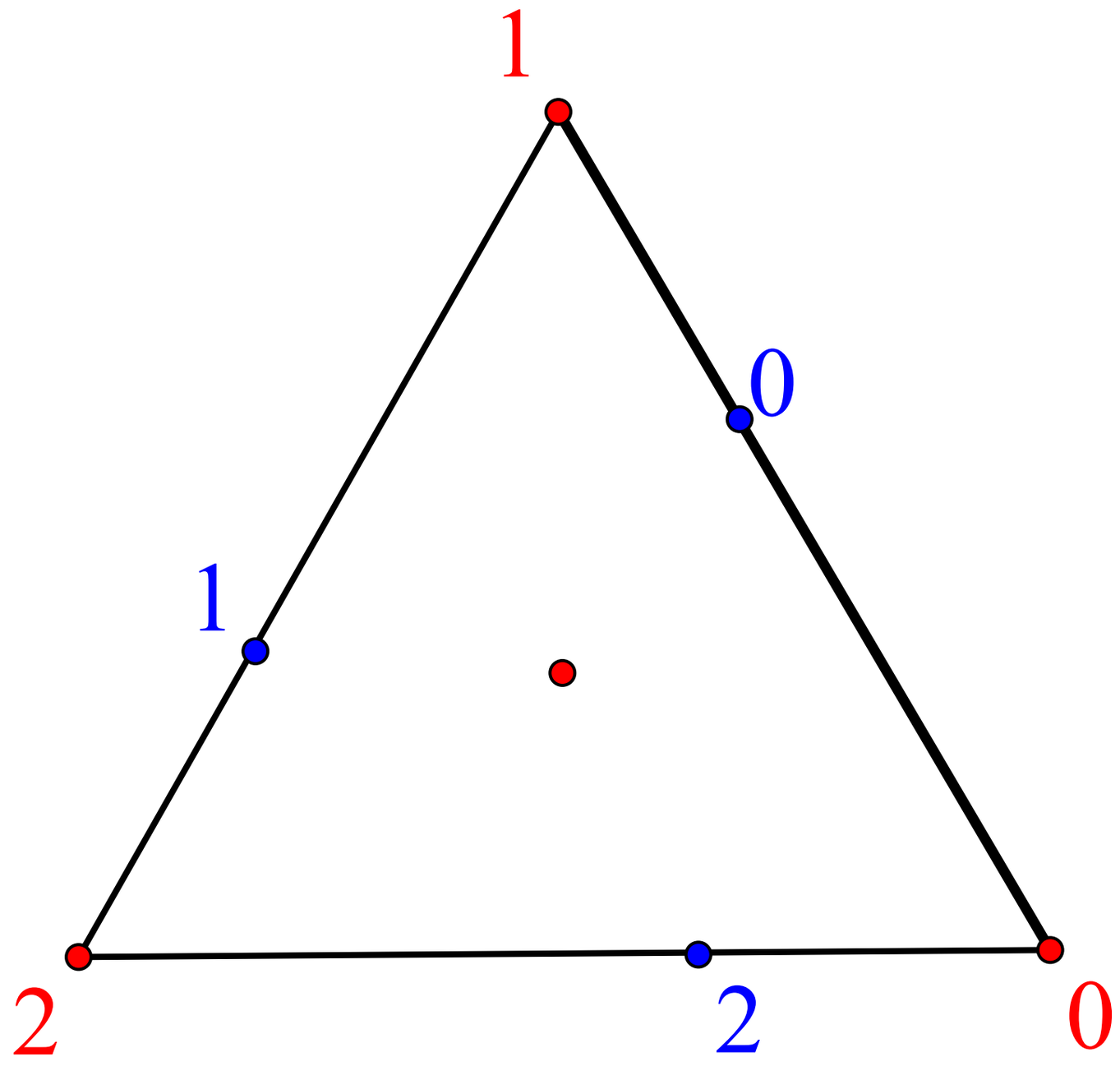

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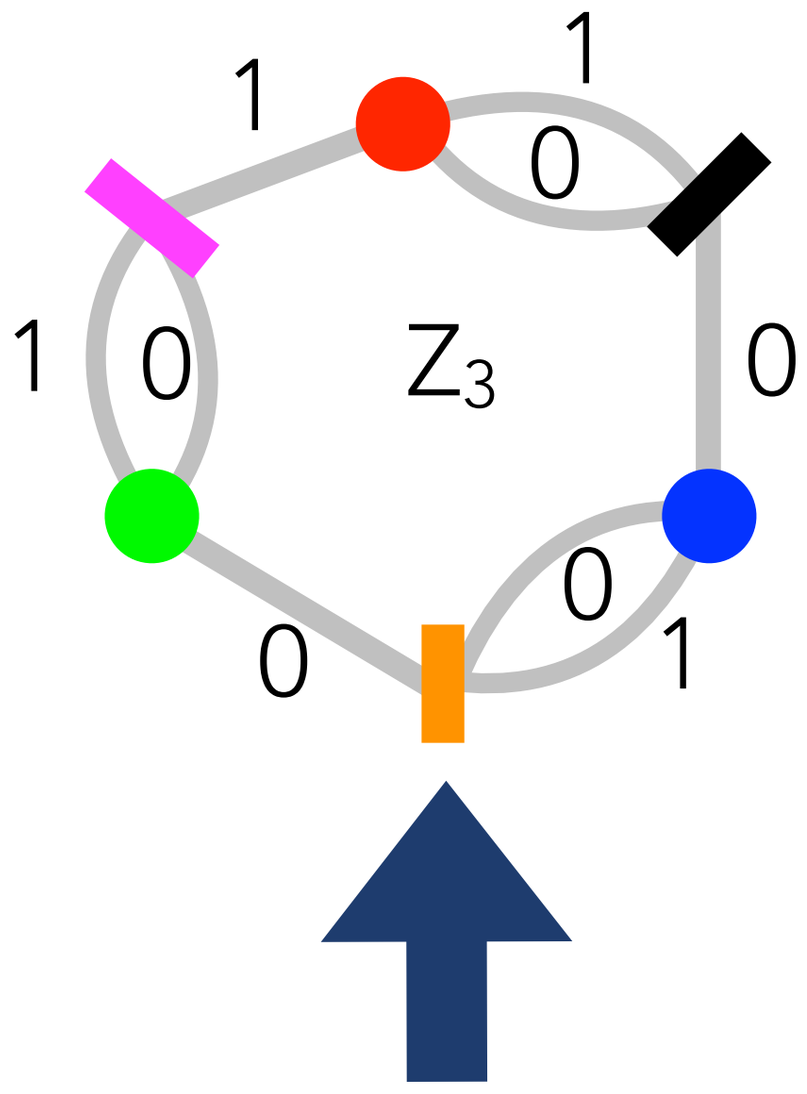

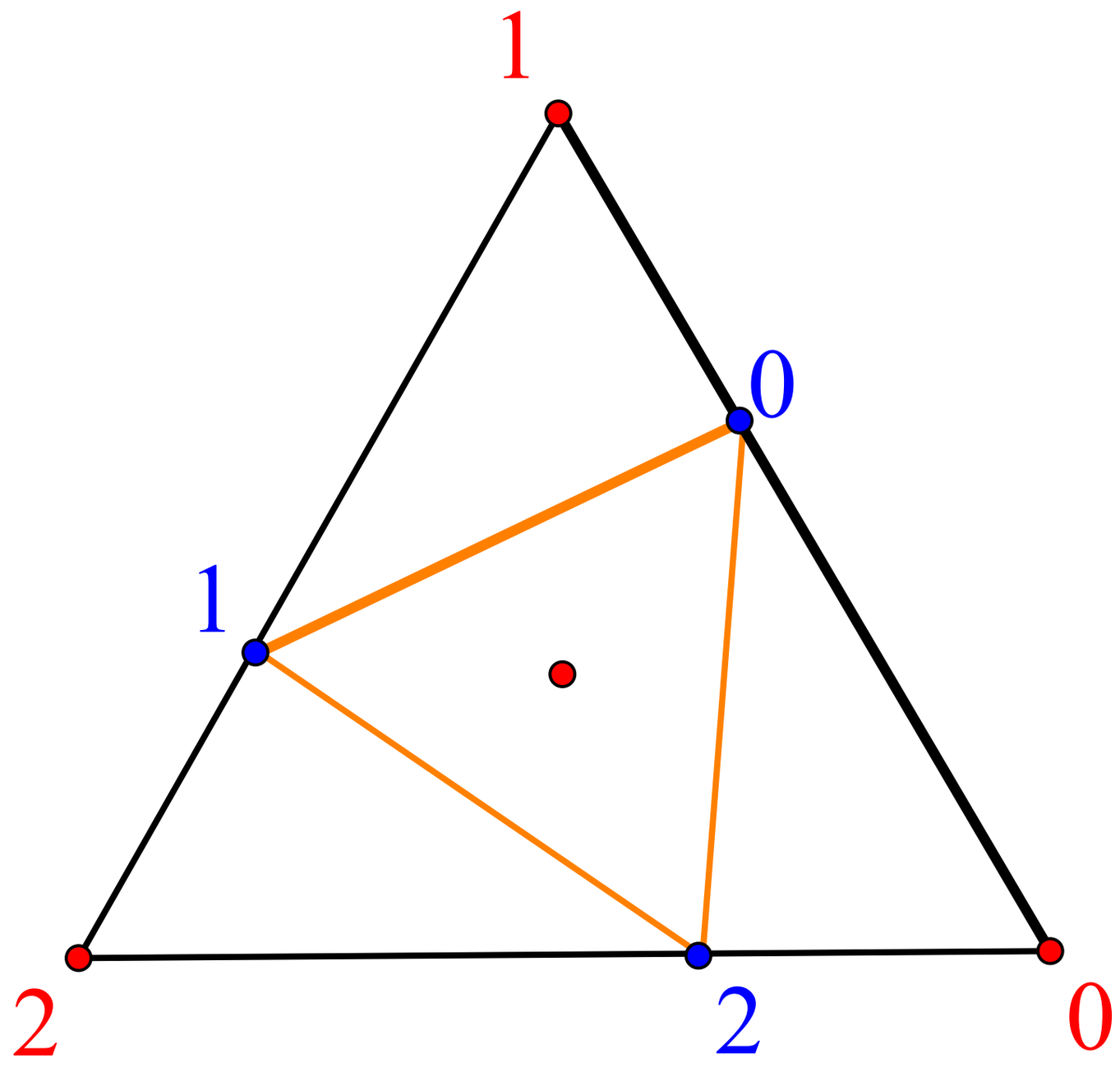

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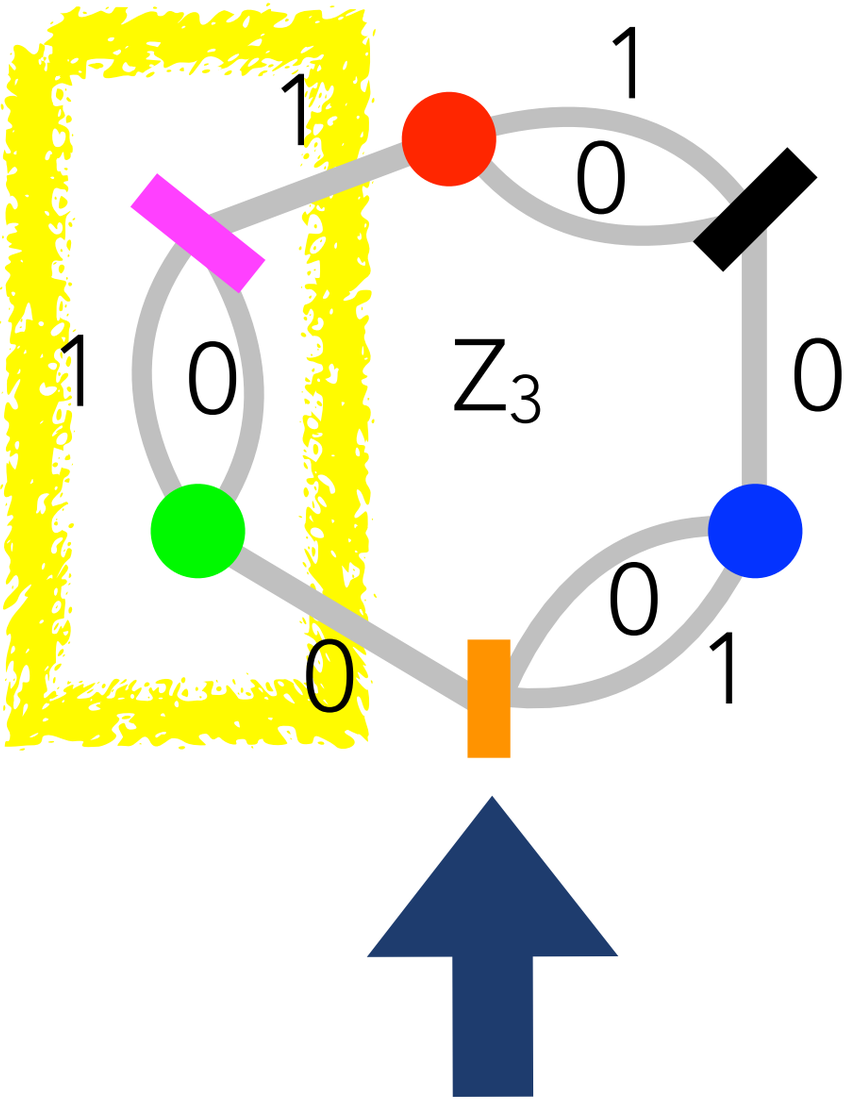

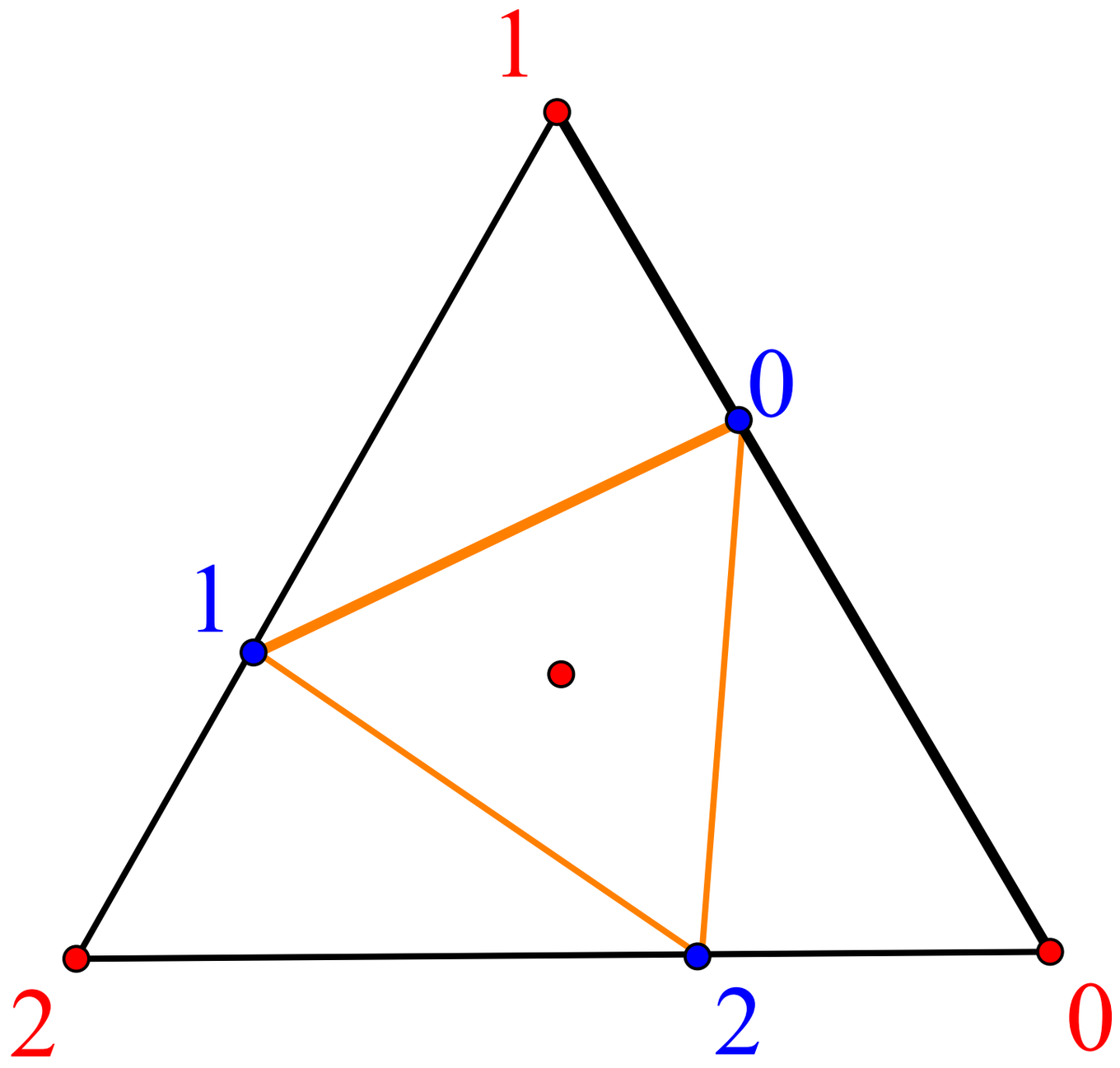

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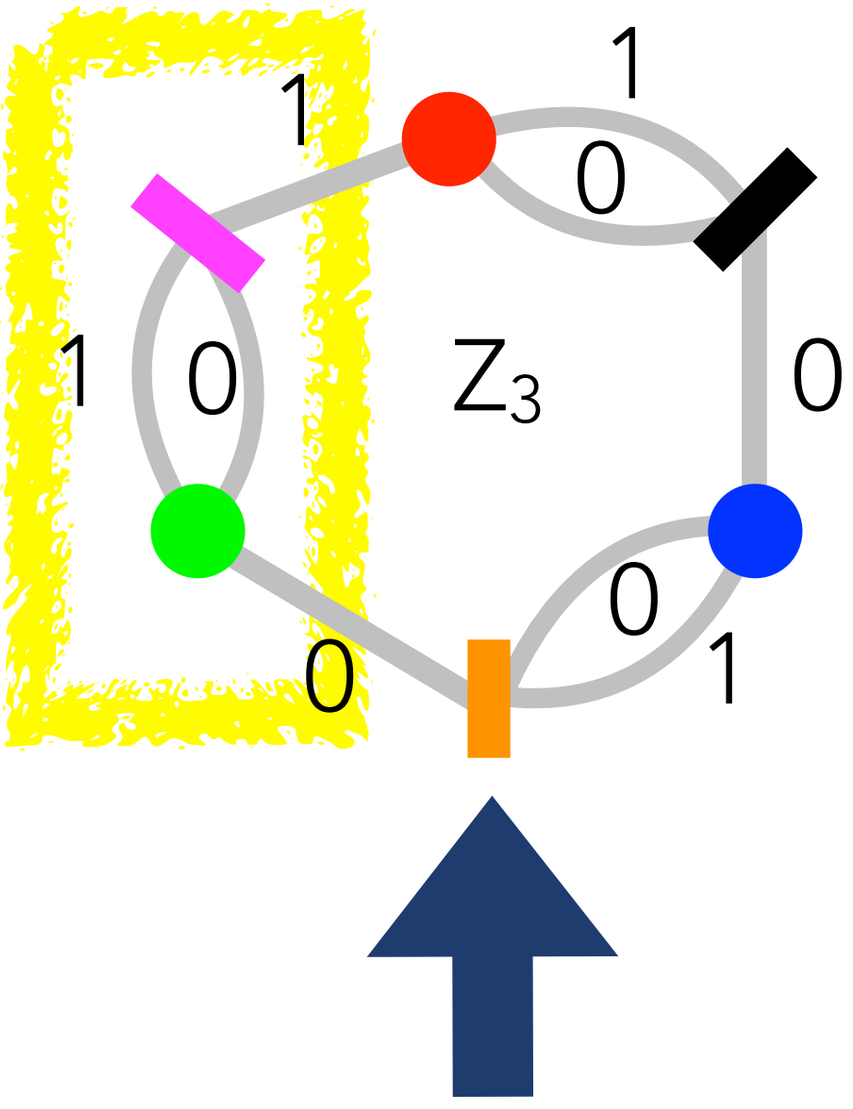

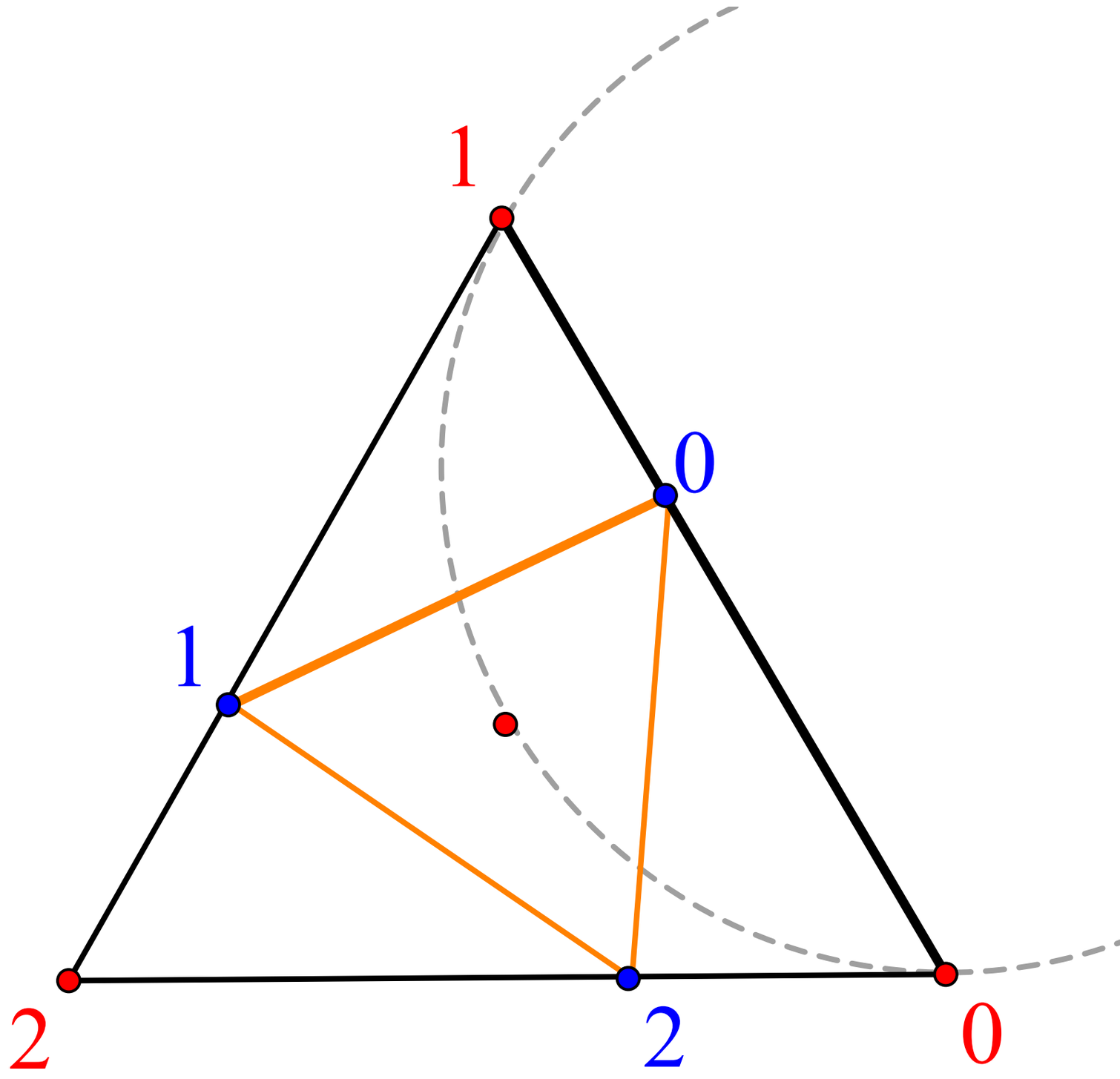

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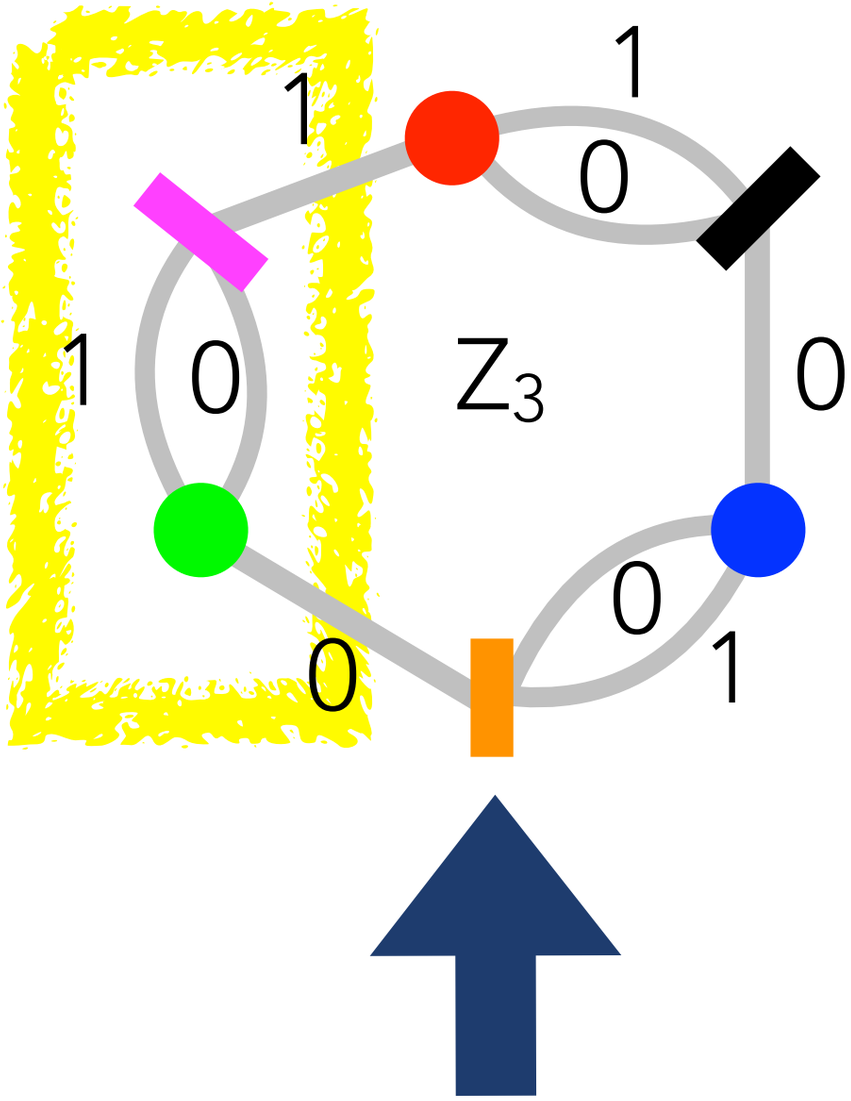
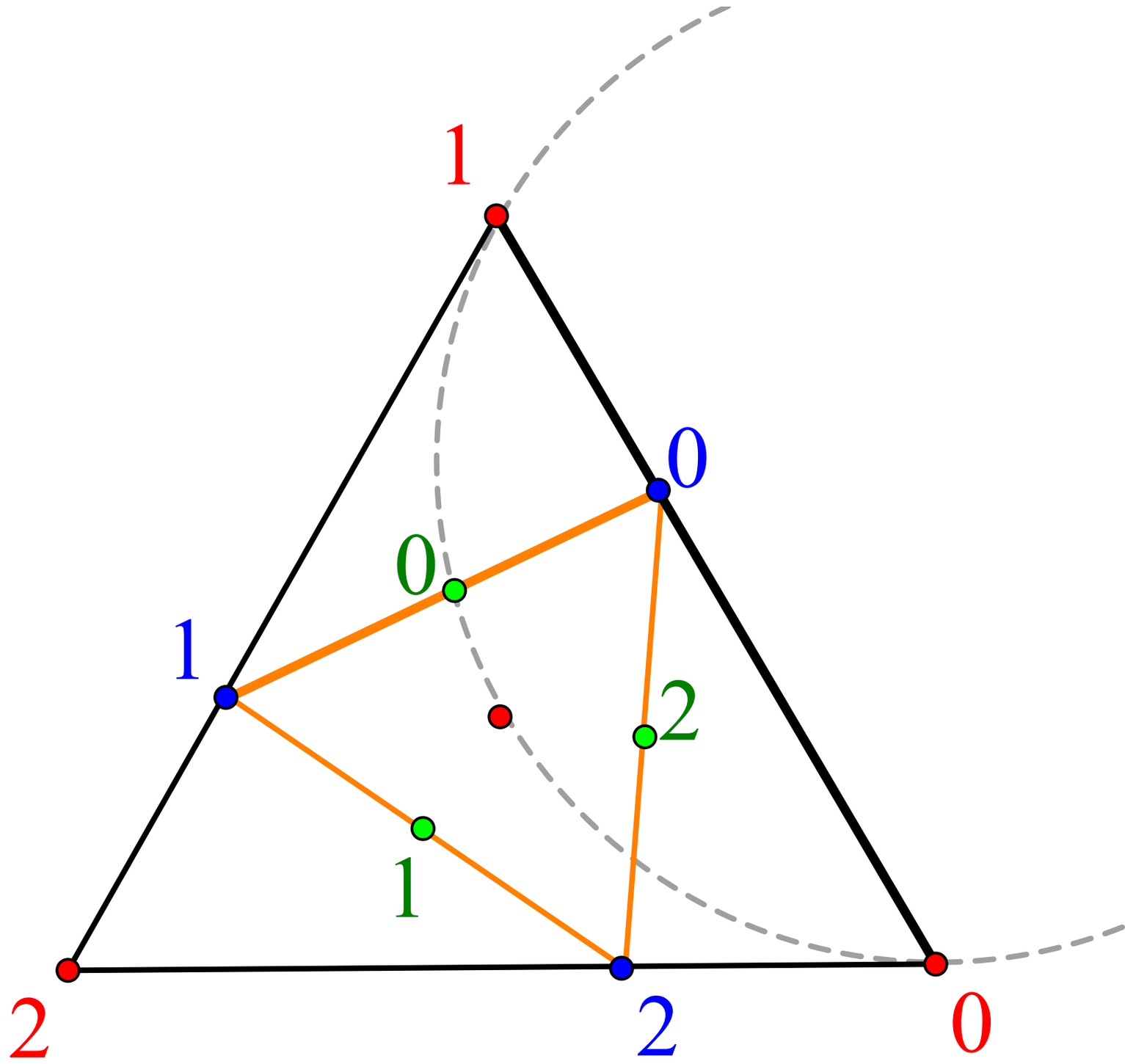
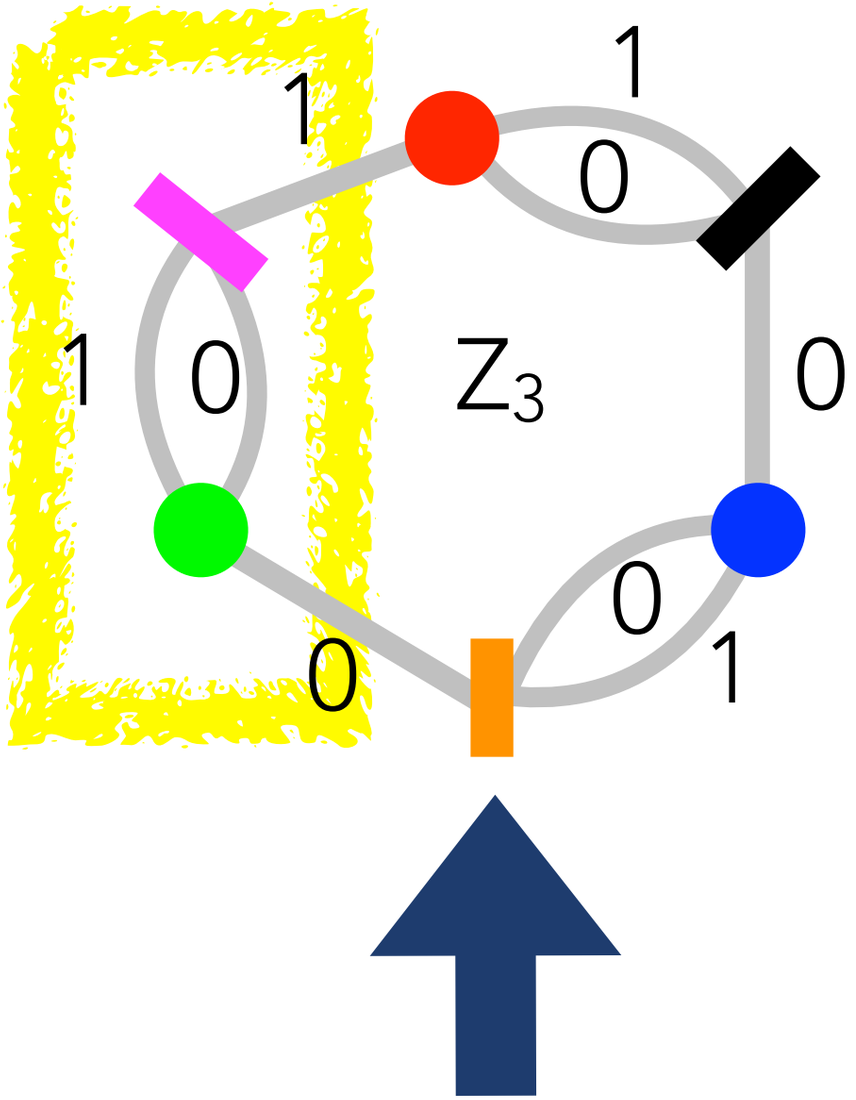
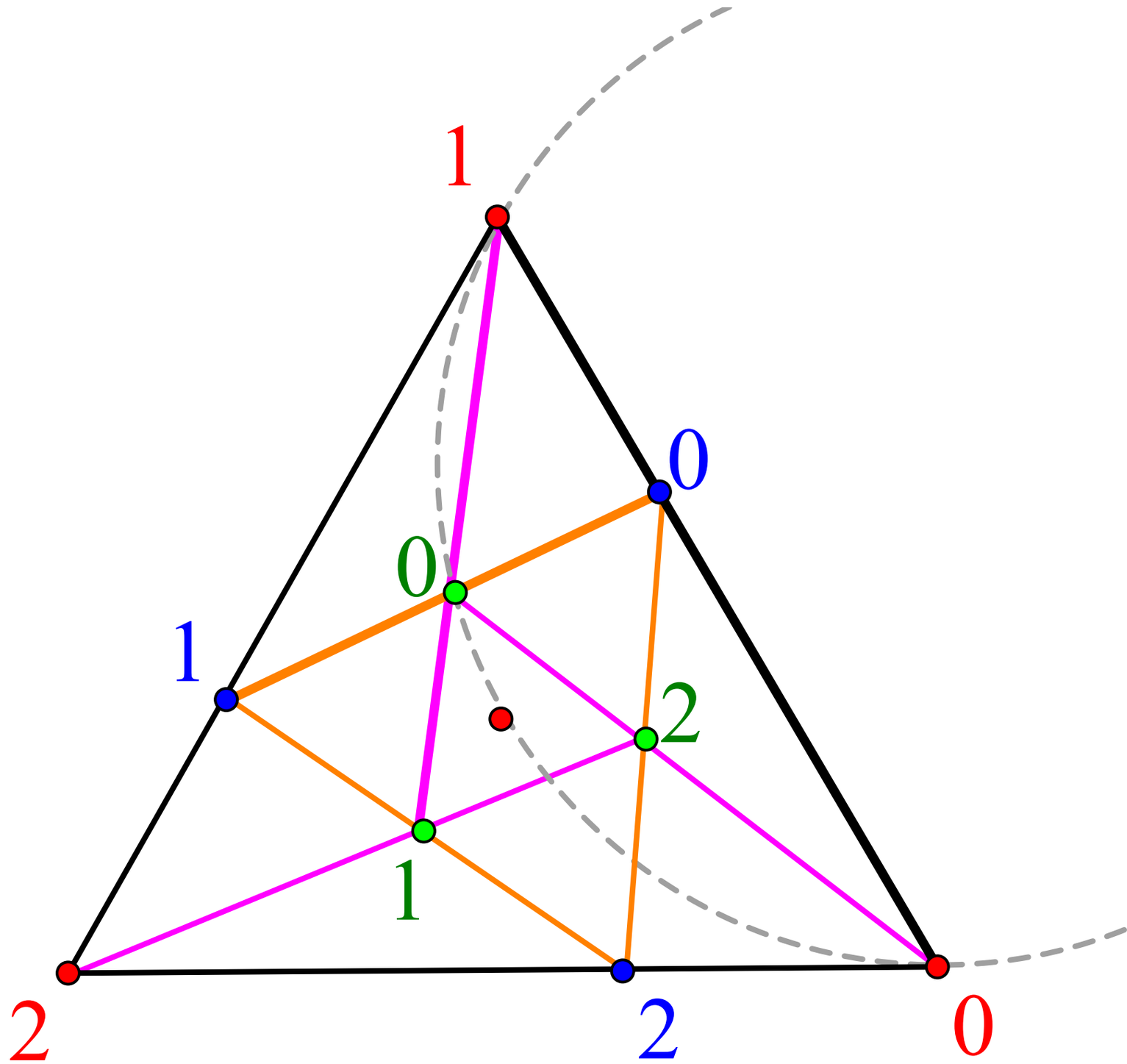


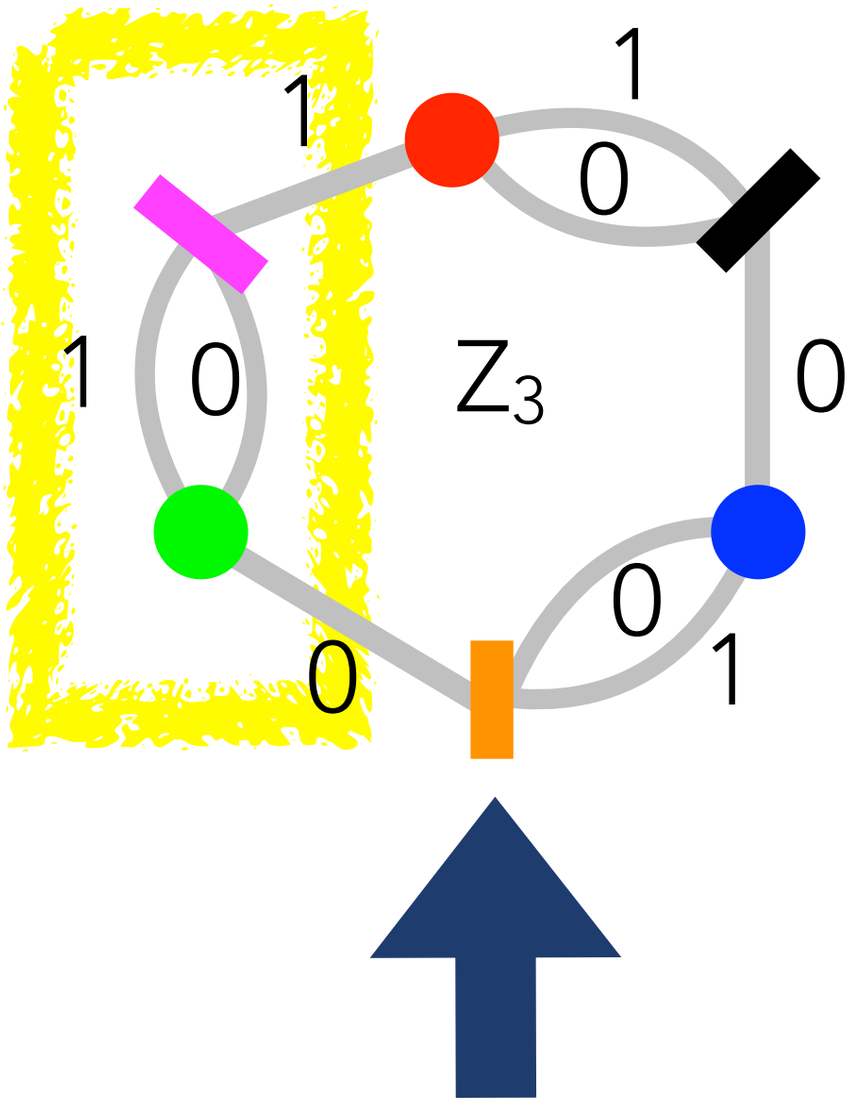


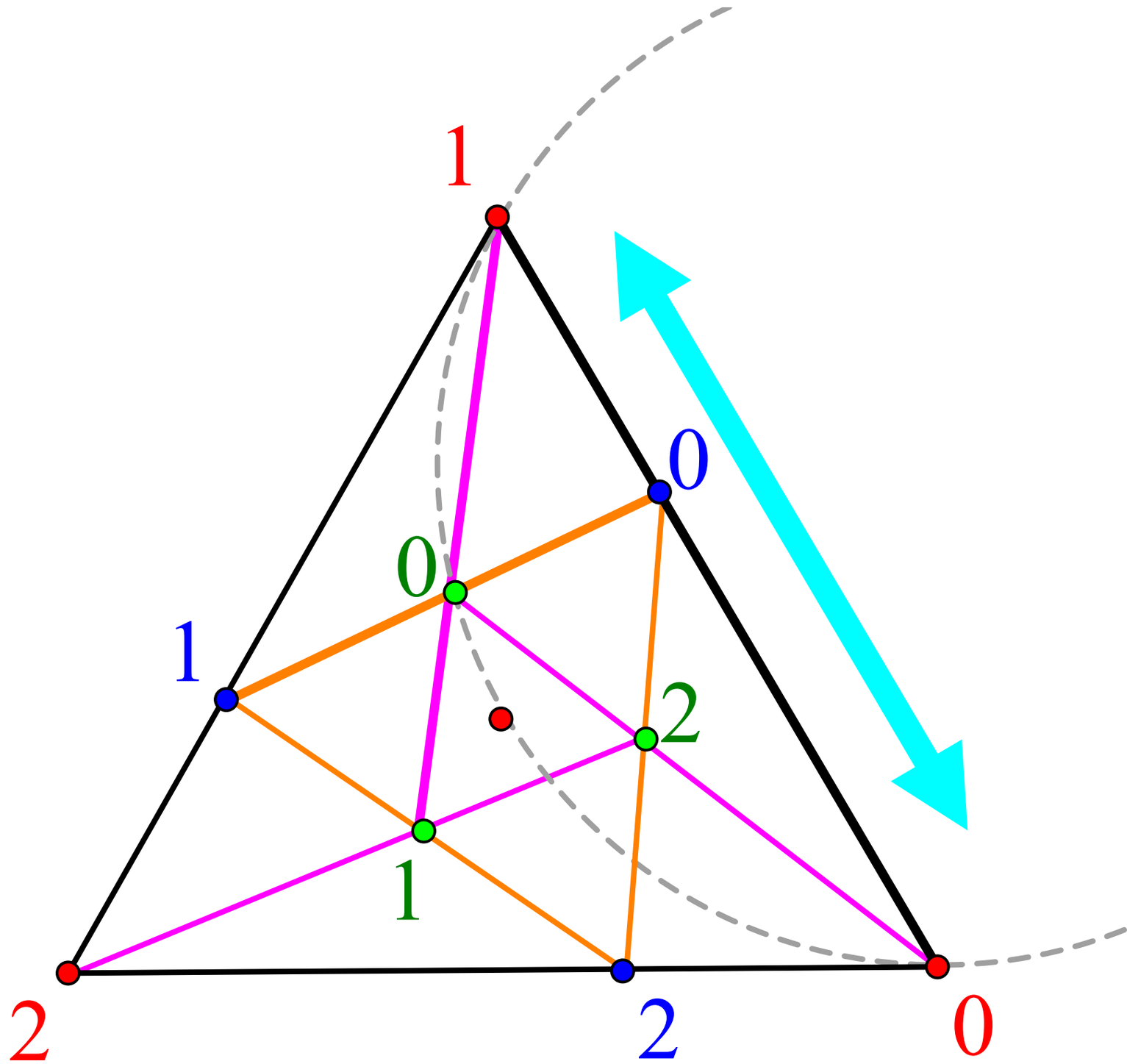
AN EXAMPLE



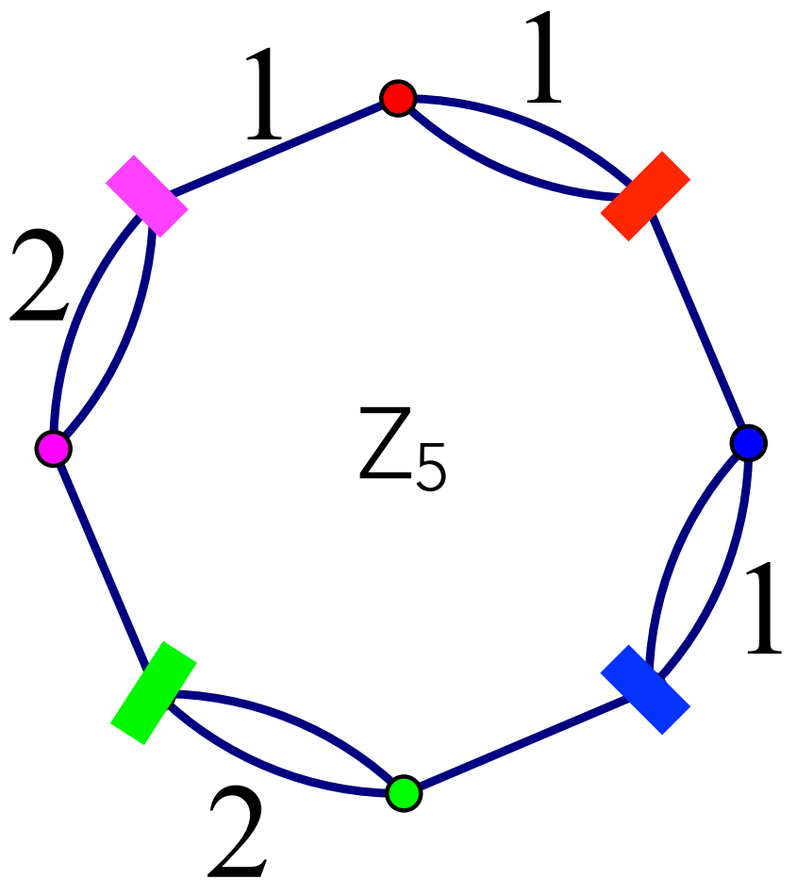


AN EXAMPLE

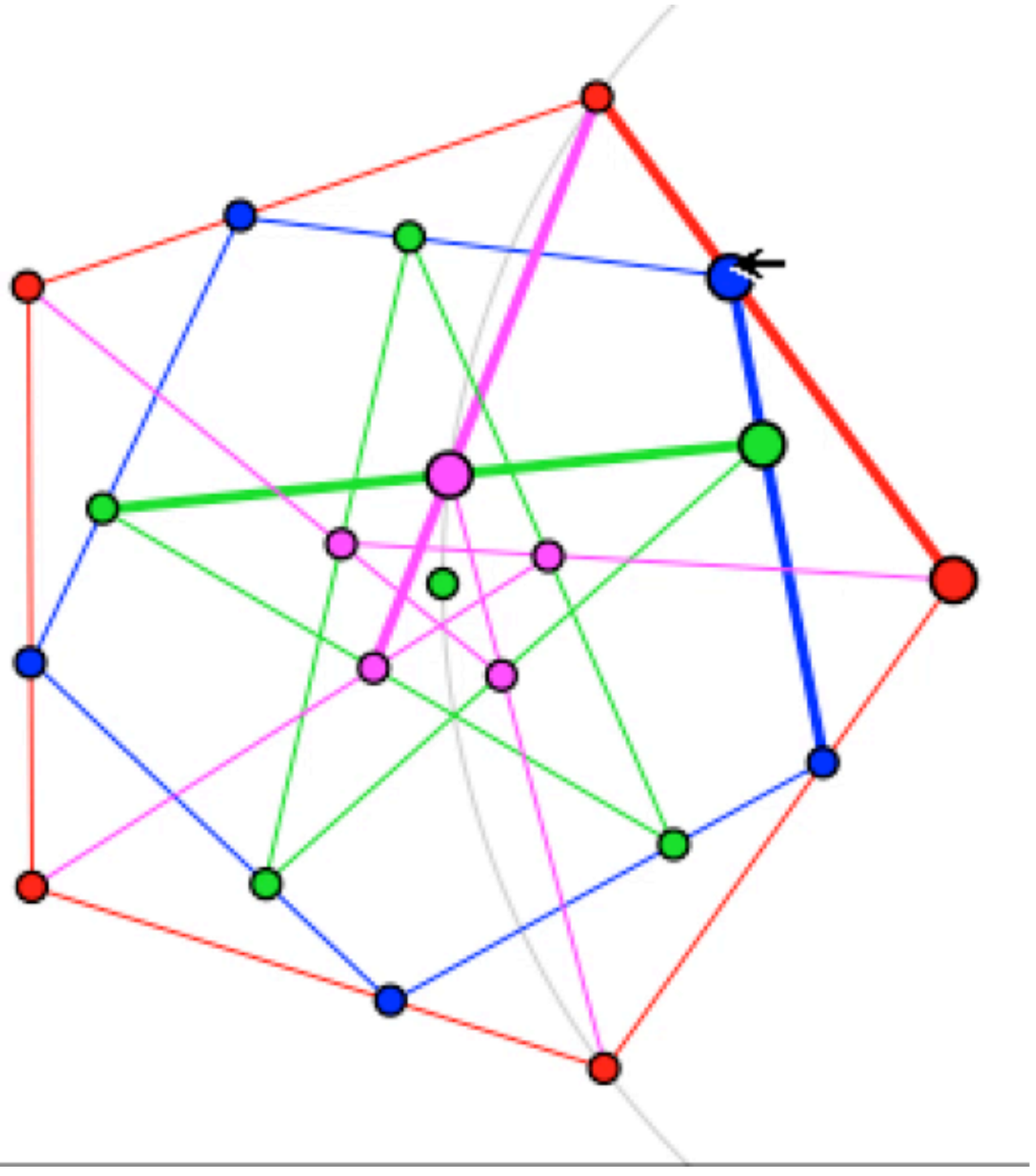




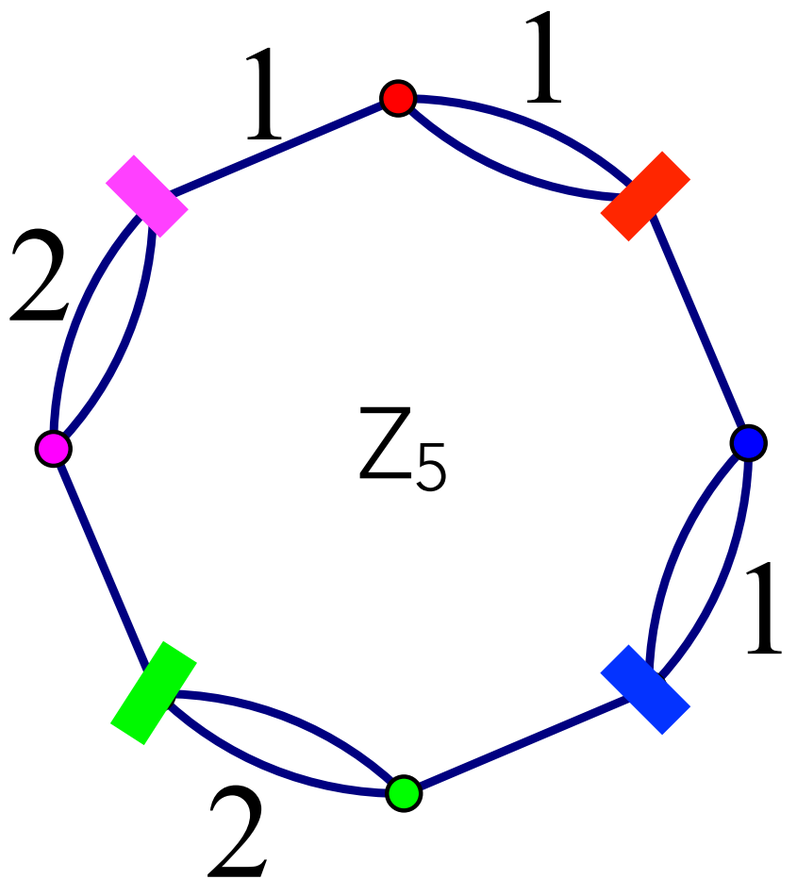
MOVABLE 3-CONFIGURATIONS



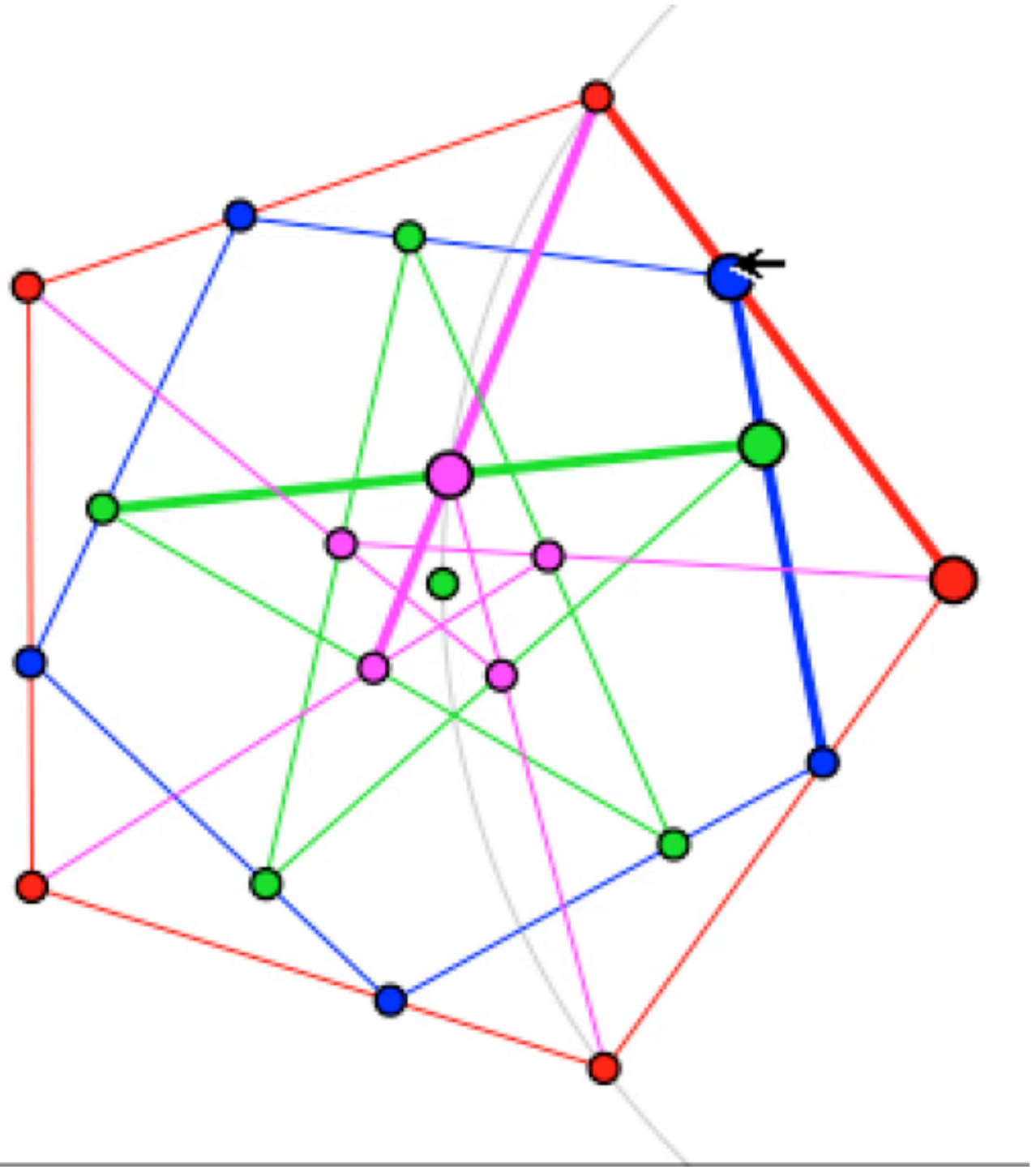
Multilateral



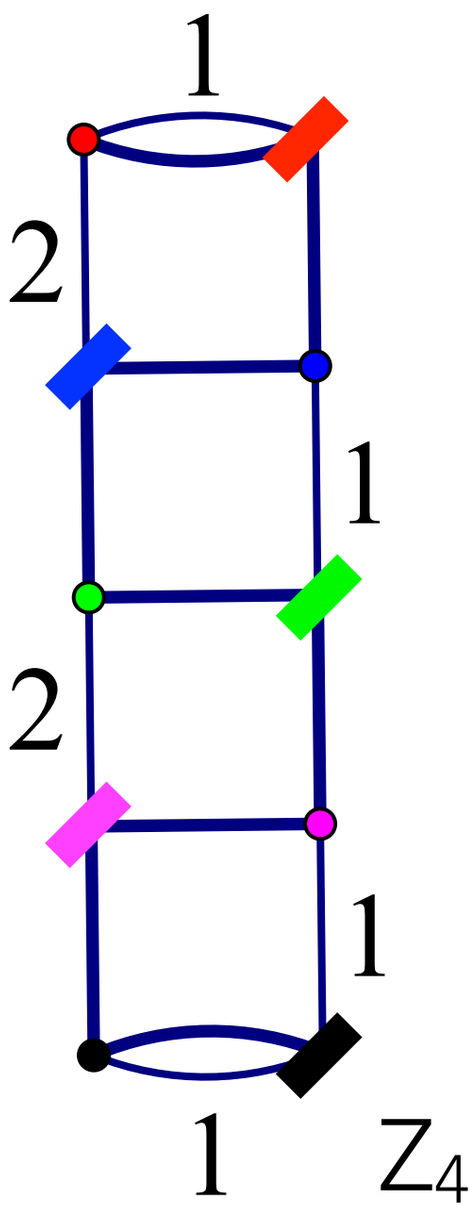
MOVABLE 3-CONFIGURATIONS



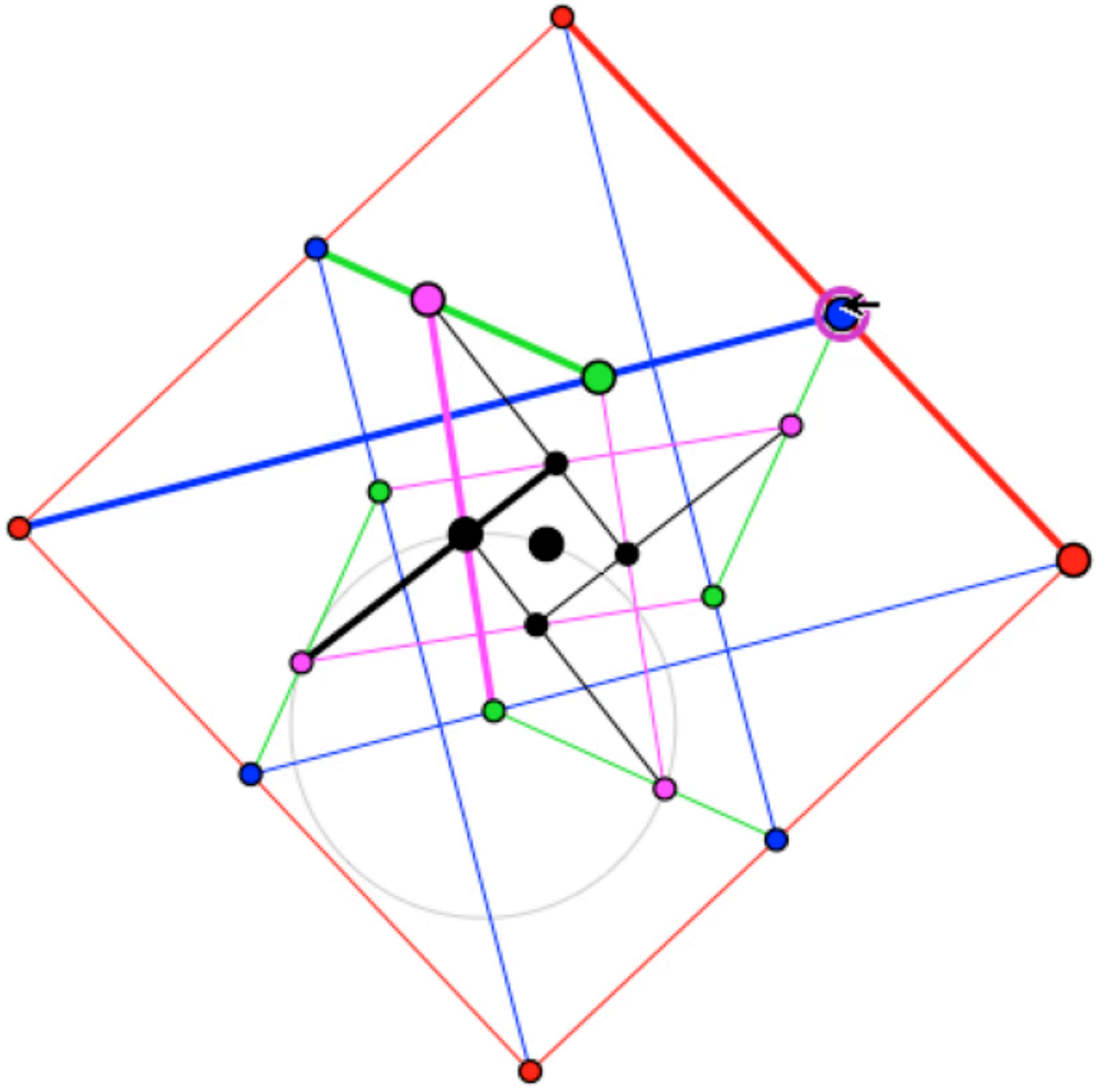
Multilateral



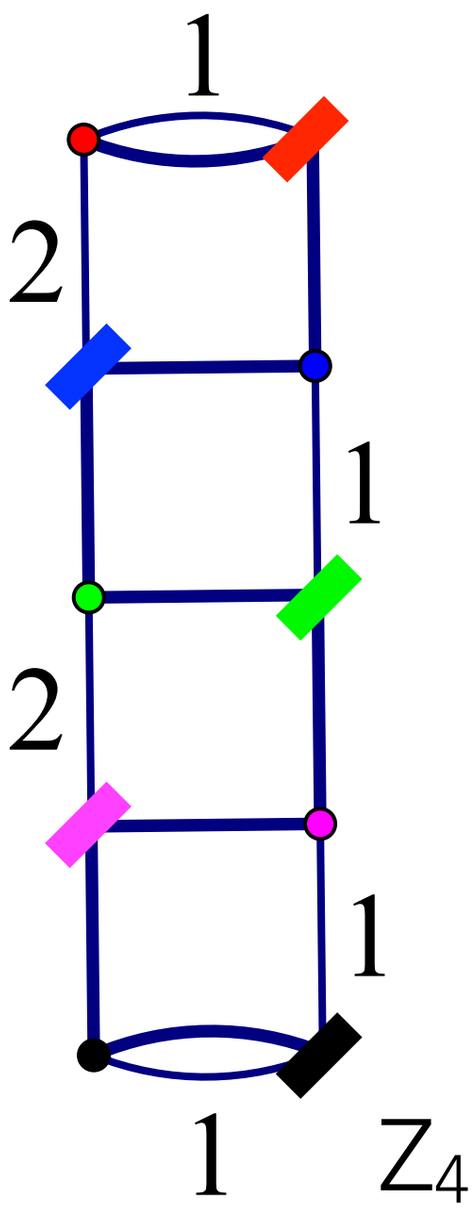
MOVABLE 3-CONFIGURATIONS



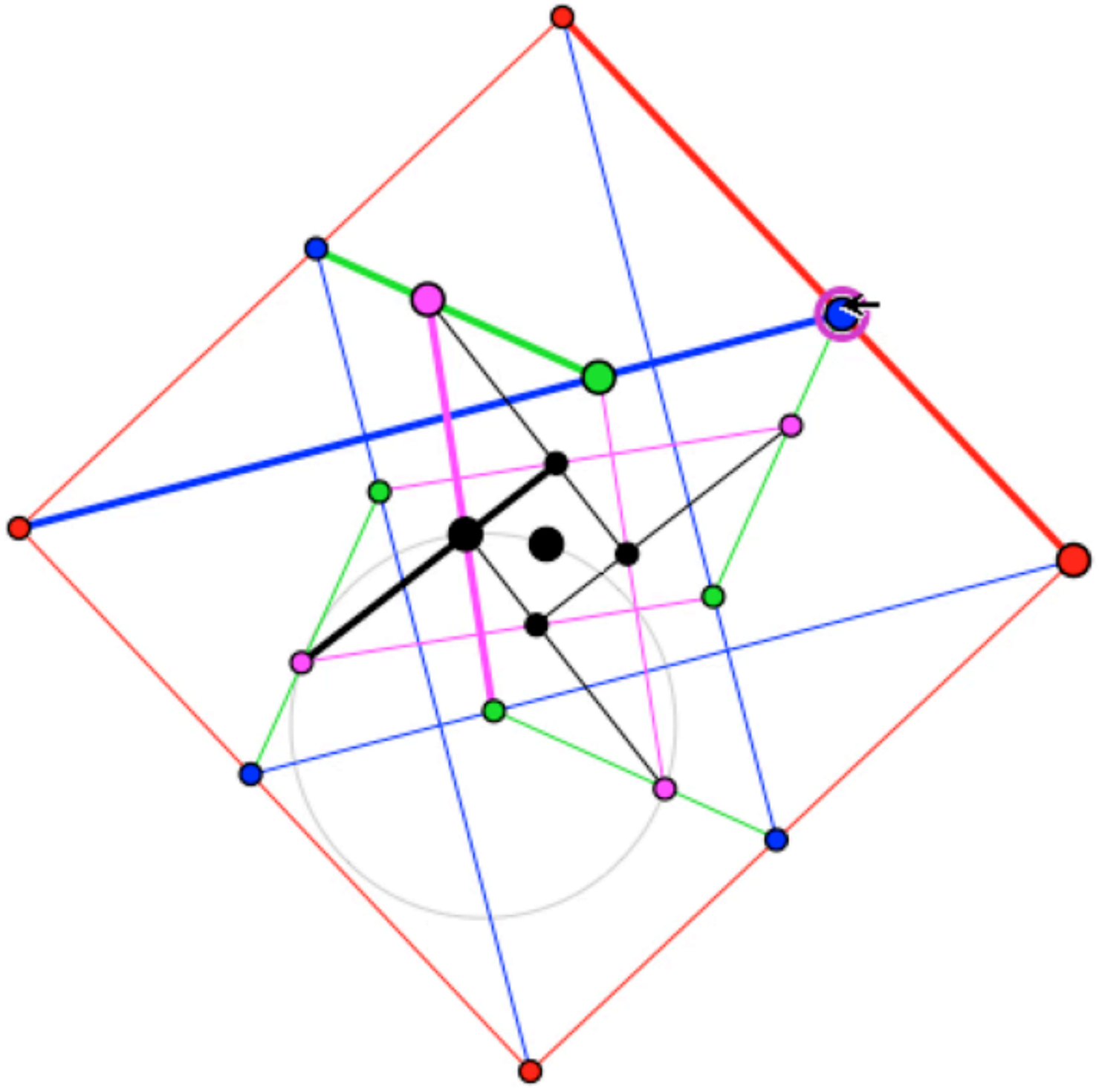
Caterpillar



MOVABLE 3-CONFIGURATIONS



Caterpillar



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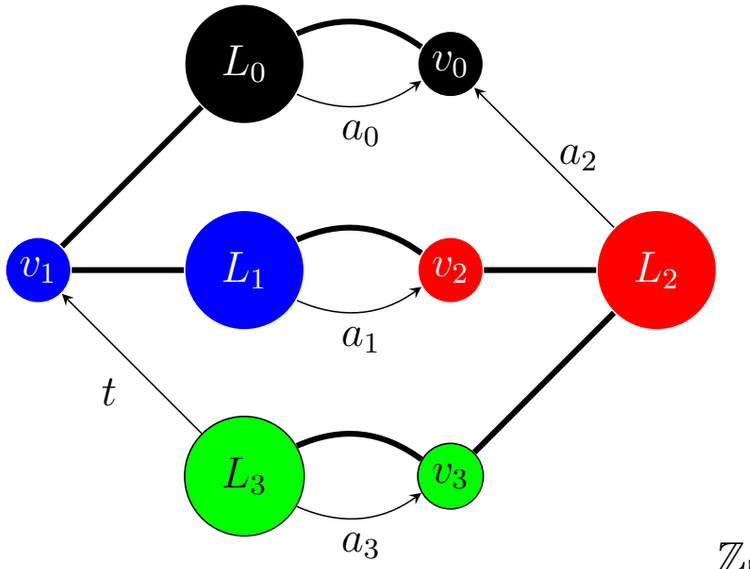
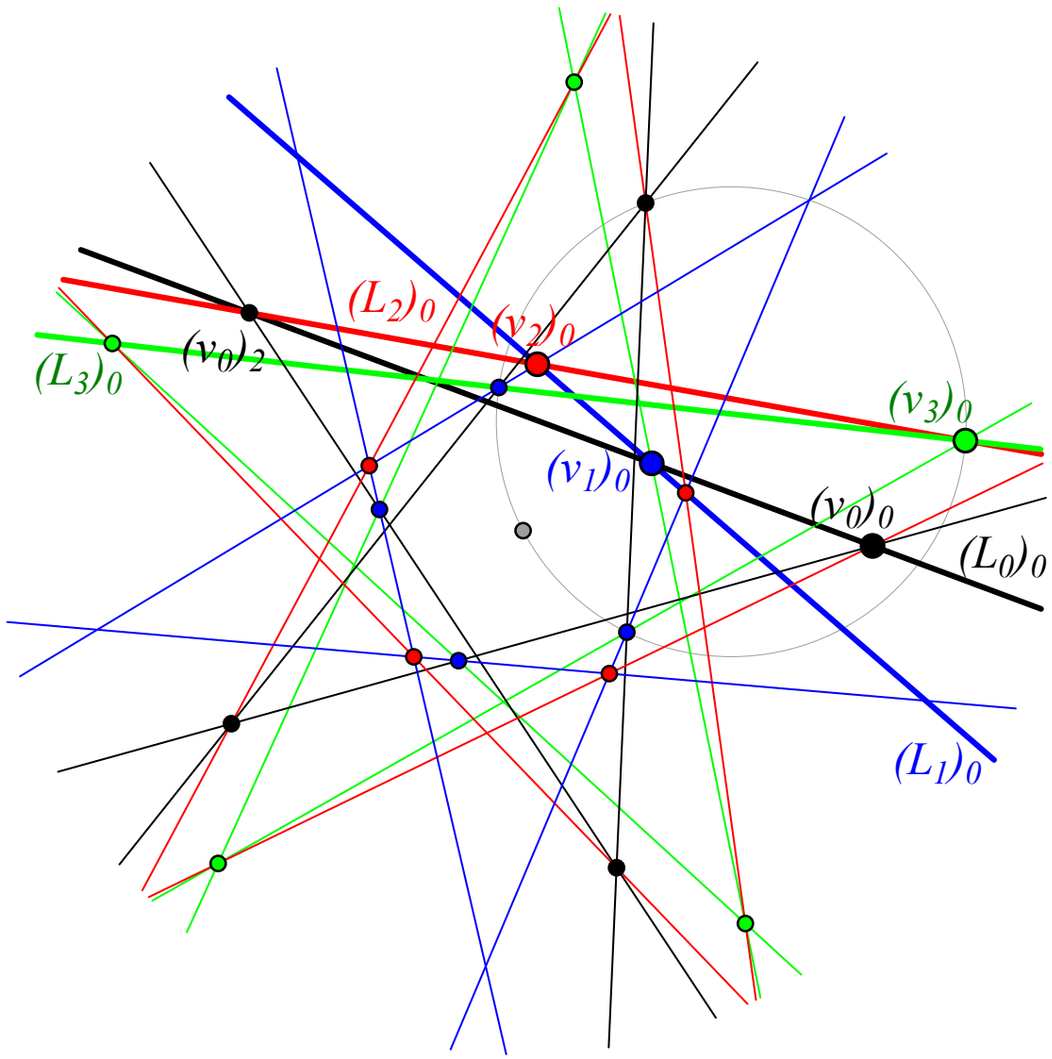
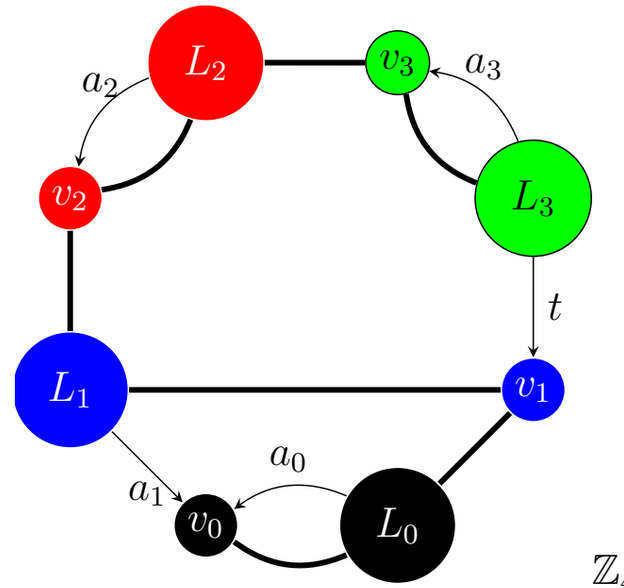
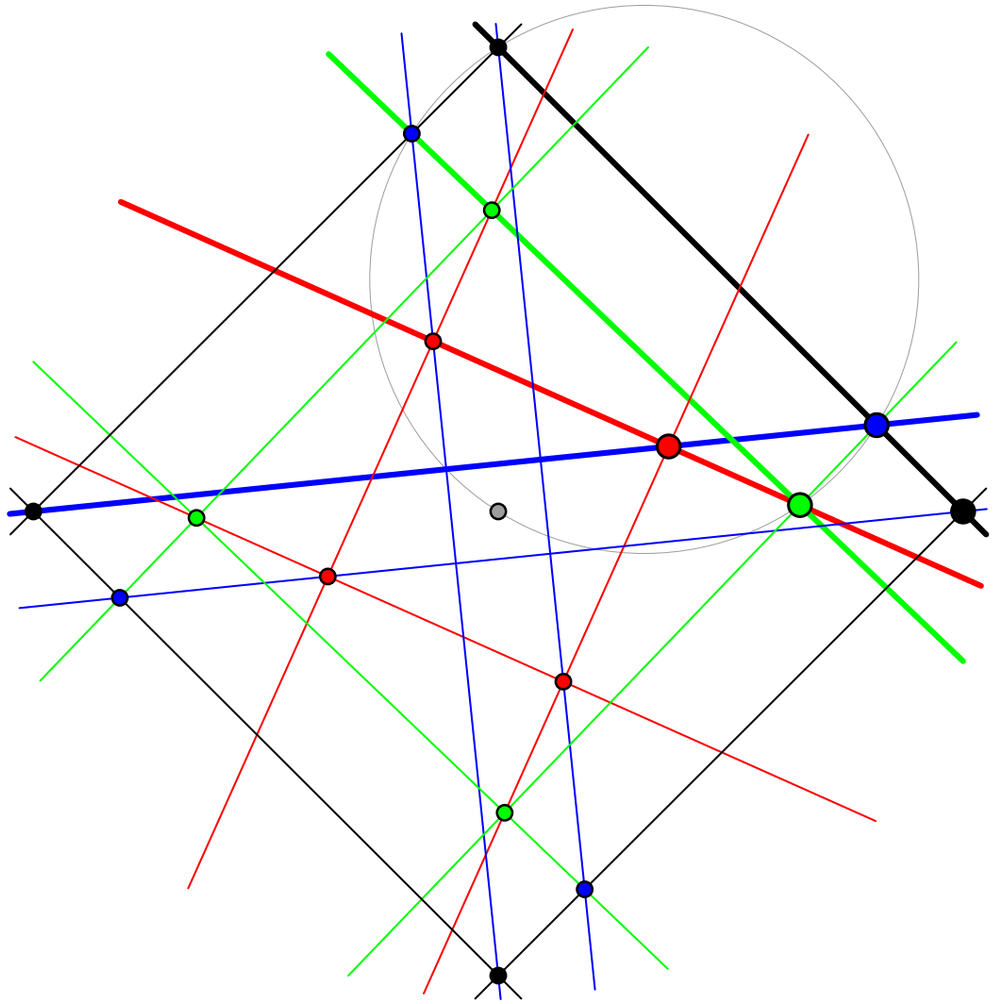


$\mathbb{Z}_5$

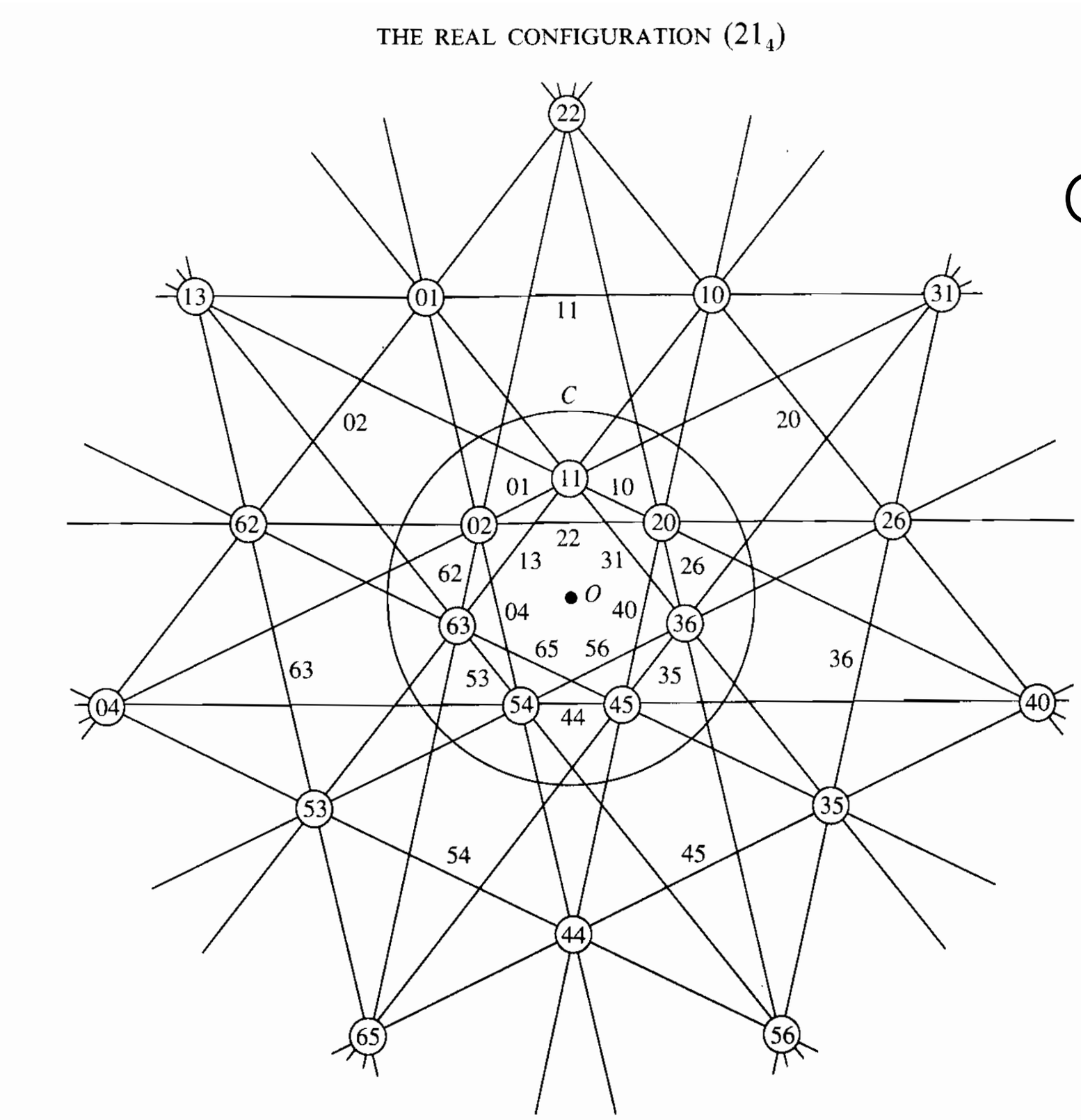




$\mathbb{Z}_4$



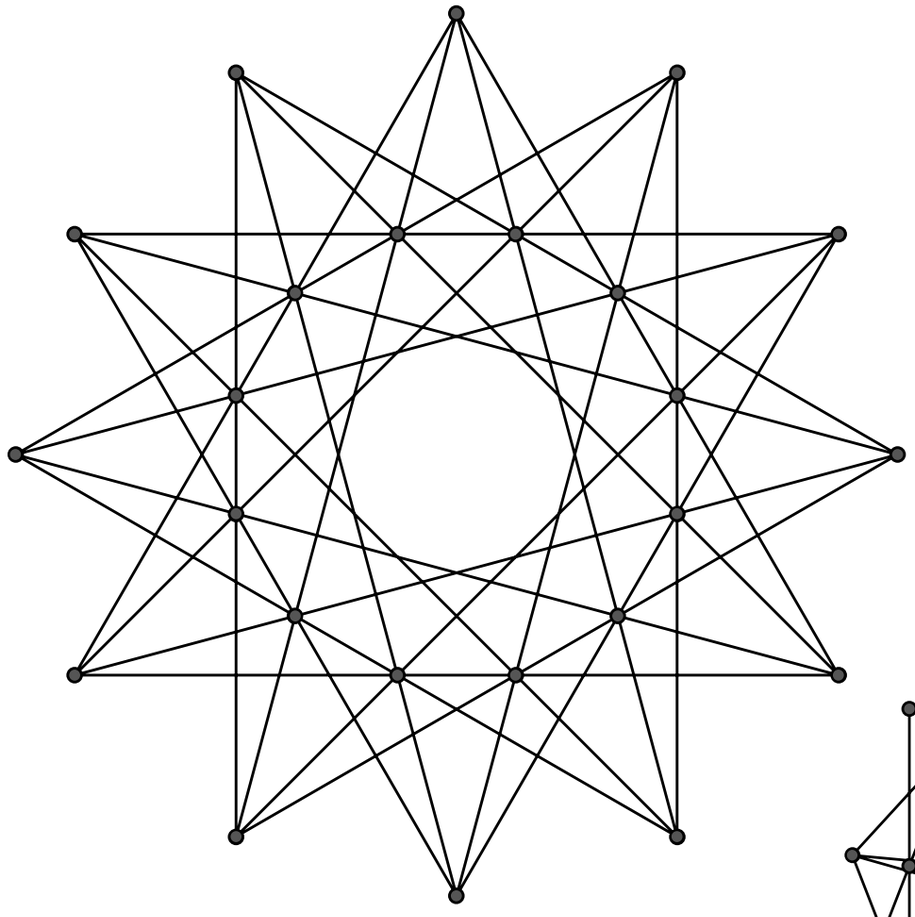
4-CONFIGURATIONS

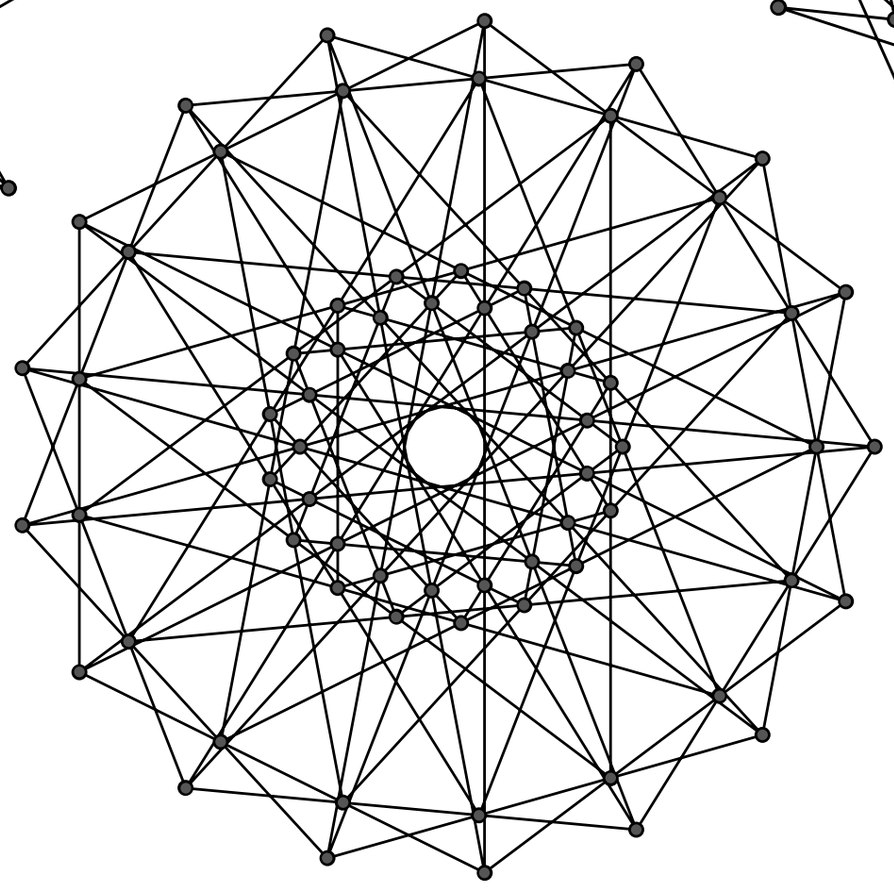


Grünbaum & Rigby, 1990

FIG. 1

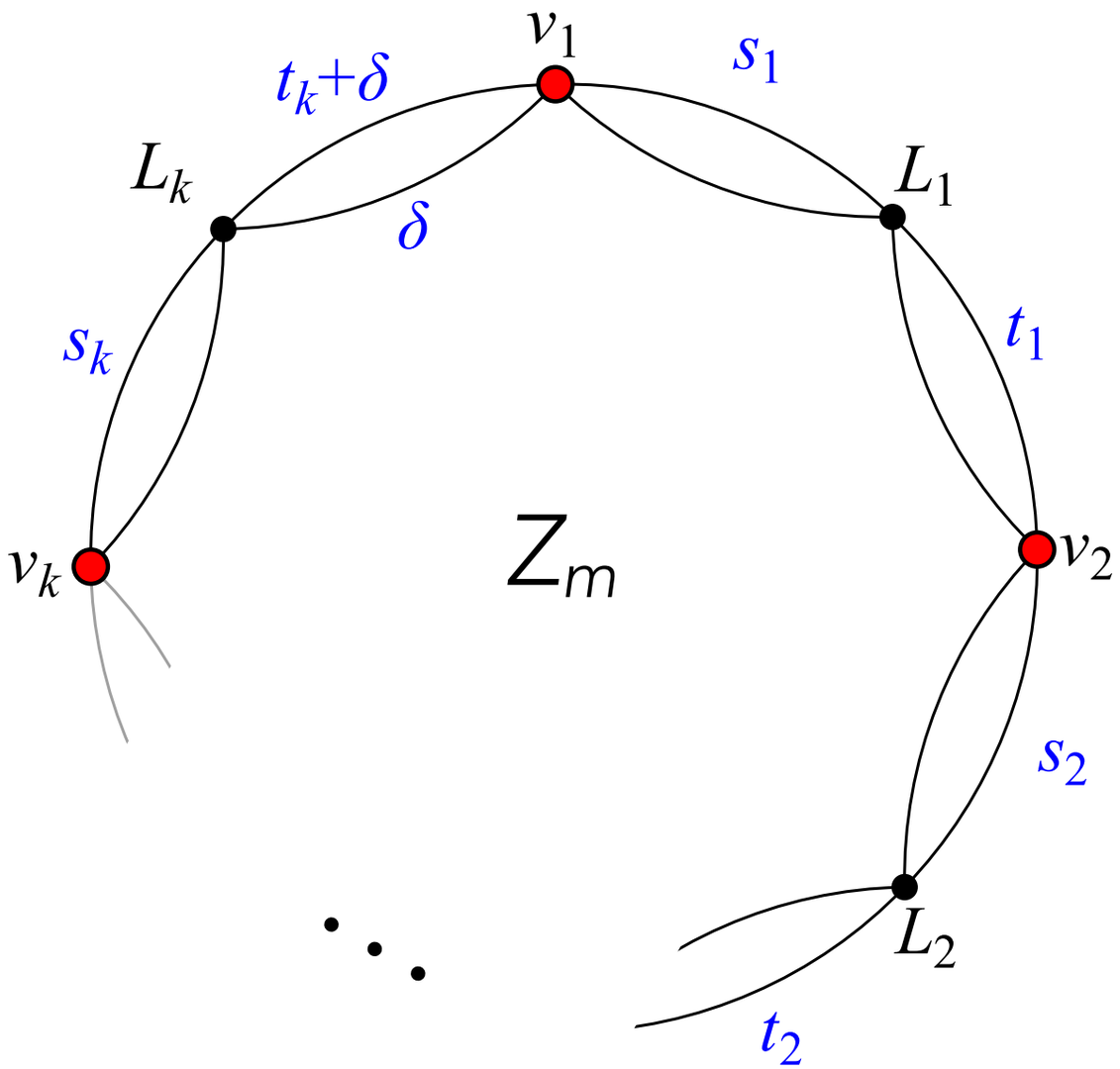
4-CONFIGURATIONS





Boben & Pisanski, 2003; Grünbaum 2000s, 2009

CELESTIAL 4-CONFIGURATIONS

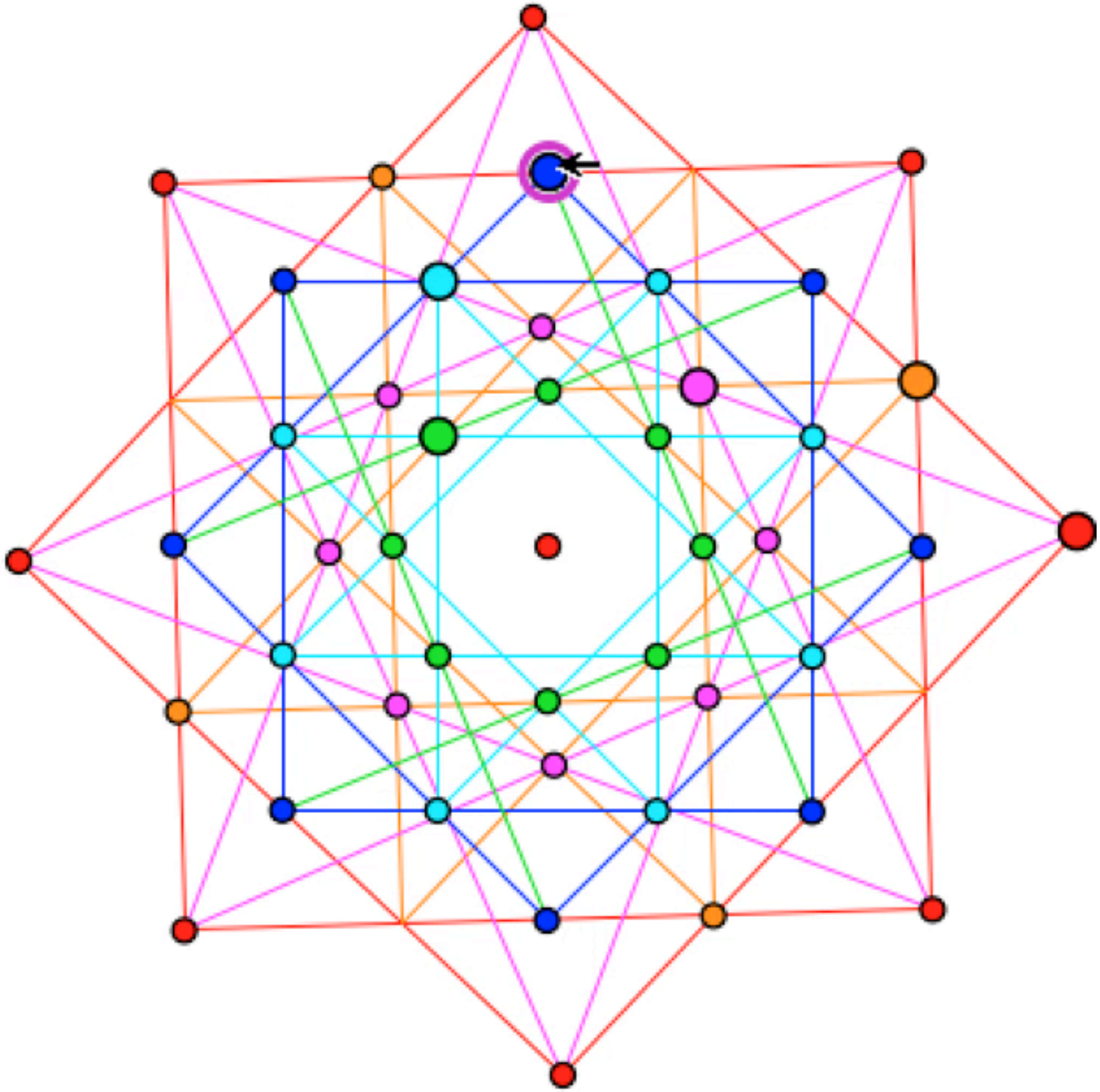


$m\#(s_1, t_1; \ldots; s_k, t_k)$

$2\delta = (s_1 - t_1) + \ldots + (s_k - t_k)$

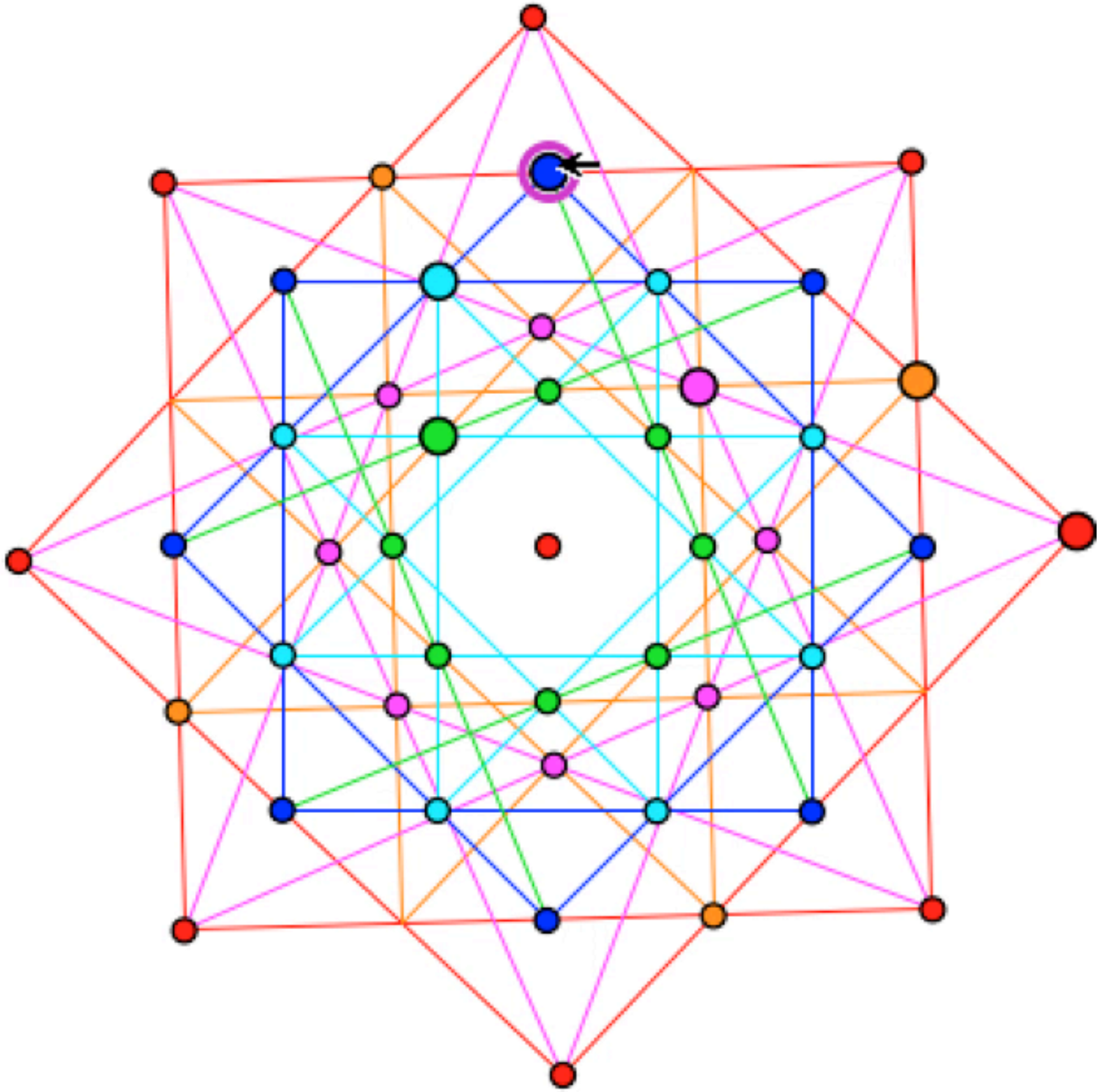
and 4 axioms

MOVABLE 4-CONFIGURATIONS



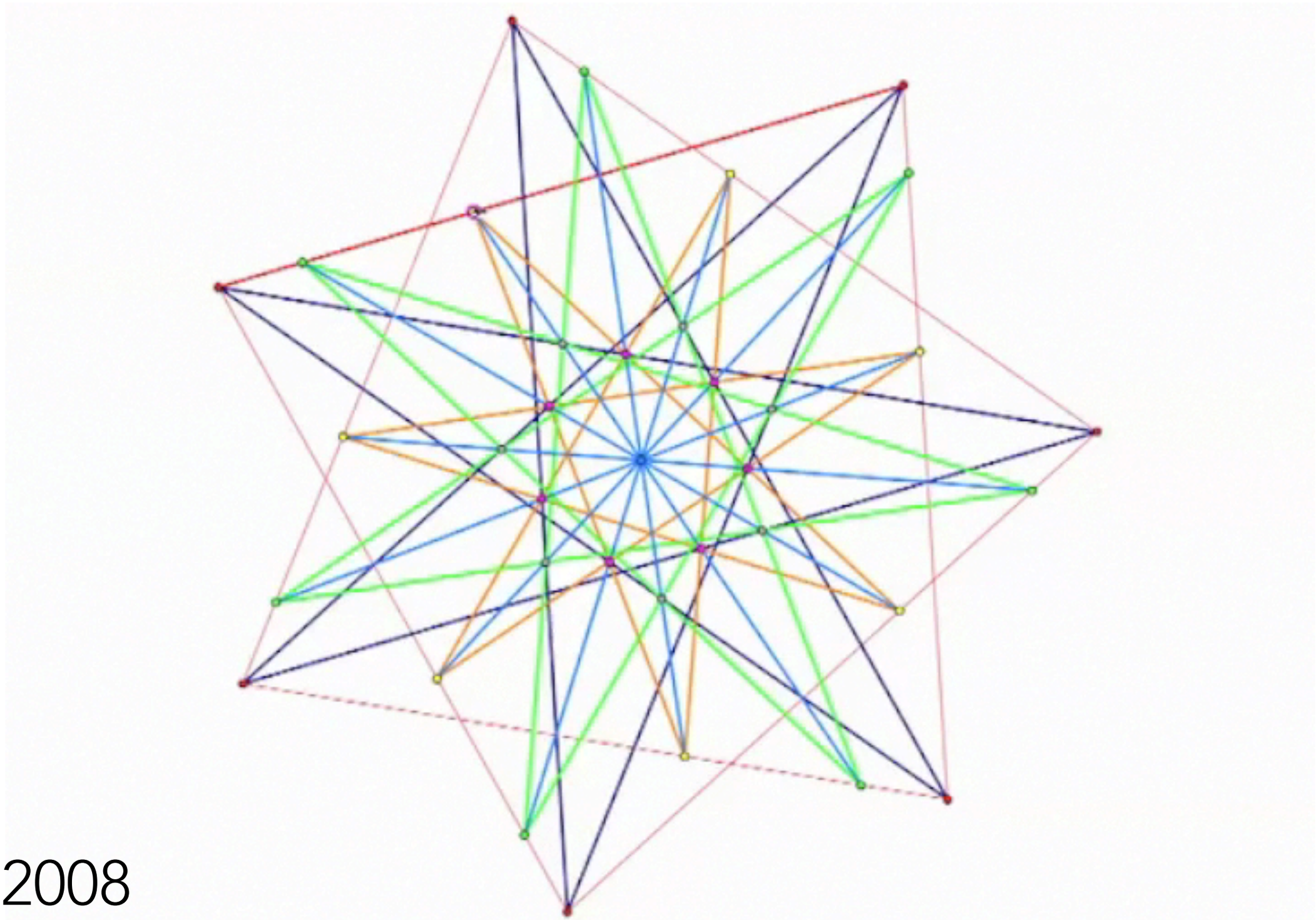
B. 2007

MOVABLE 4-CONFIGURATIONS



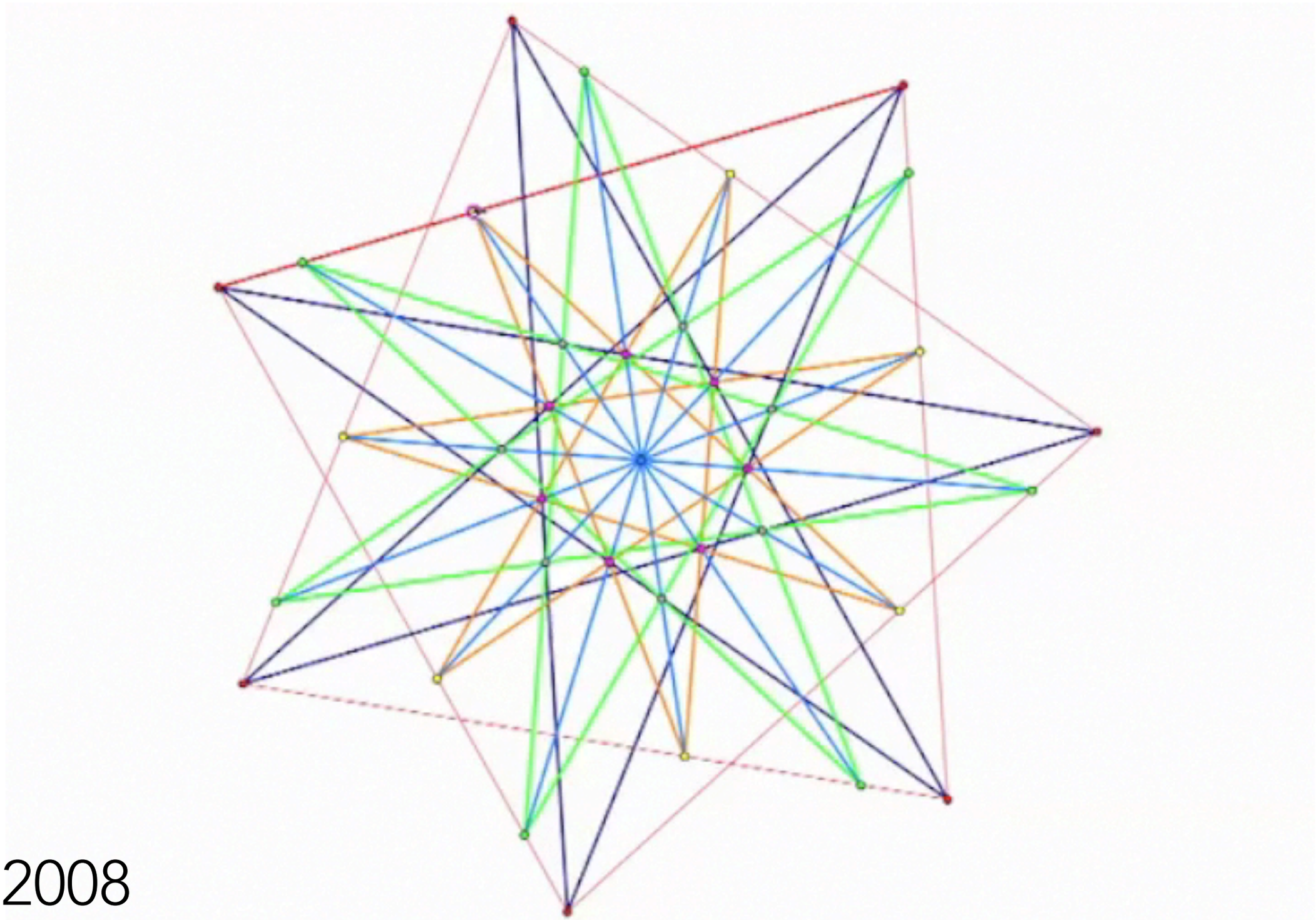
B. 2007

MOVABLE 4-CONFIGURATIONS



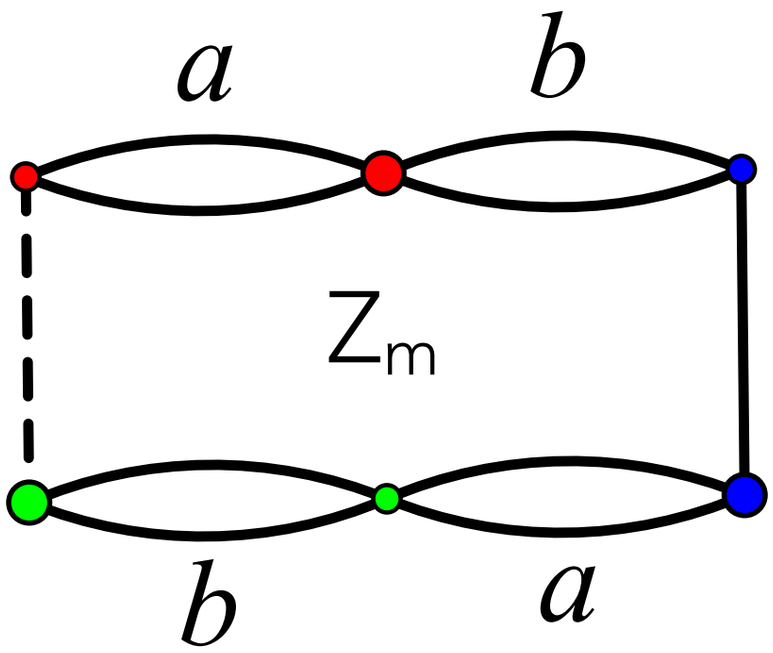
B. 2008

MOVABLE 4-CONFIGURATIONS

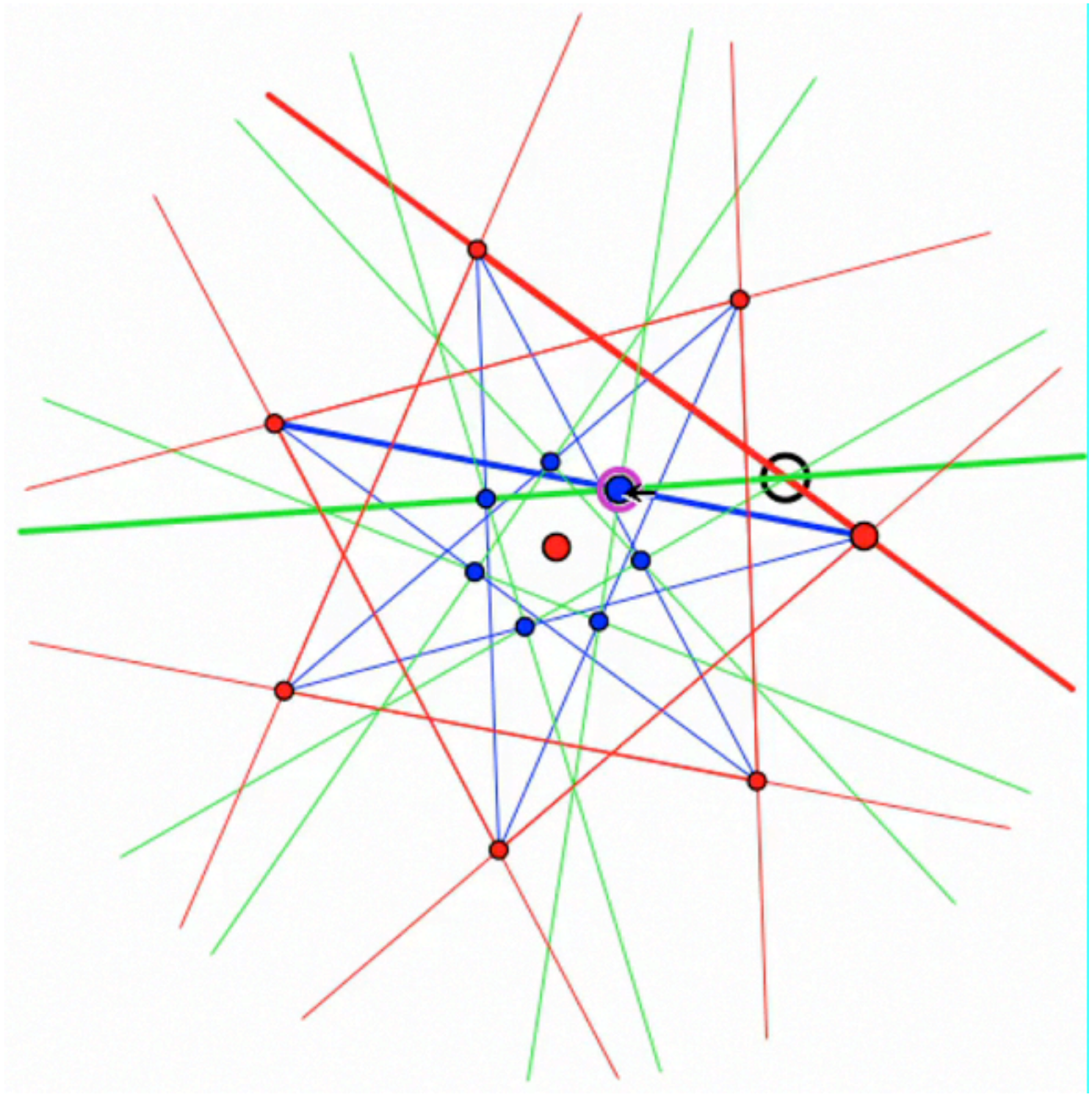


B. 2008

CROSSING SPANS LEMMA

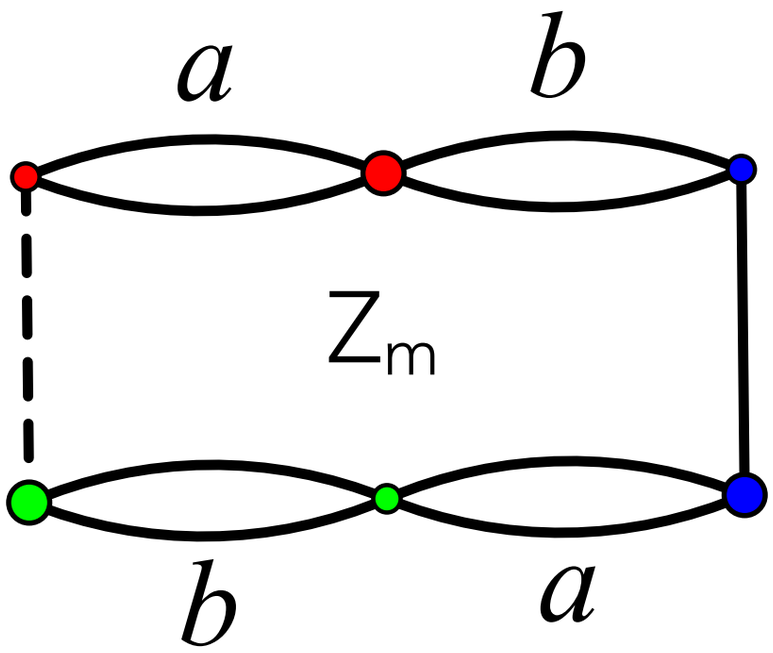


$m=7$, $a=2$, $b=3$

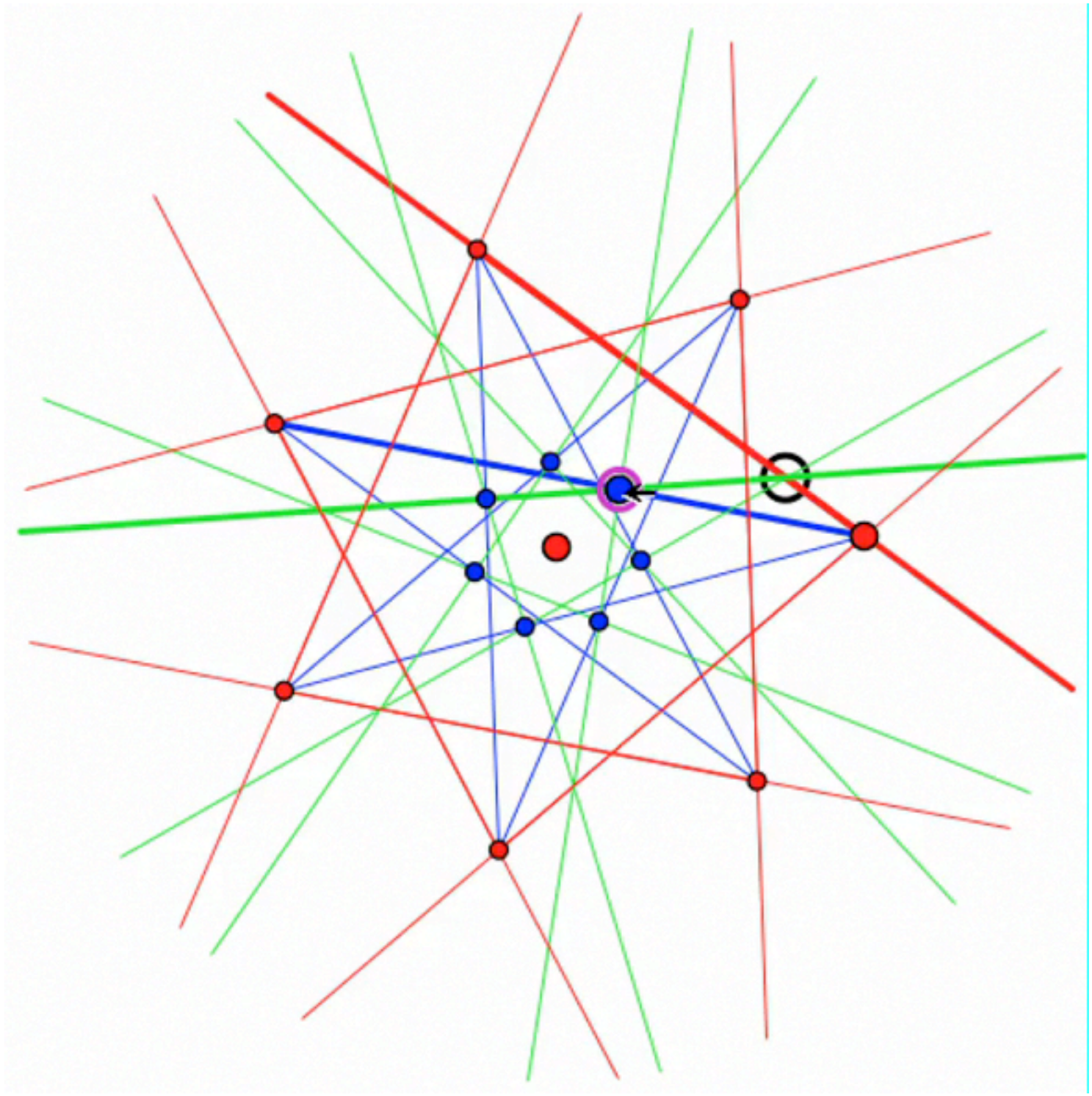


B., 2006

CROSSING SPANS LEMMA

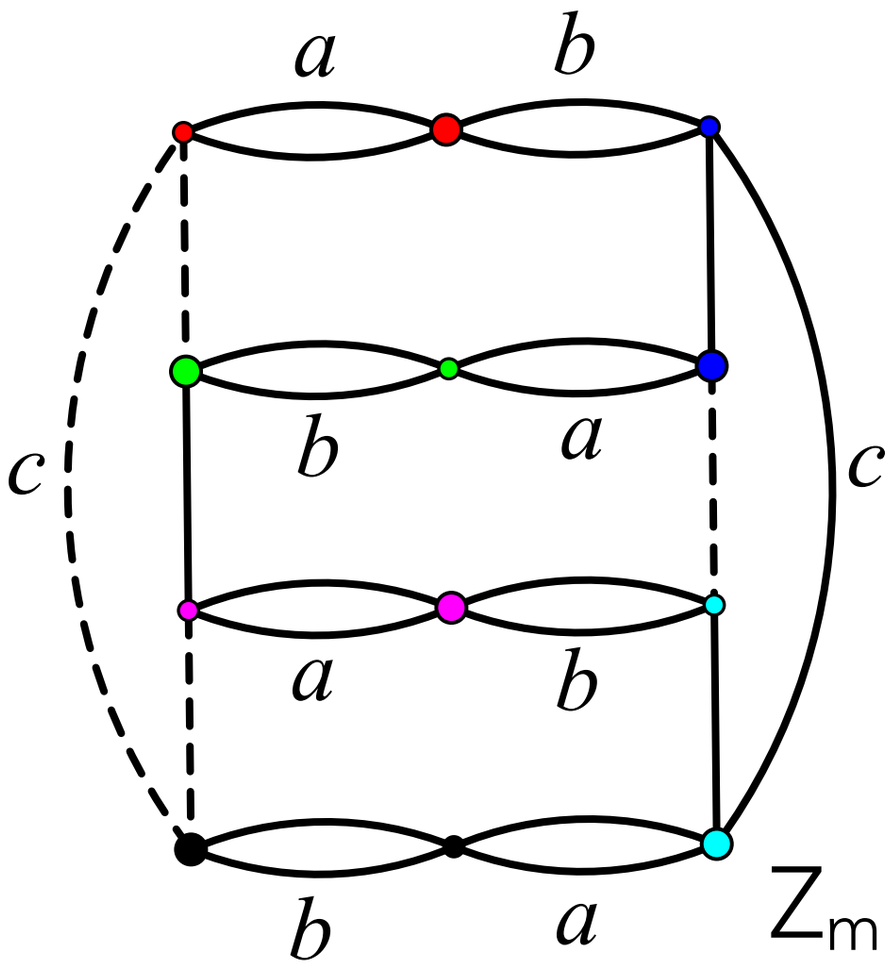


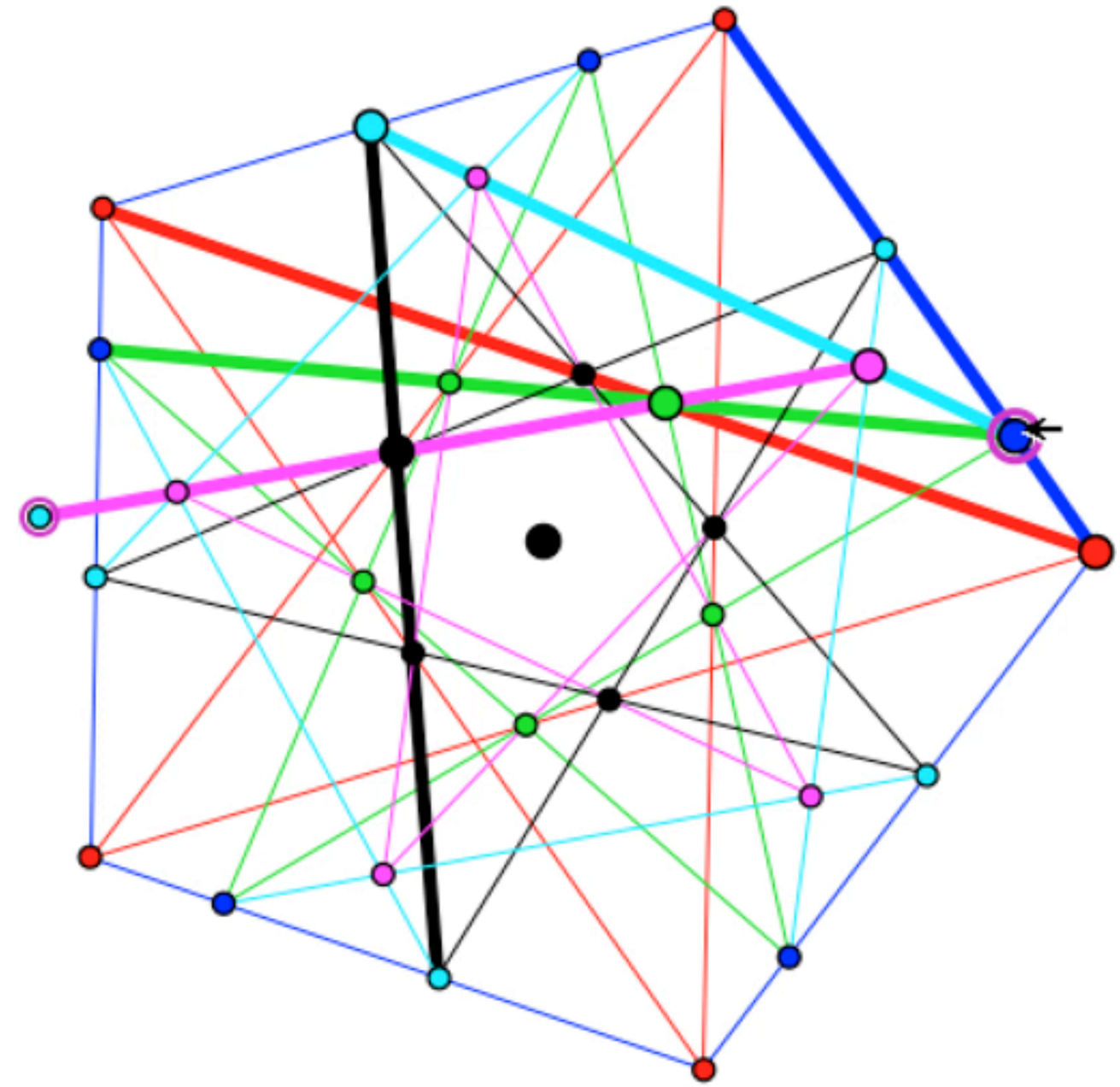
$m=7$, $a=2$, $b=3$



B., 2006

MOVABLE 4-CONFIGURATIONS

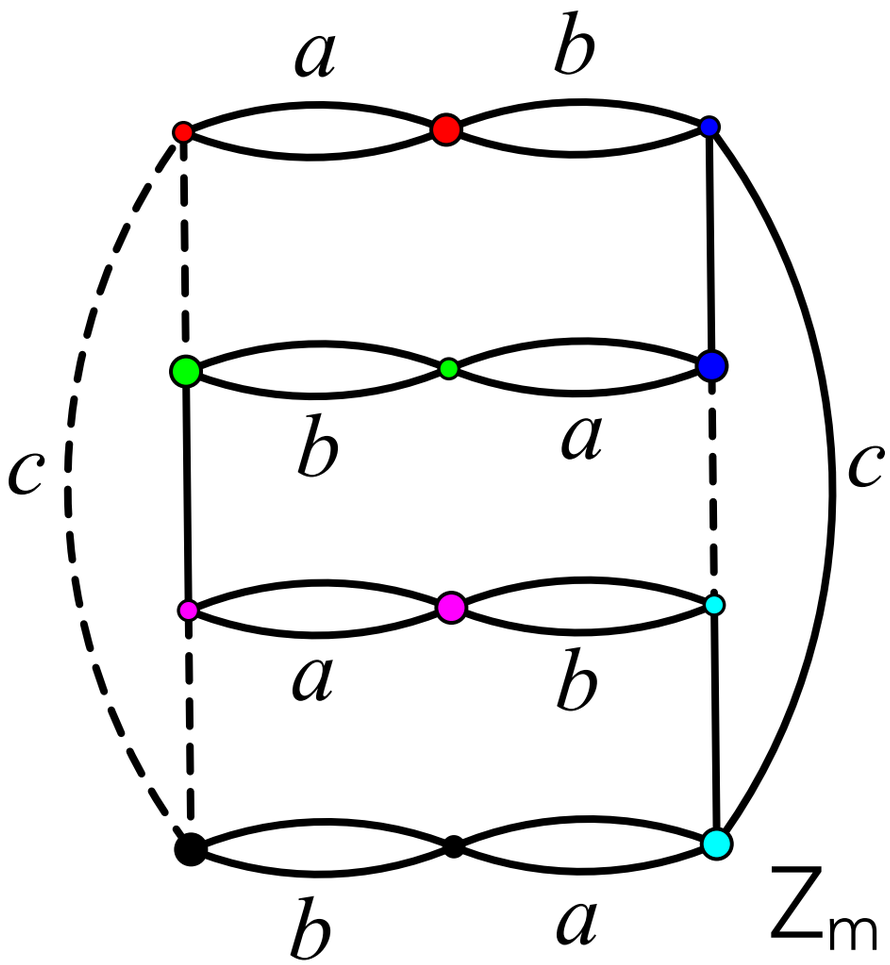


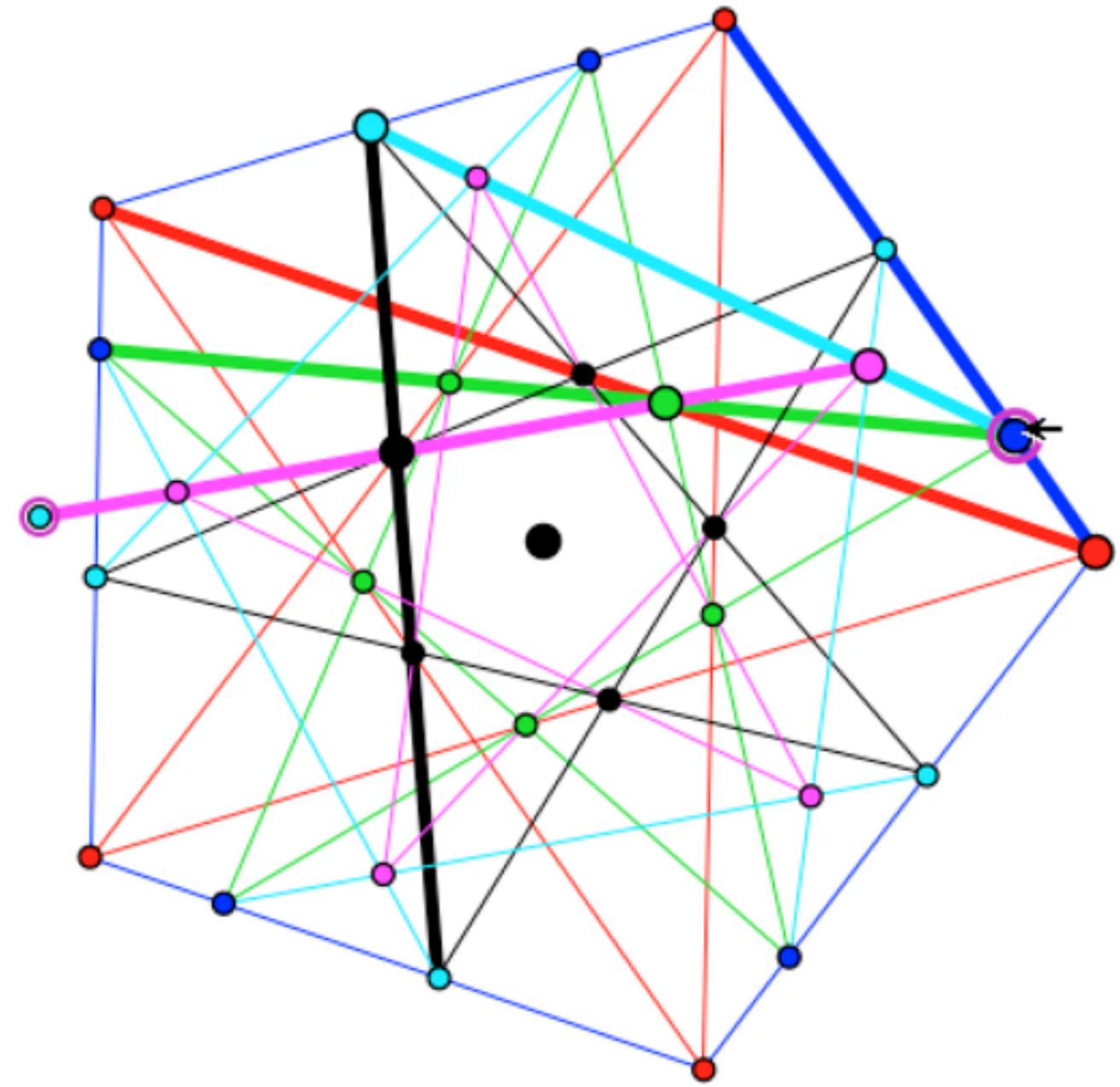


m=5, a=2, b=1, c=1

B., J. Faudree, T. Pisanski, in review

MOVABLE 4-CONFIGURATIONS

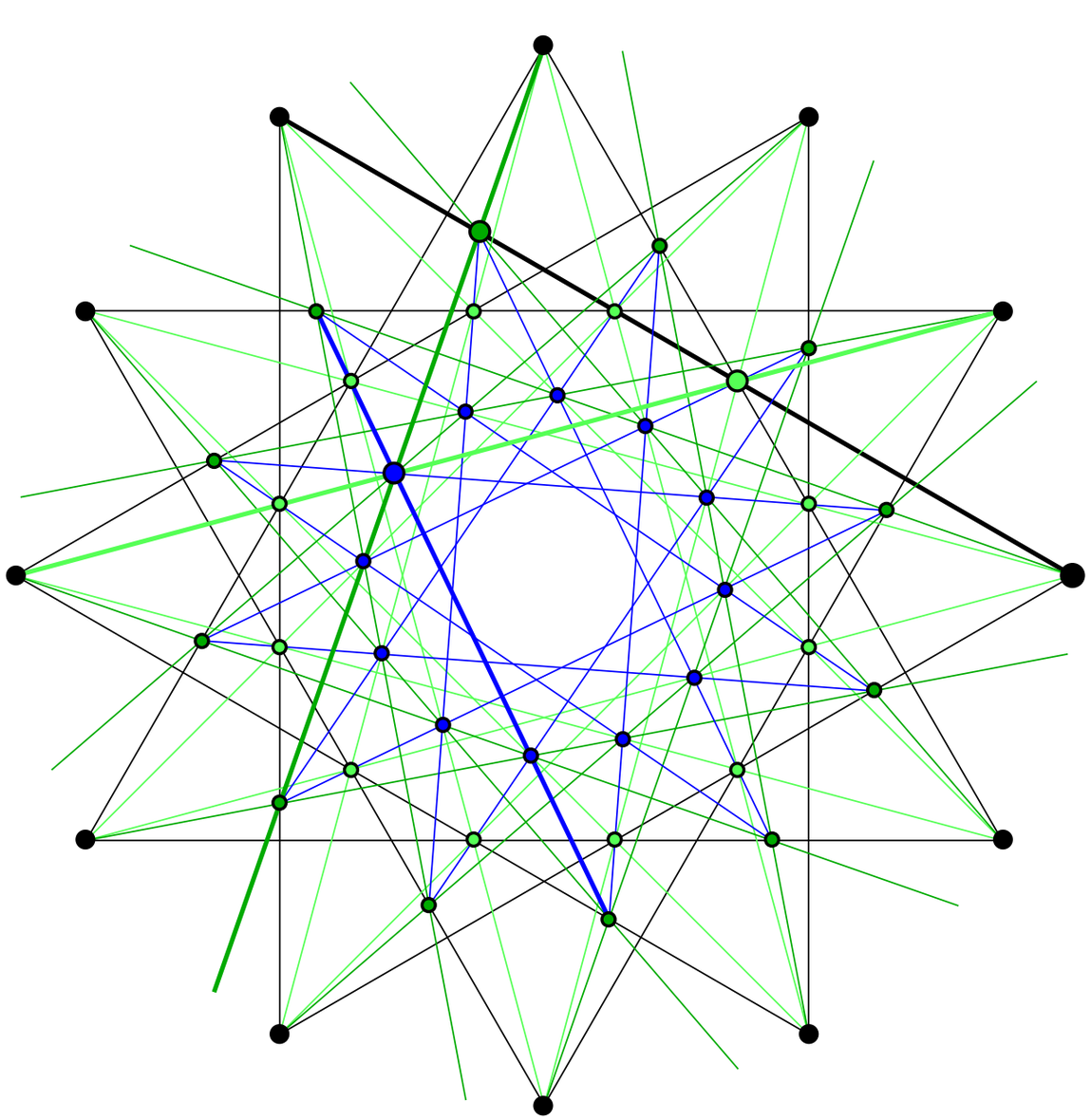




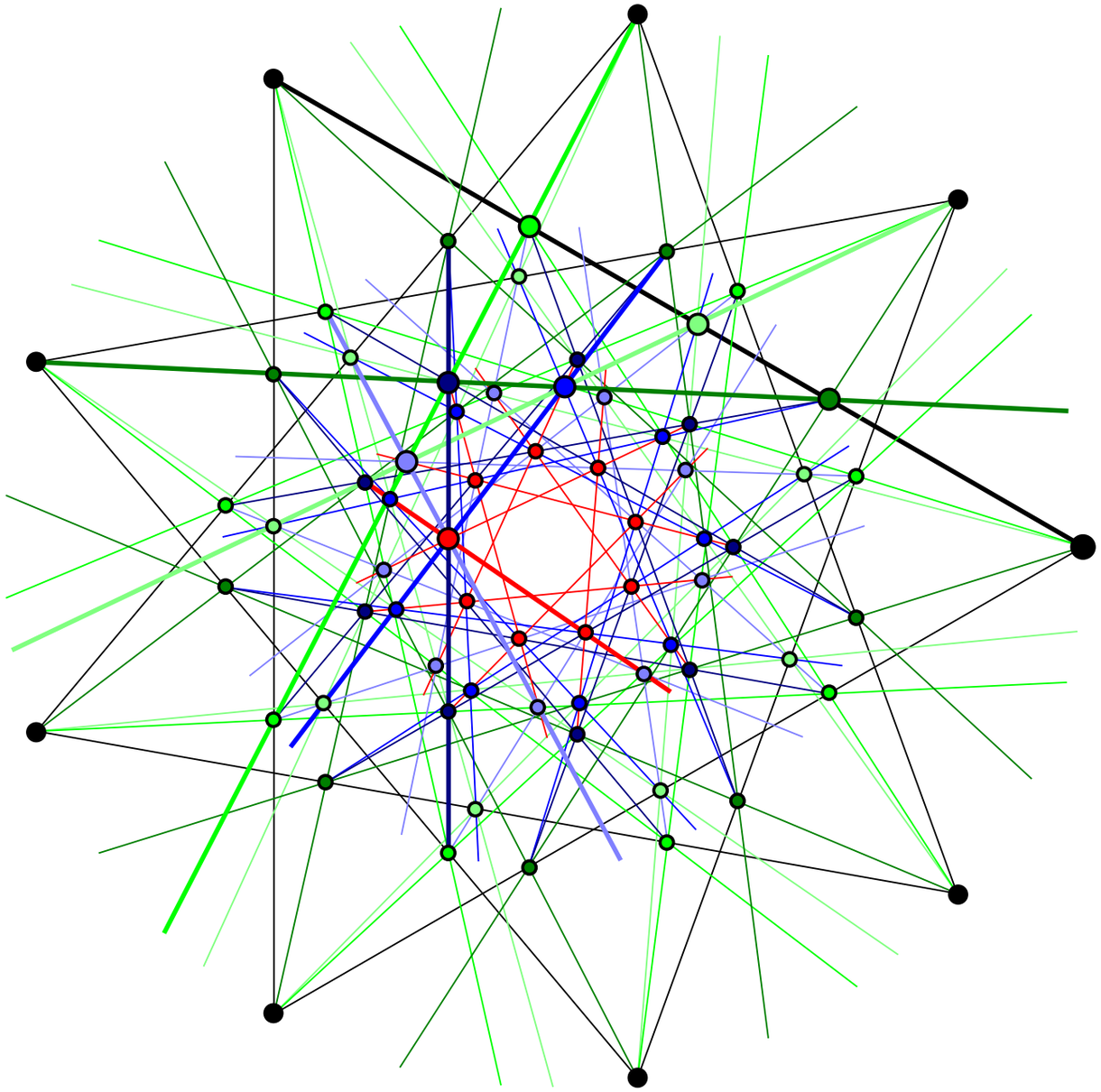
m=5, a=2, b=1, c=1

B., J. Faudree, T. Pisanski, in review

5-CONFIGURATIONS



B. & Ng, 2010

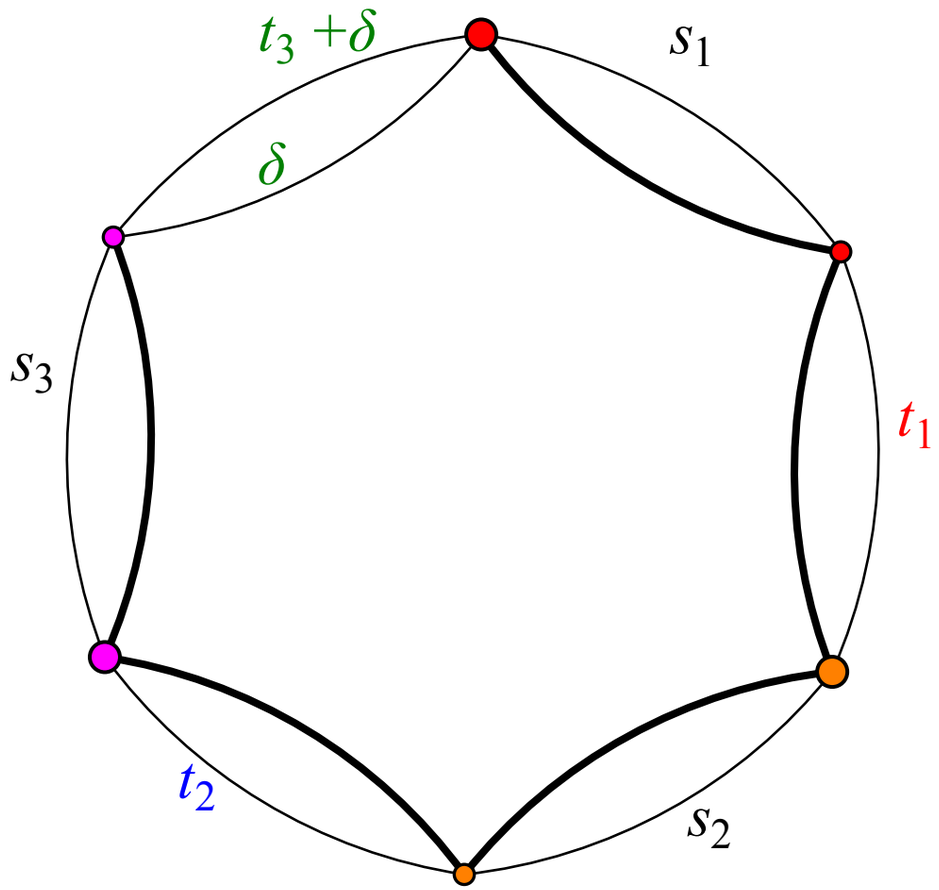


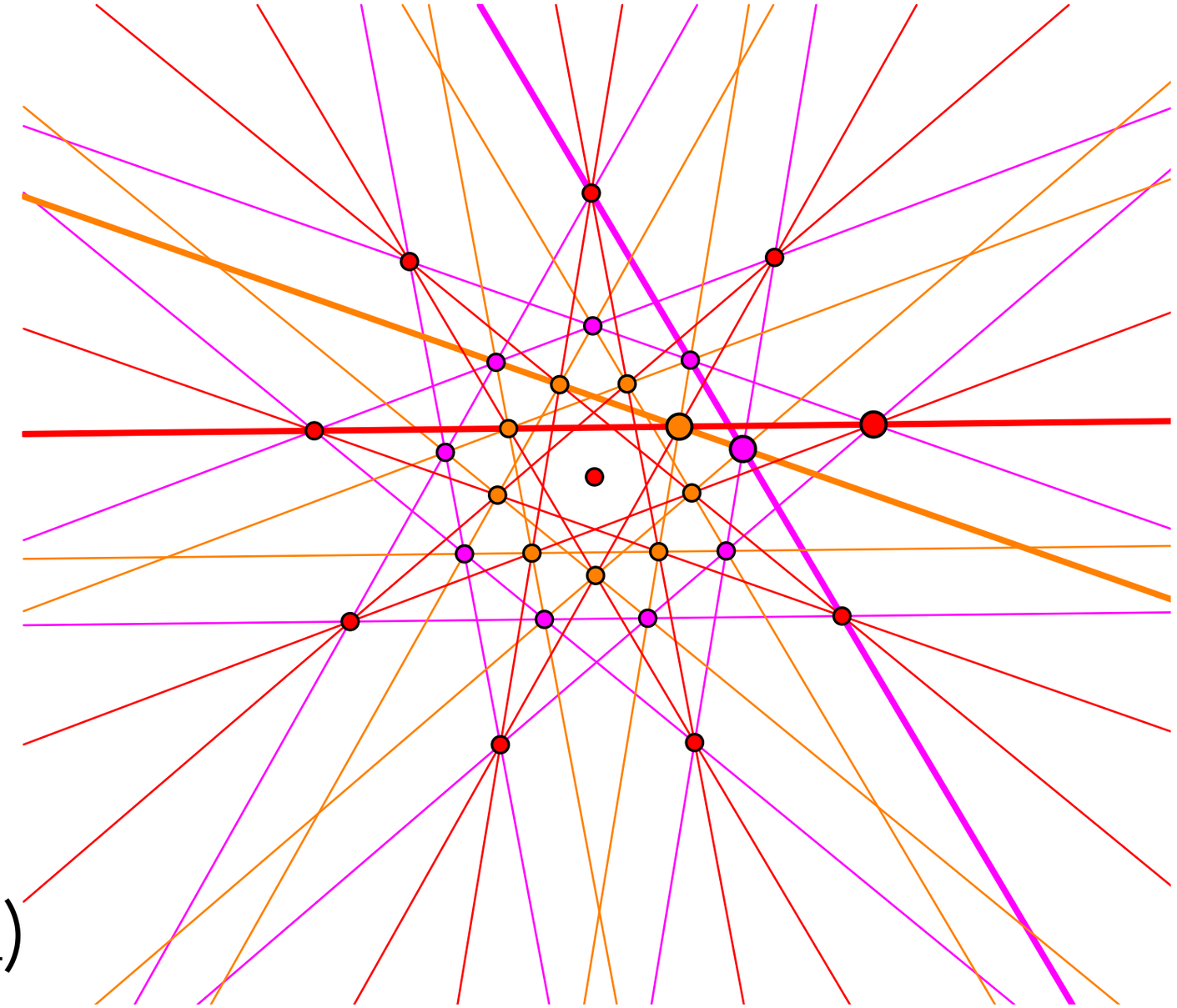
B. & J. Faudree, 2013

Static!

MOVABLE 5-CONFIGURATIONS

Joint with Lander VerHoef



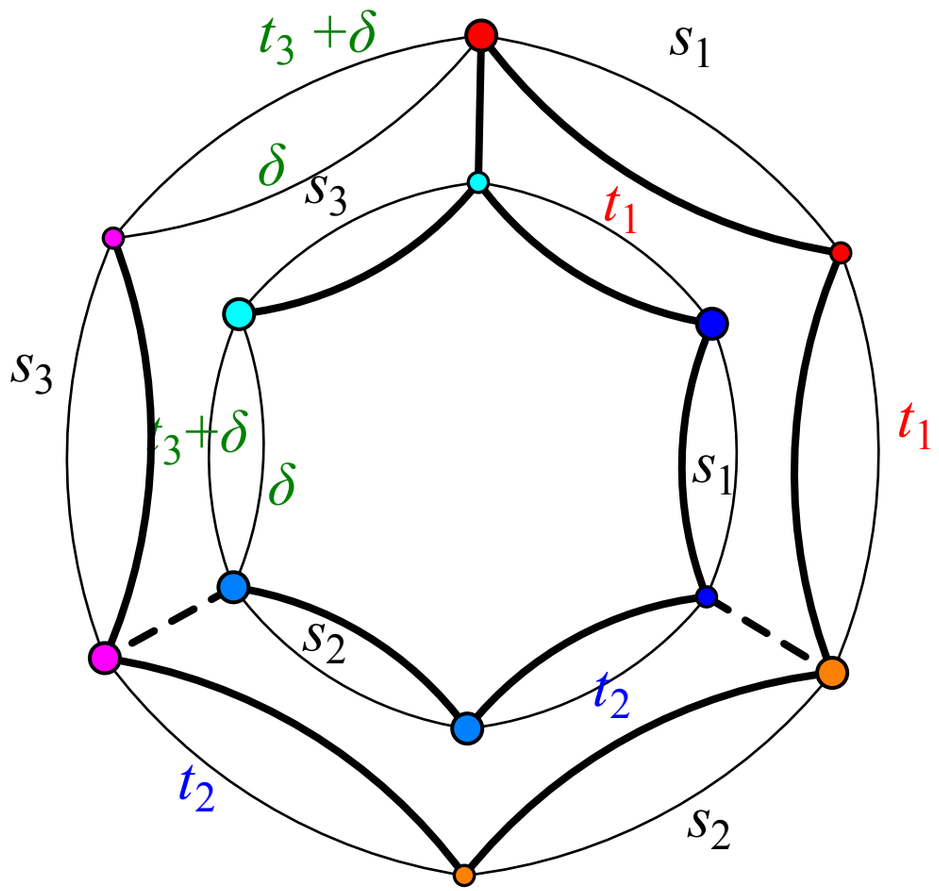


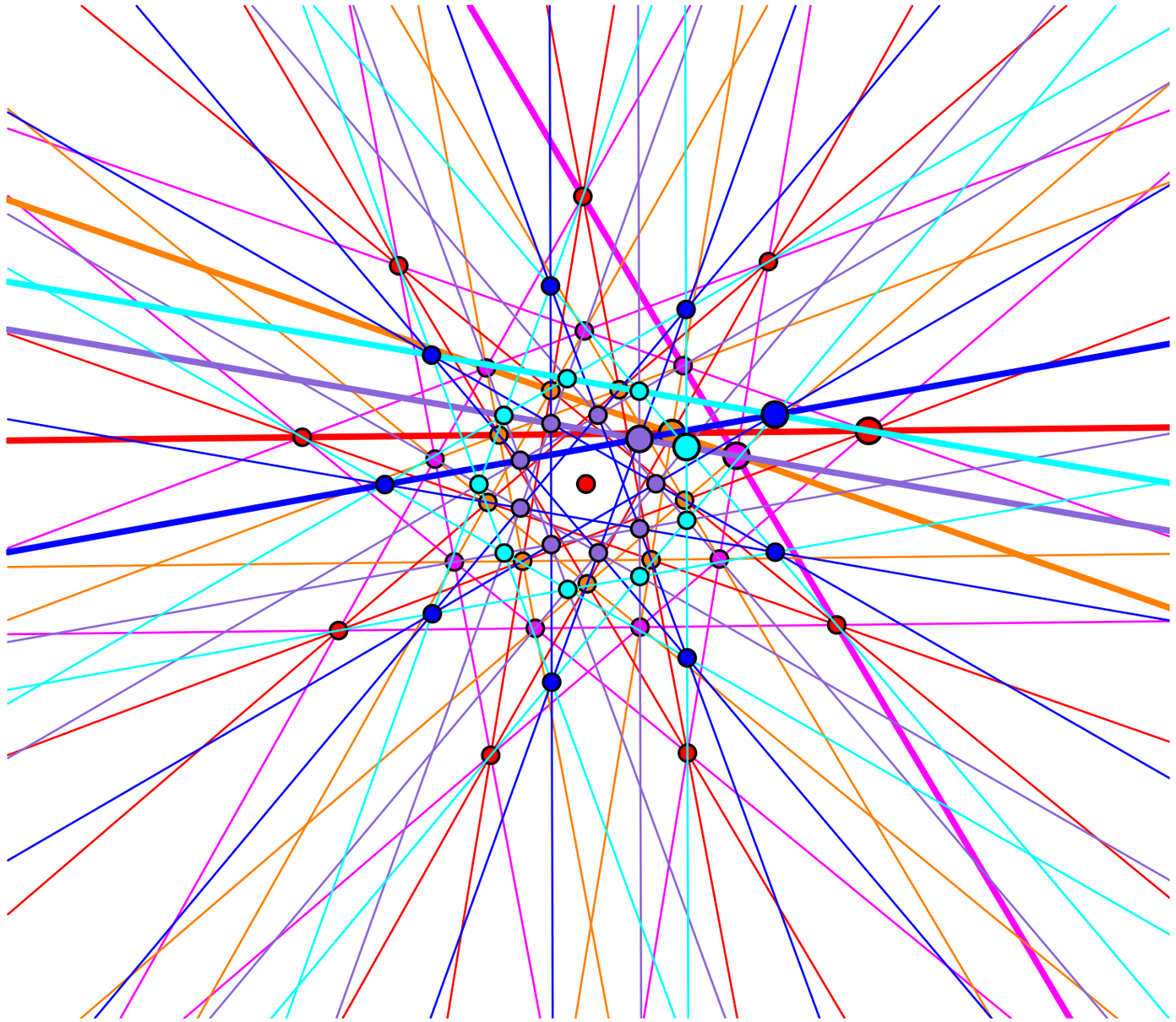
(27_4)

Celestial Configuration m#(s_1, t_1; s_2, t_2; s_3, t_3); $S \cap T = \varnothing$

MOVABLE 5-CONFIGURATIONS

Joint with Lander VerHoef

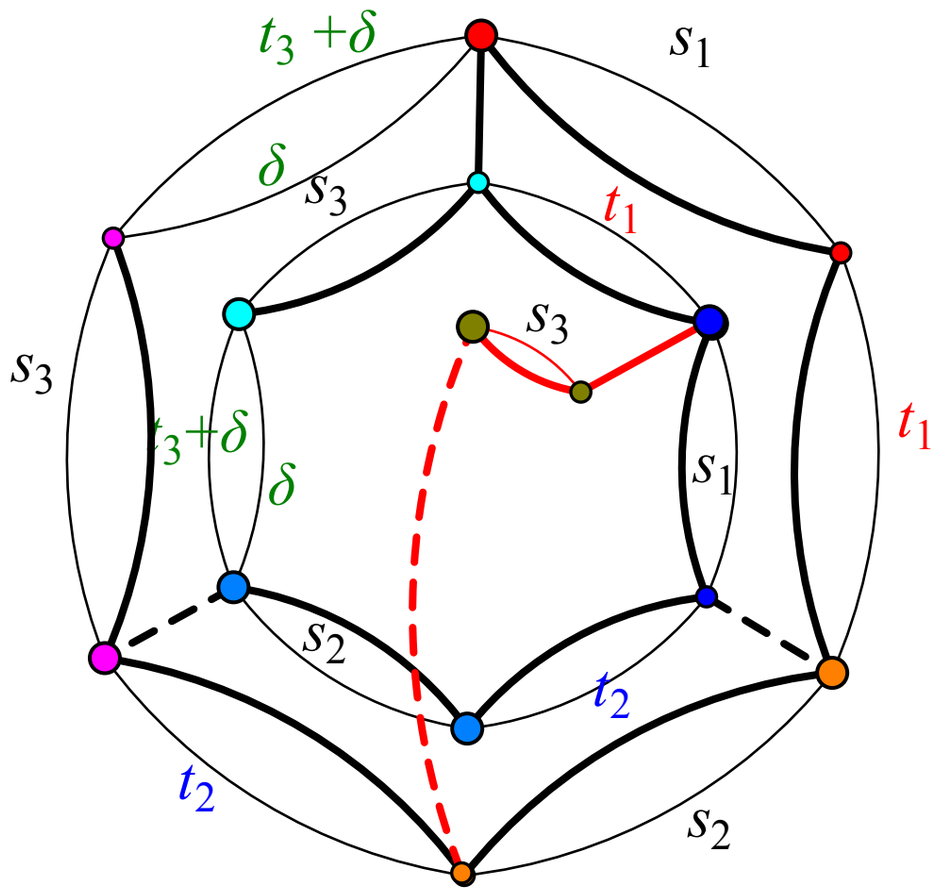


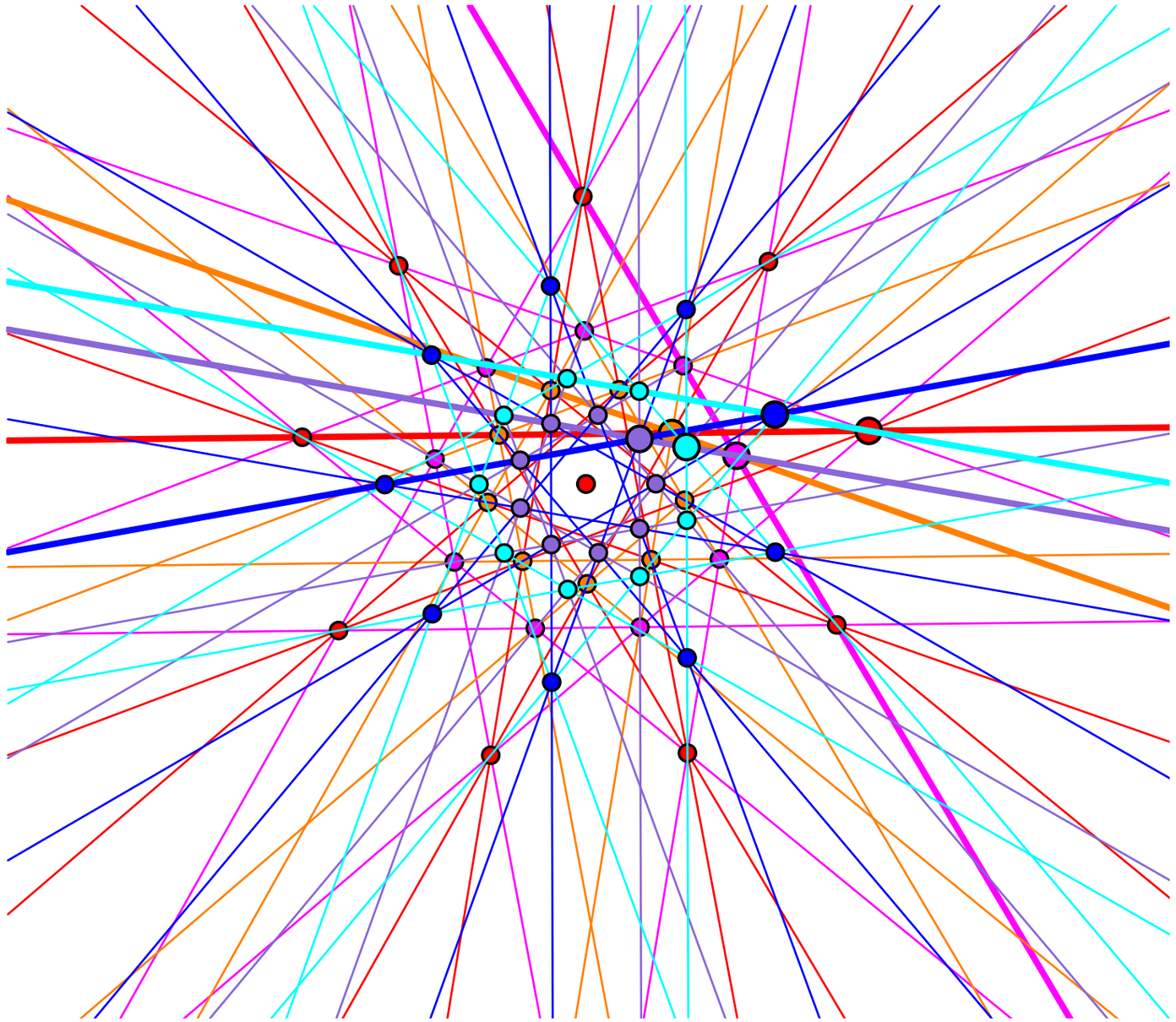


Celestial Configuration m#(s_1, t_1; s_2, t_2; s_3, t_3); $S \cap T = \varnothing$

MOVABLE 5-CONFIGURATIONS

Joint with Lander VerHoef

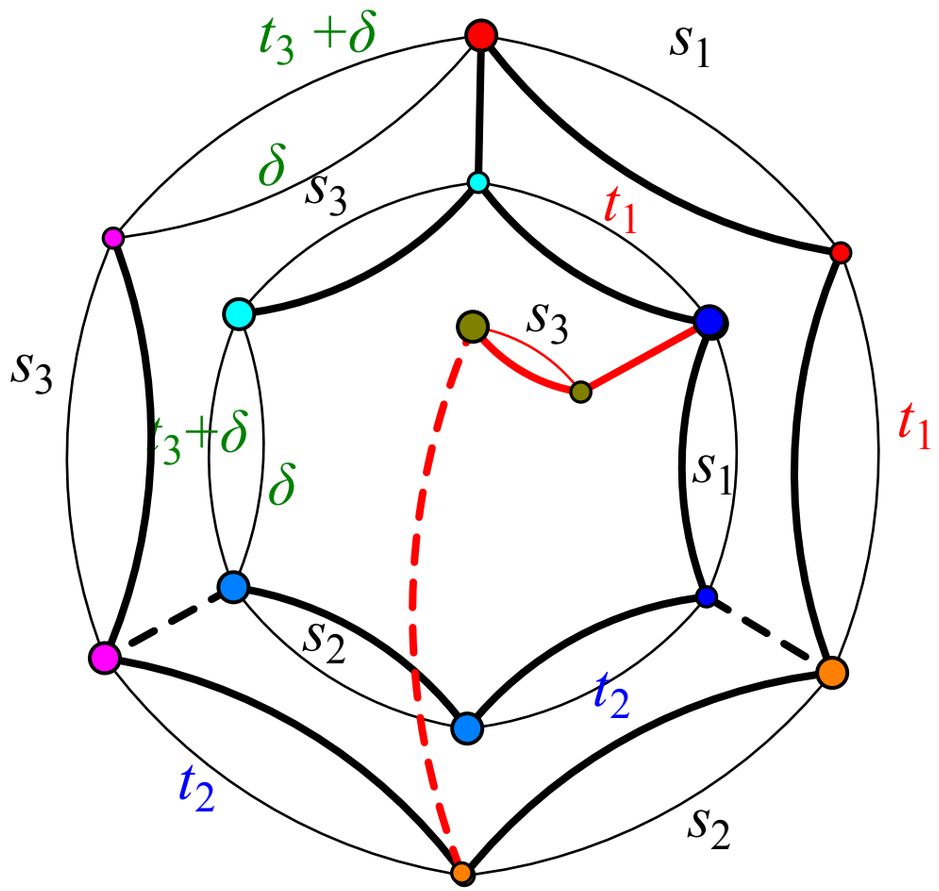


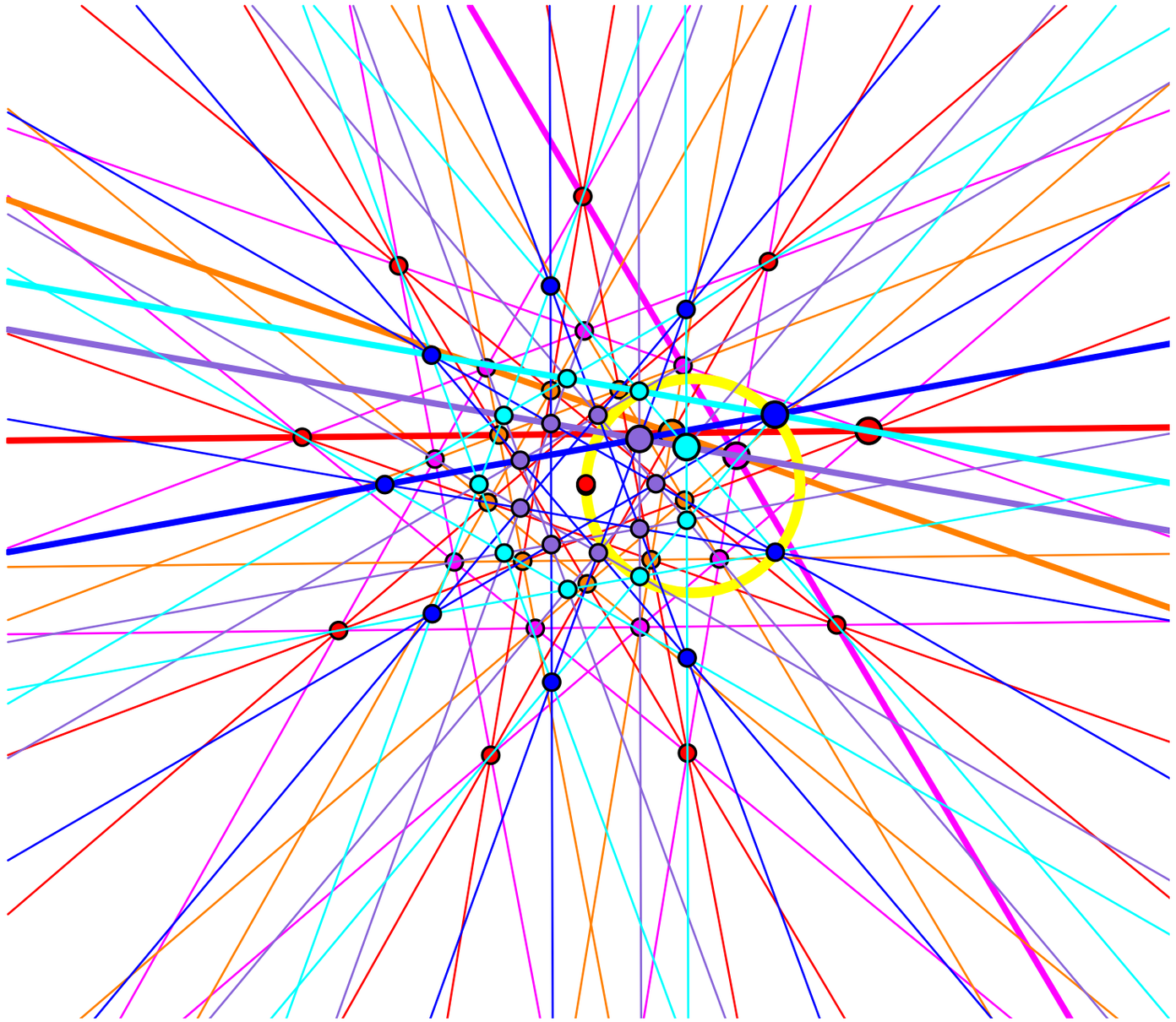


Celestial Configuration m#(s_1, t_1; s_2, t_2; s_3, t_3); $S \cap T = \varnothing$

MOVABLE 5-CONFIGURATIONS

Joint with Lander VerHoef

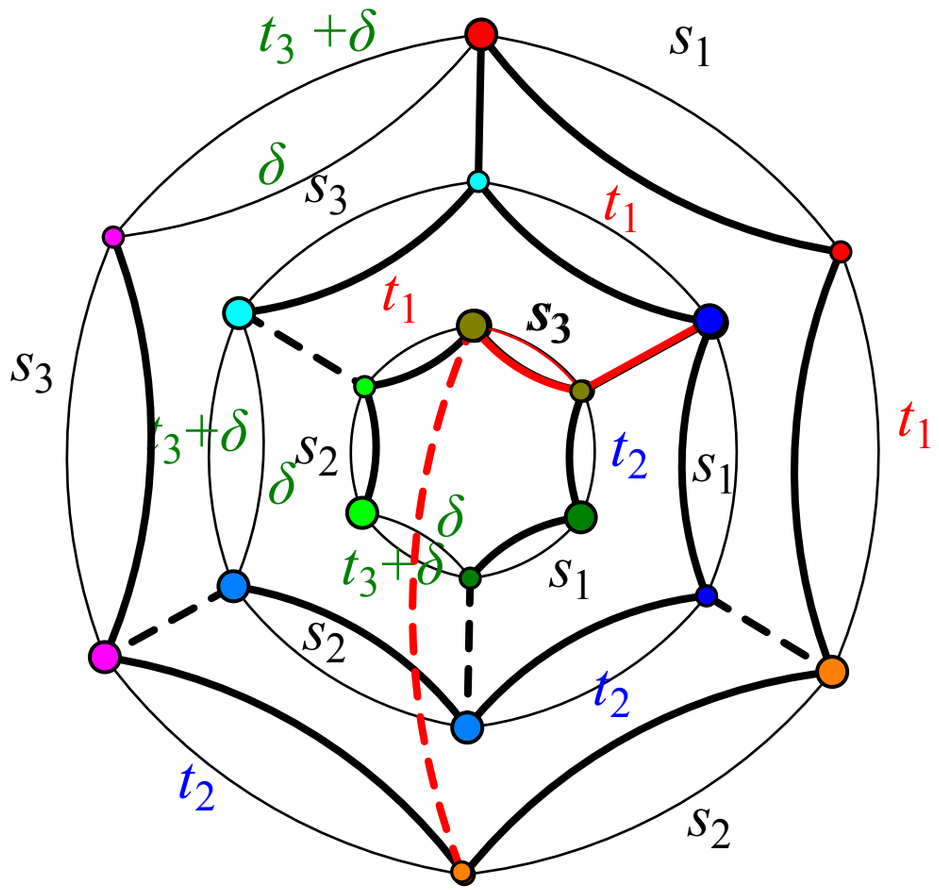


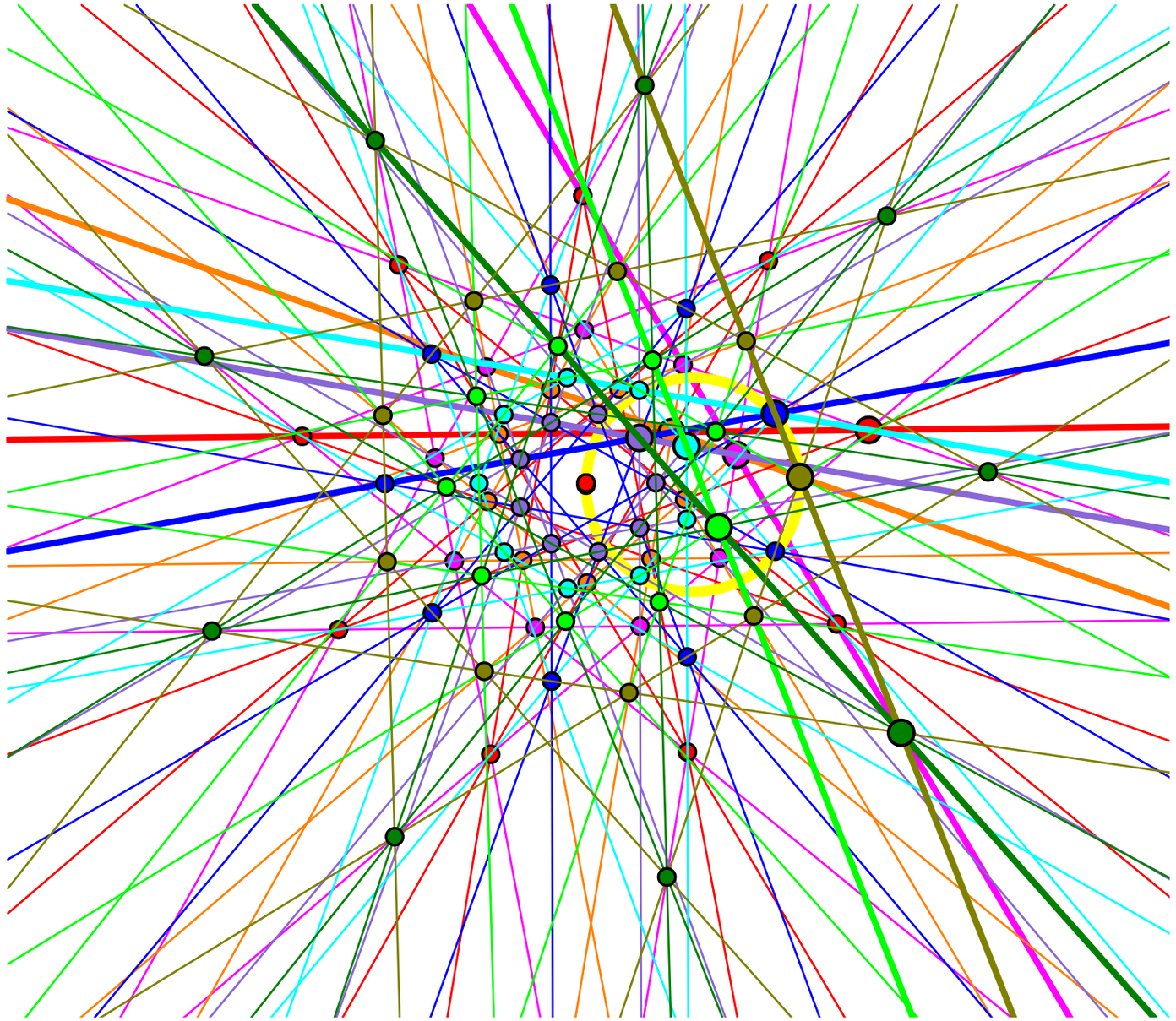


Celestial Configuration m#(s_1, t_1; s_2, t_2; s_3, t_3); $S \cap T = \varnothing$

MOVABLE 5-CONFIGURATIONS

Joint with Lander VerHoef

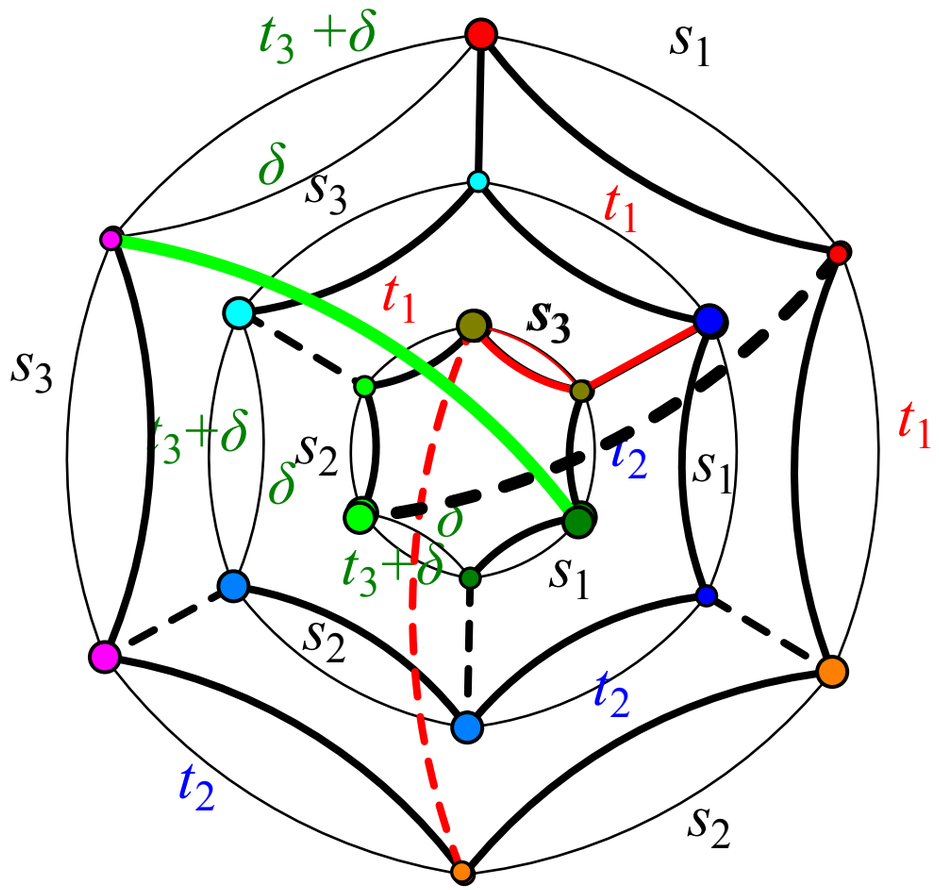


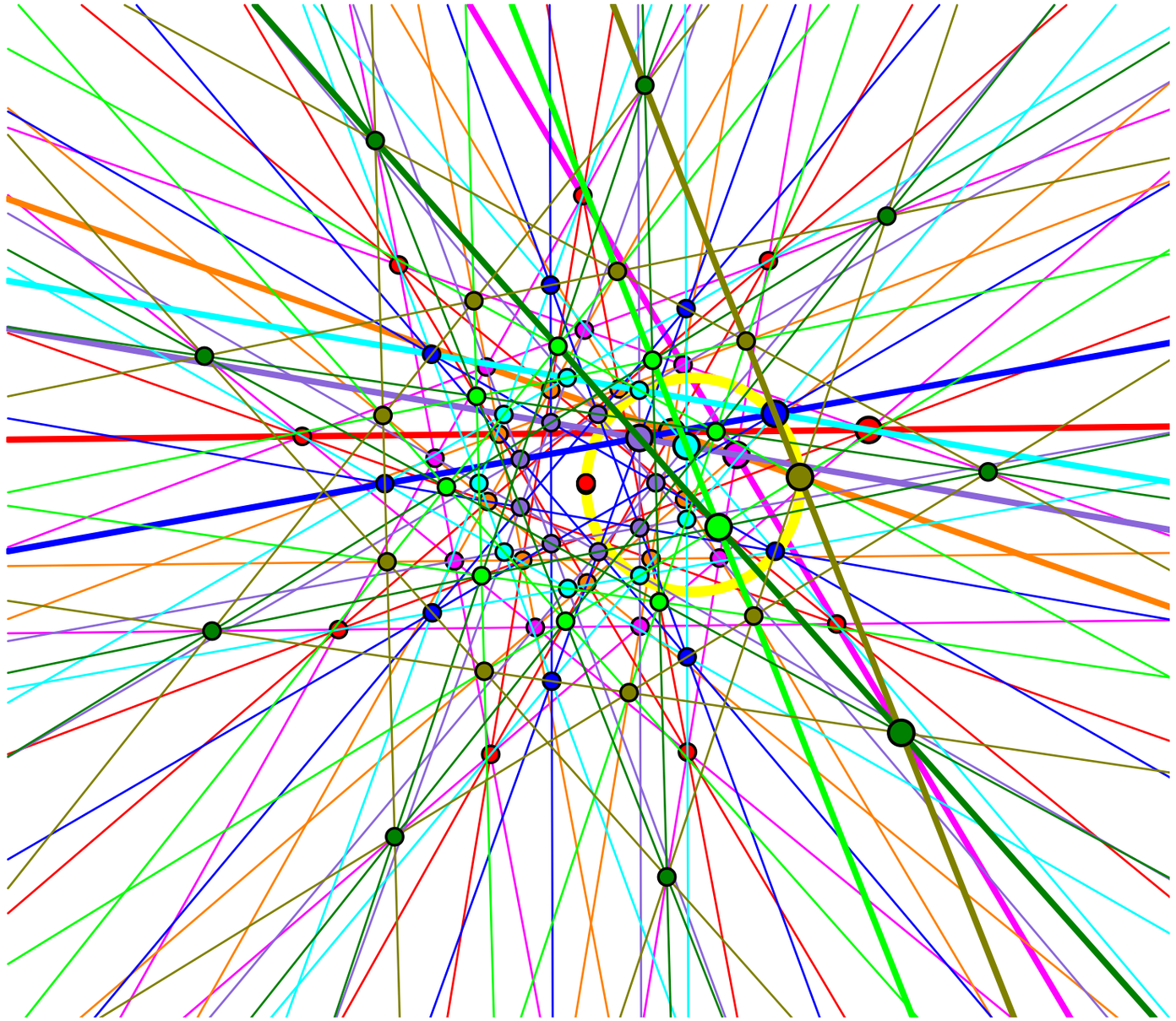


Celestial Configuration m#(s_1, t_1; s_2, t_2; s_3, t_3); $S \cap T = \varnothing$

MOVABLE 5-CONFIGURATIONS

Joint with Lander VerHoef

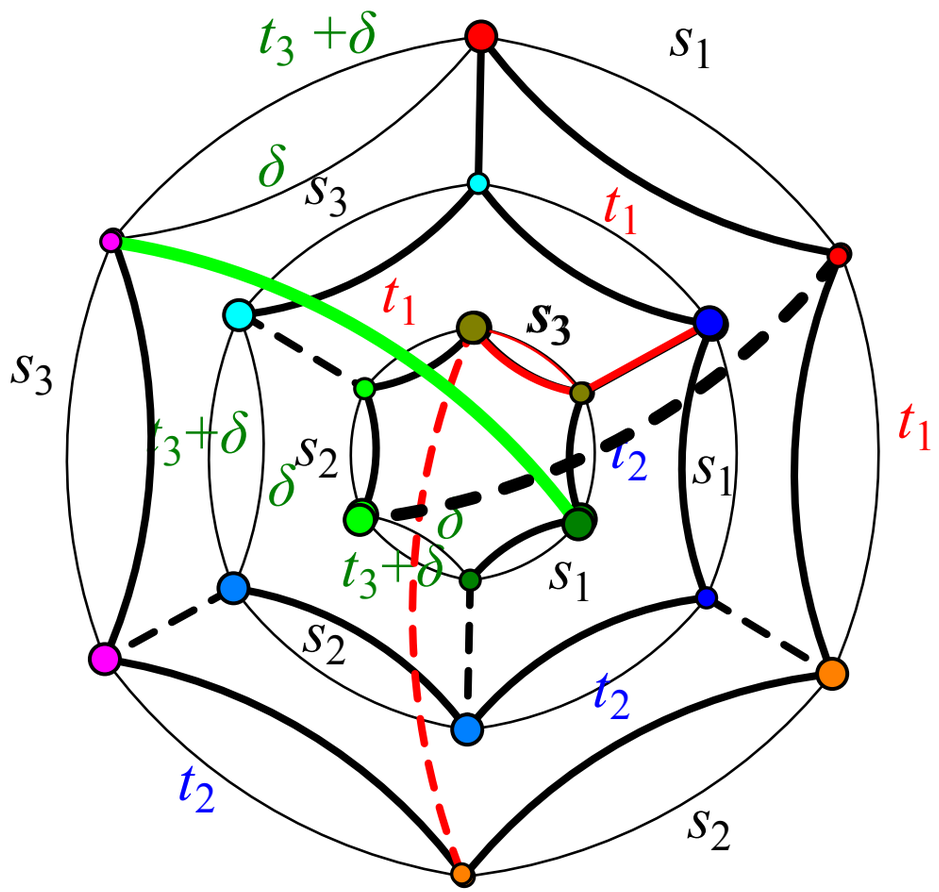


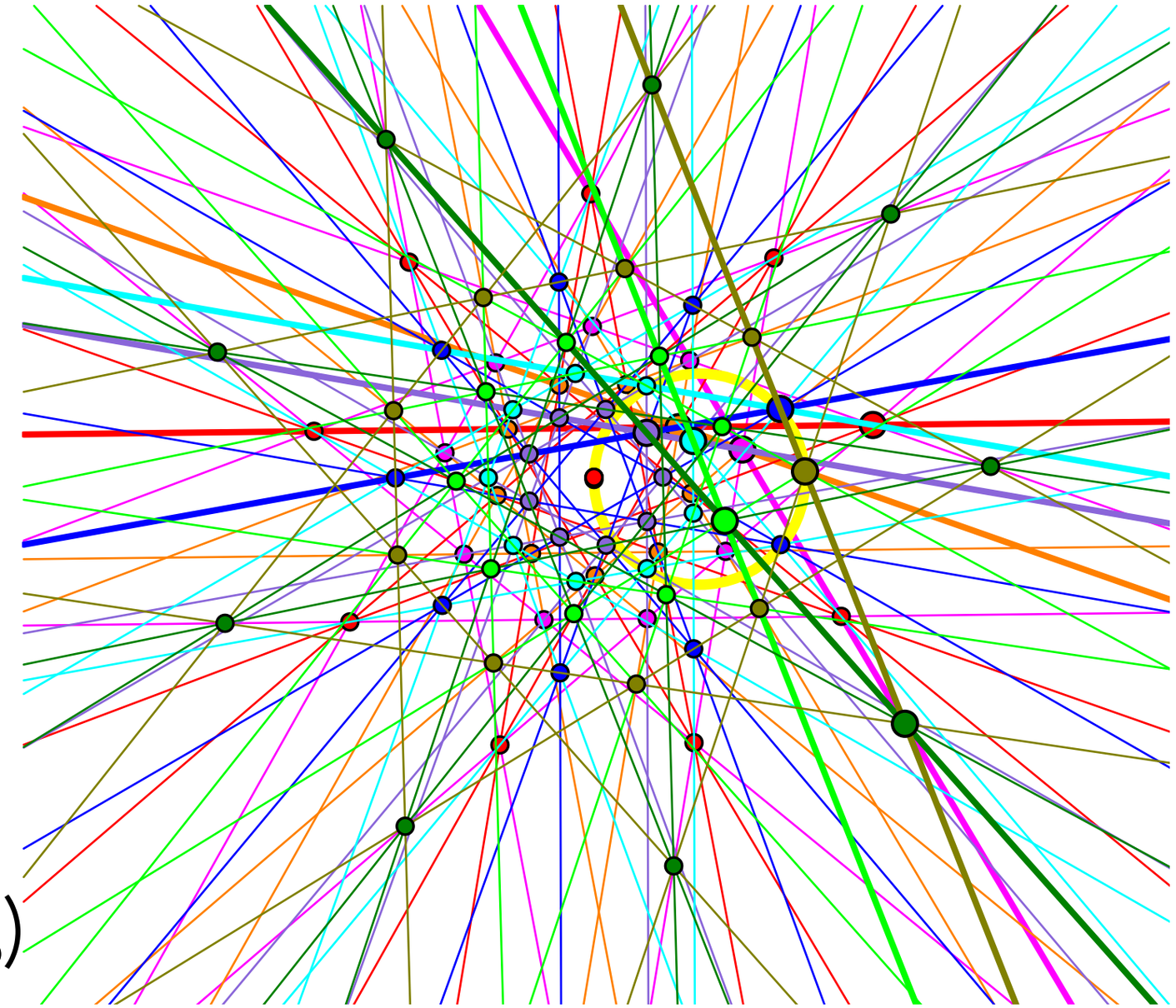


Celestial Configuration m#(s_1, t_1; s_2, t_2; s_3, t_3); $S \cap T = \varnothing$

MOVABLE 5-CONFIGURATIONS

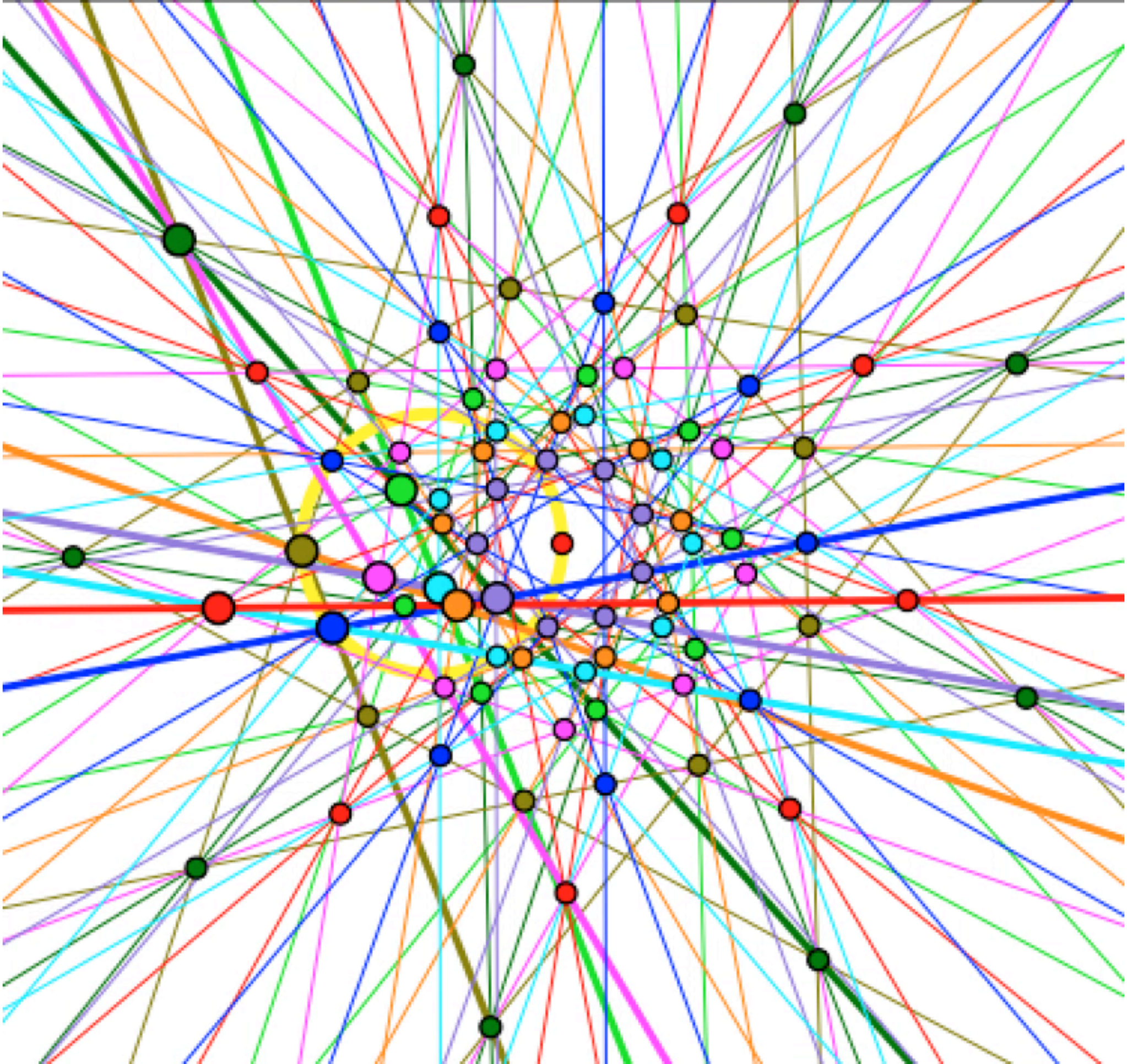
Joint with Lander VerHoef

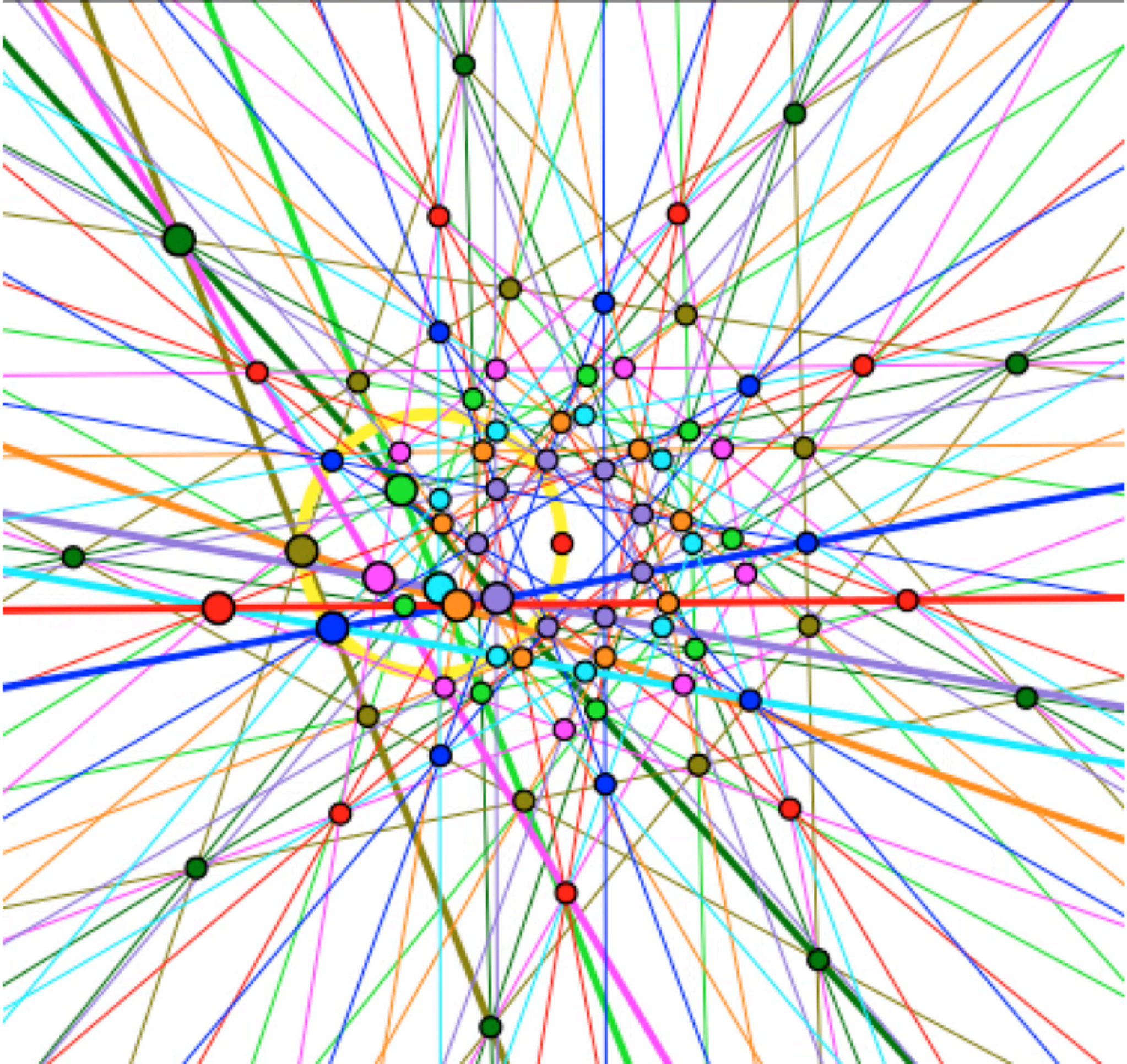




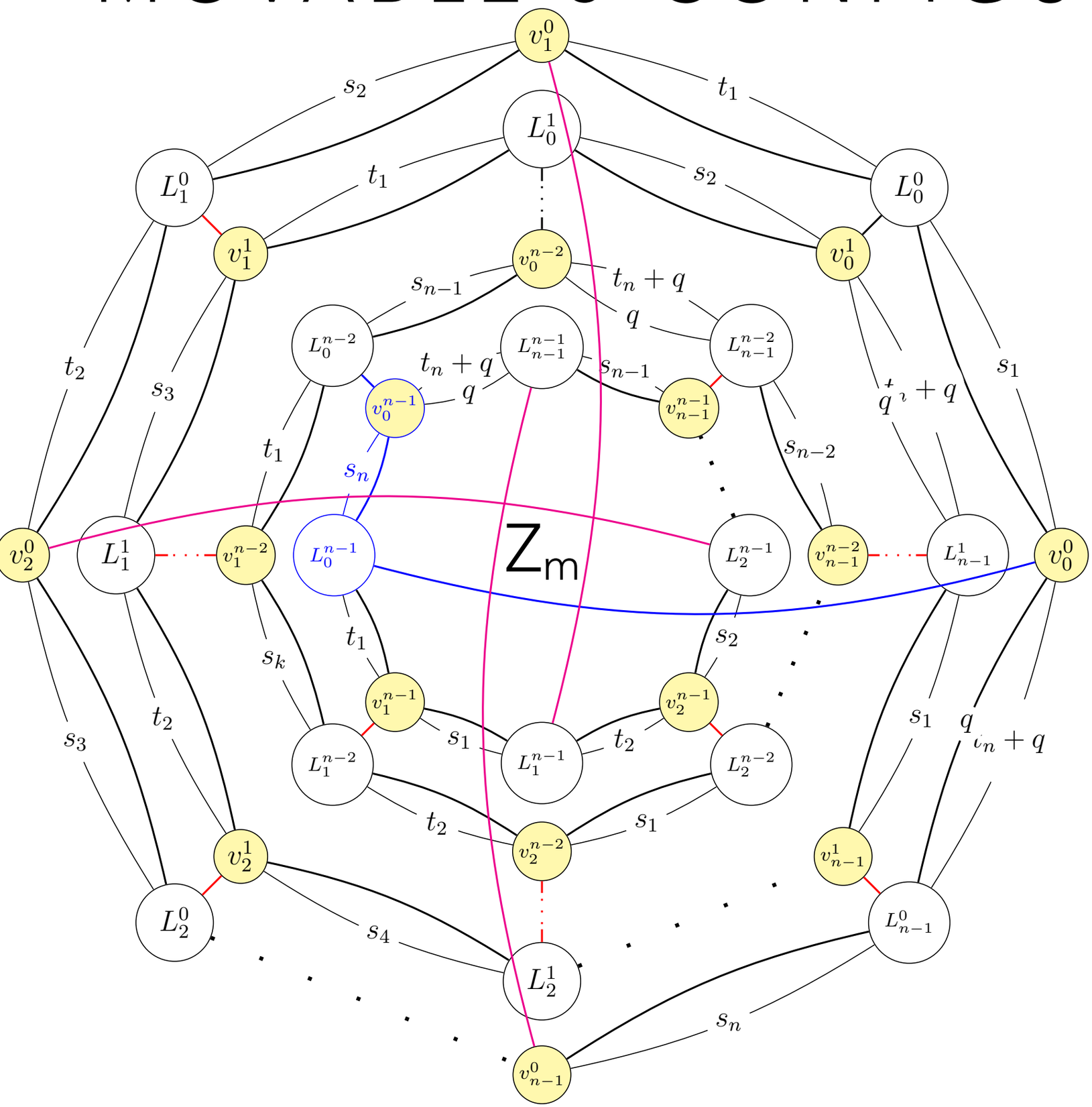
(81_5)

Celestial Configuration m#(s_1, t_1; s_2, t_2; s_3, t_3); $S \cap T = \varnothing$





MOVABLE 5-CONFIGURATION

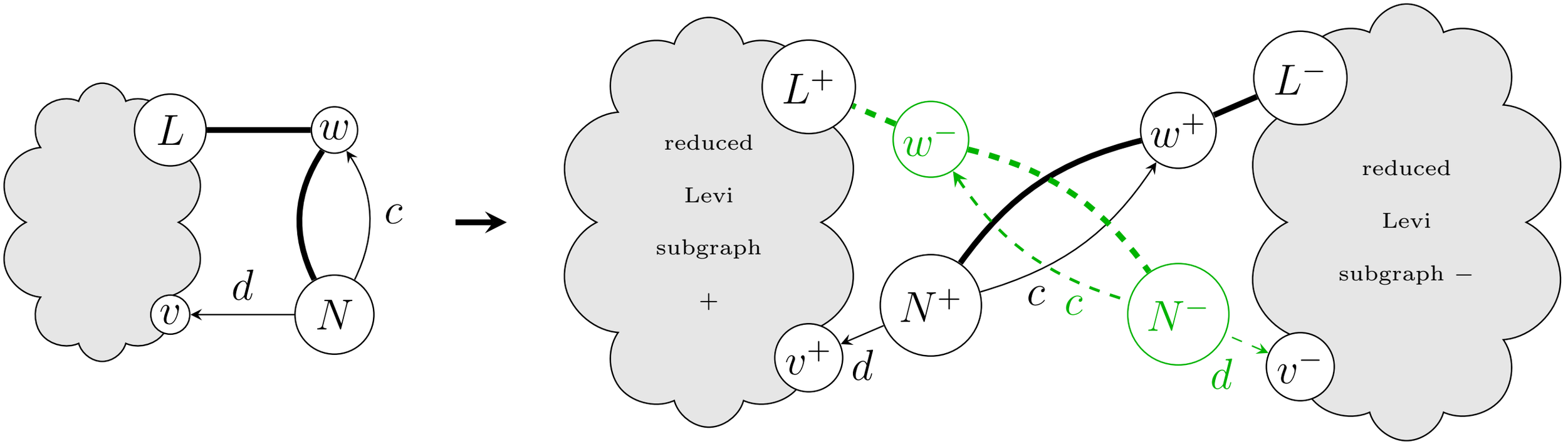


k-celestial 4-configuration (mk_4)

↓

k rings; k-2 degrees of freedom ($(mk^2)_5$)

DIHEDRALIZATION



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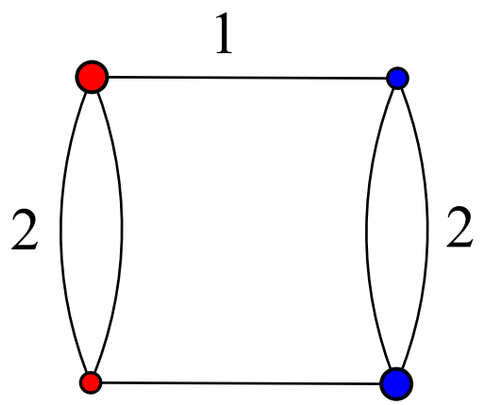

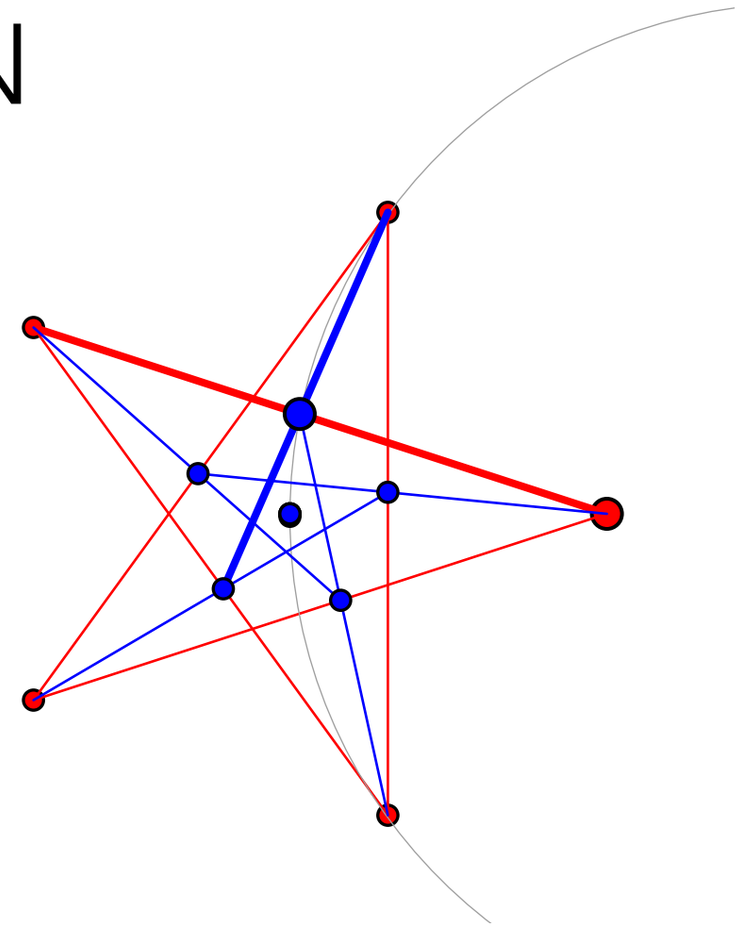

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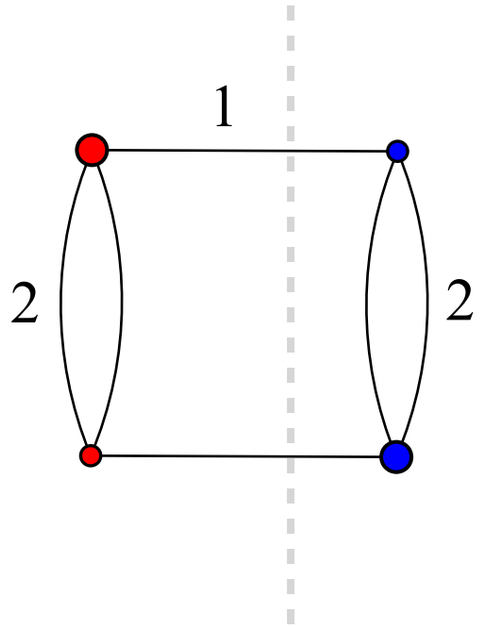

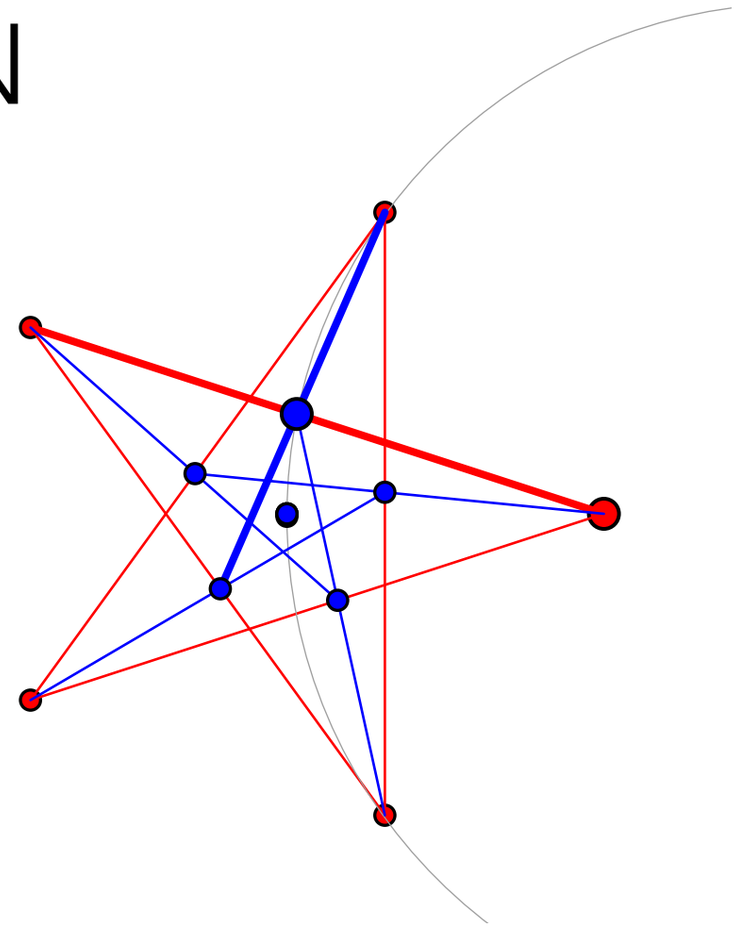

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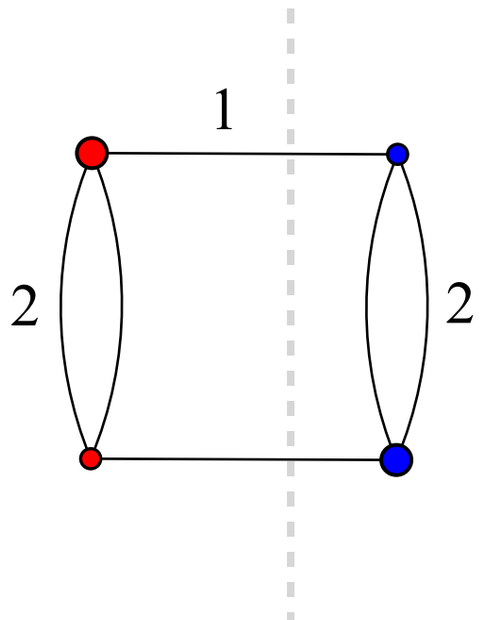

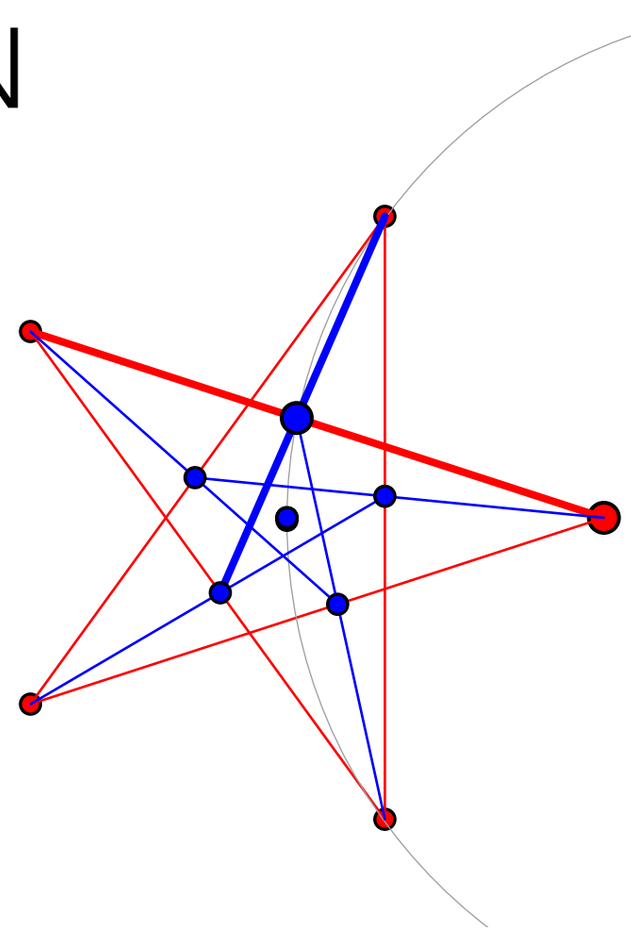

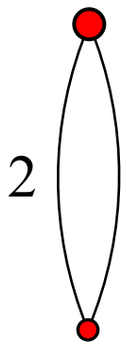

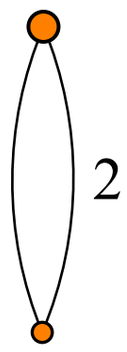

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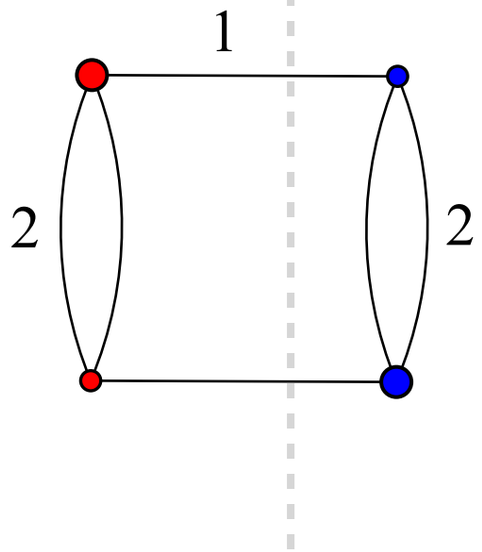

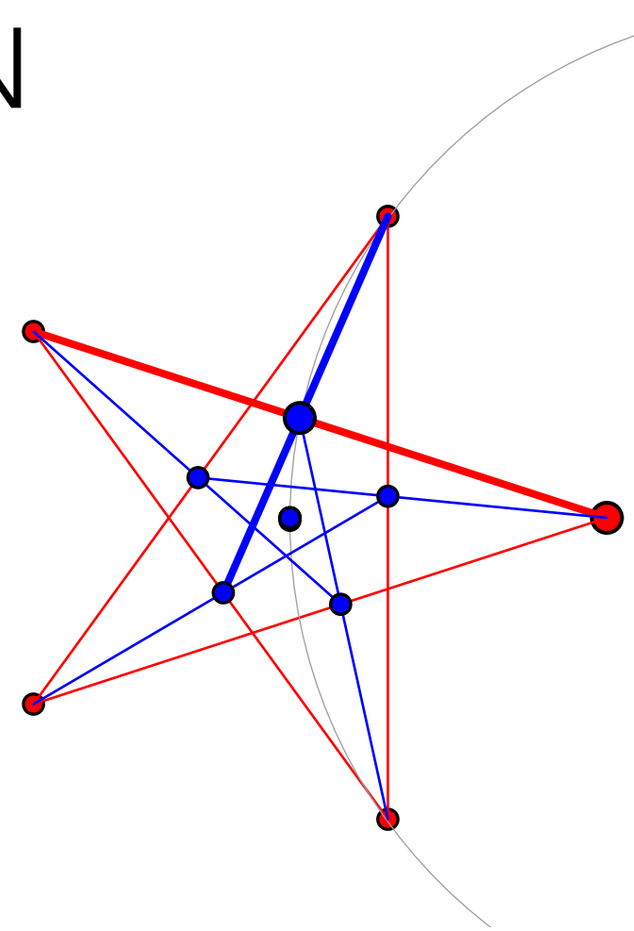

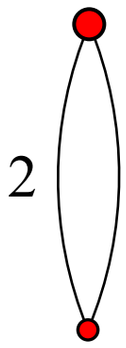

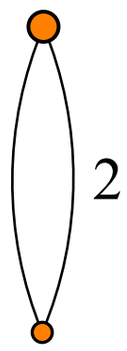

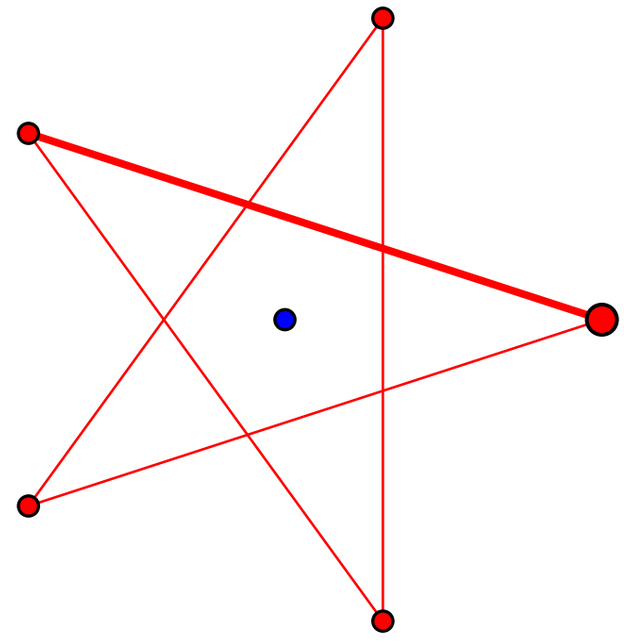

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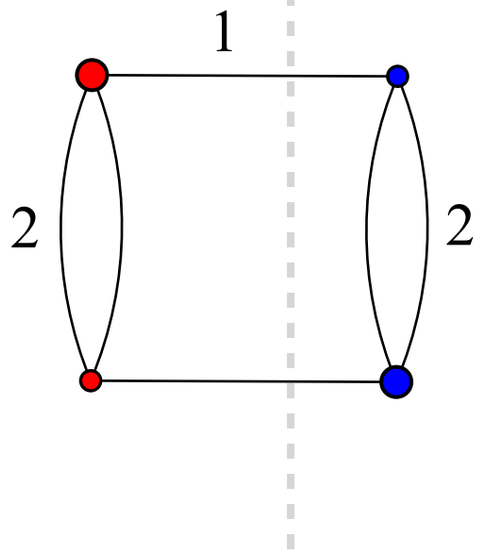

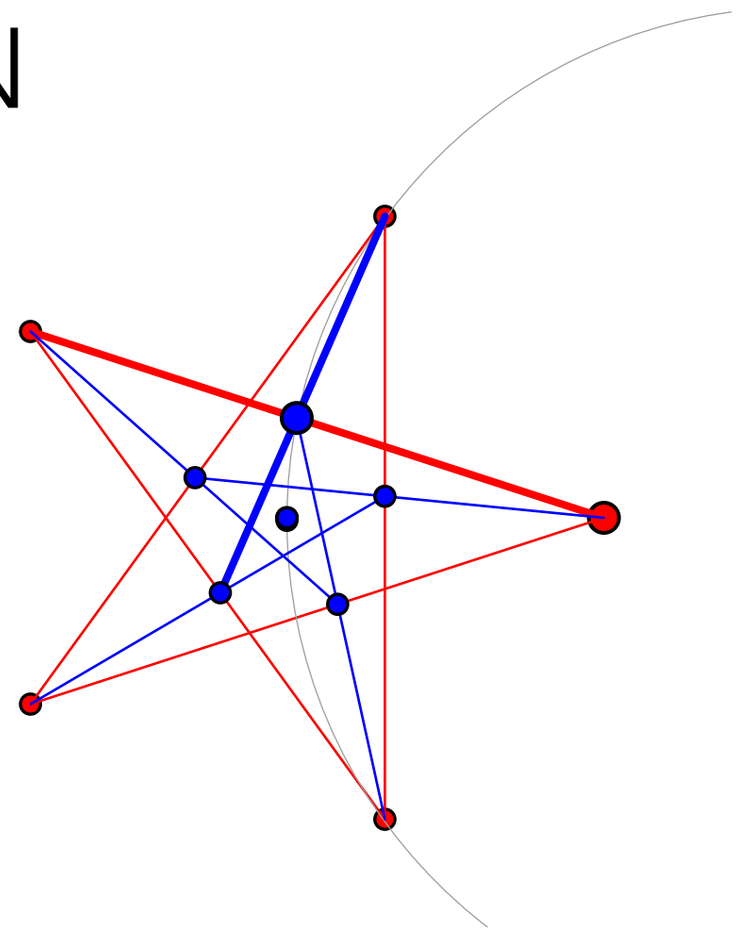

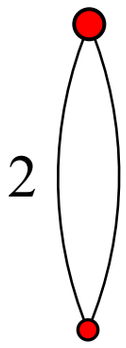

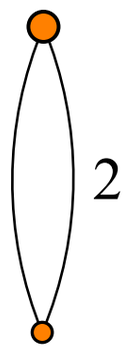

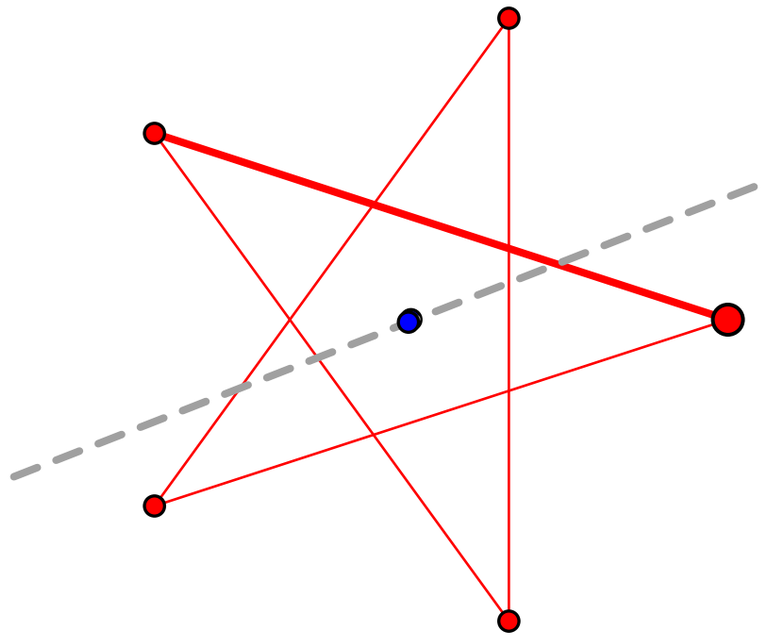

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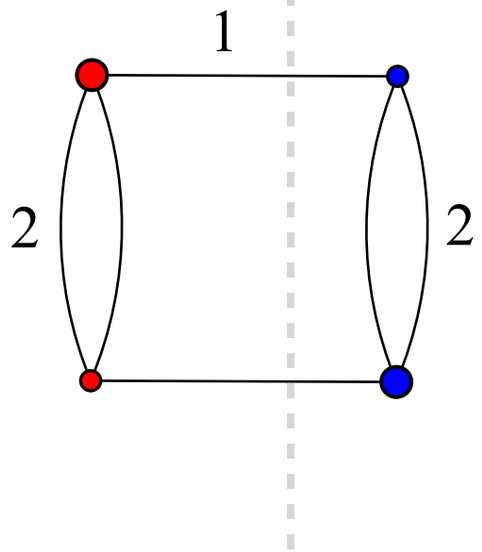

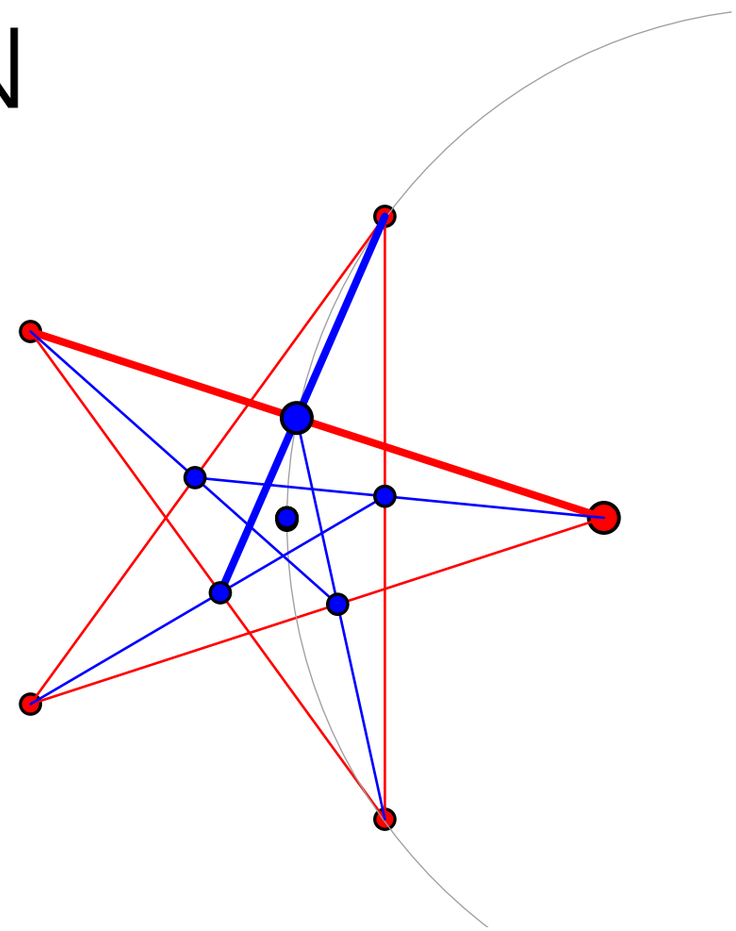

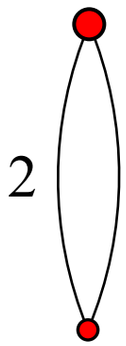

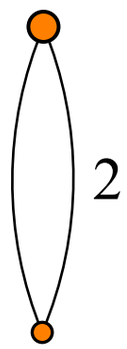

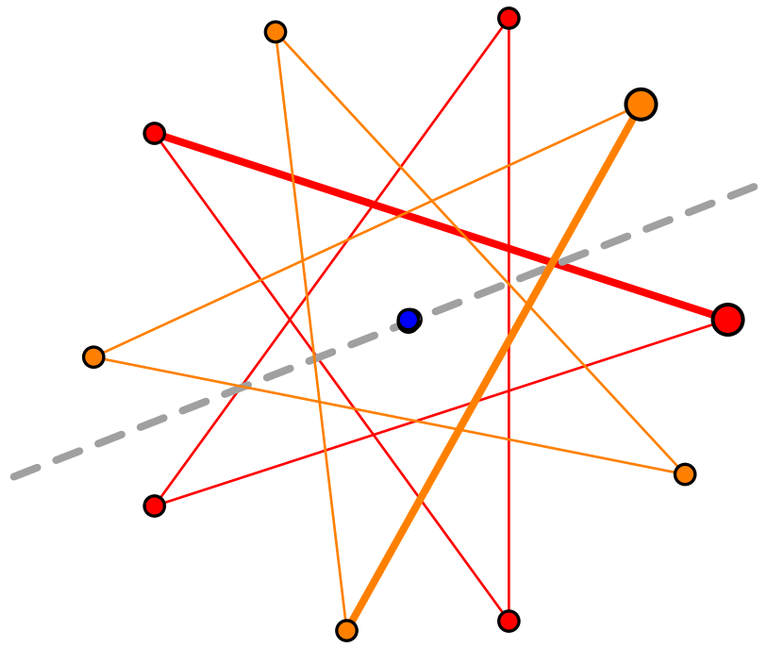

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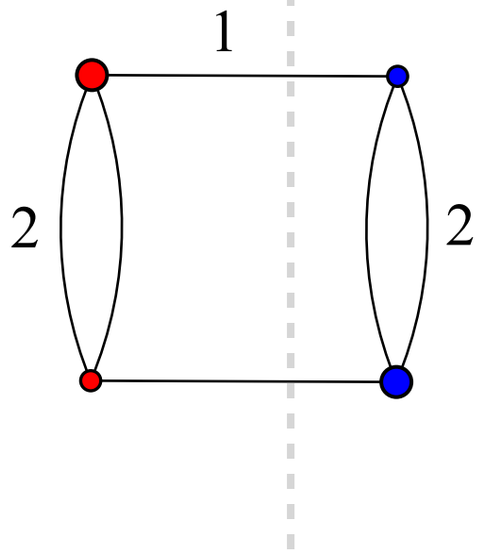

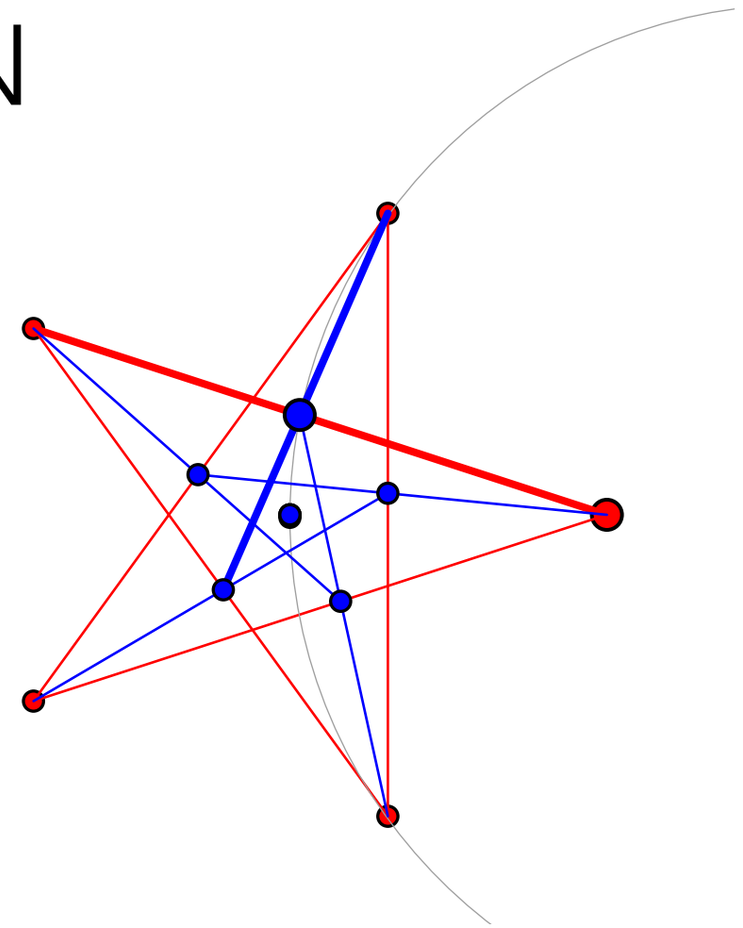

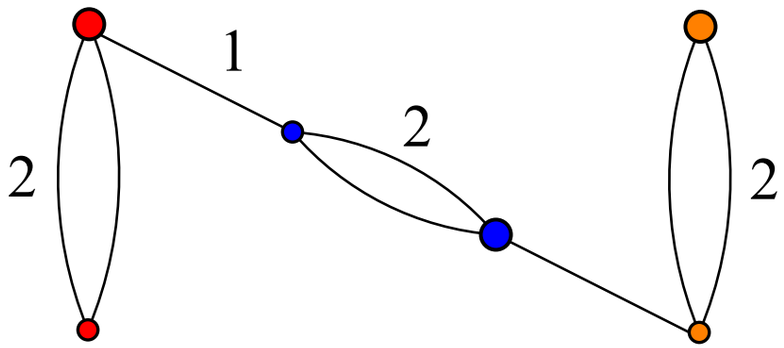

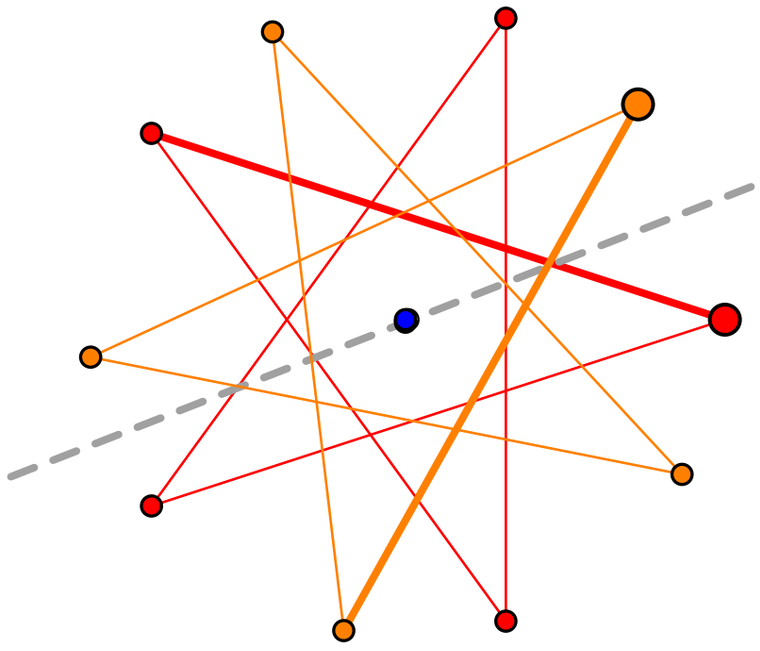

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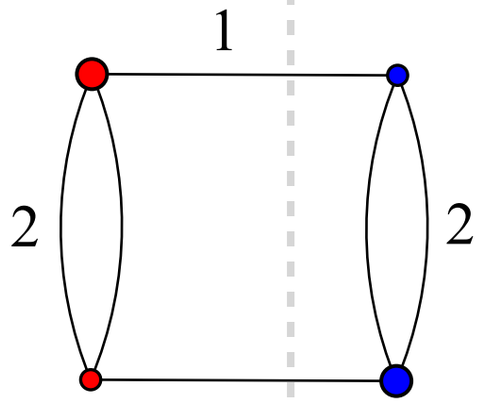

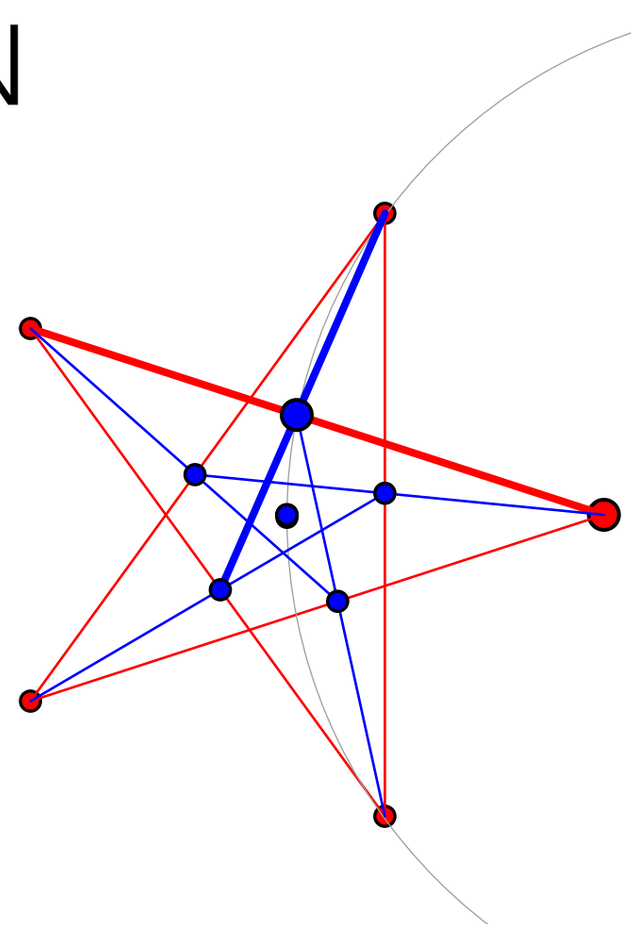

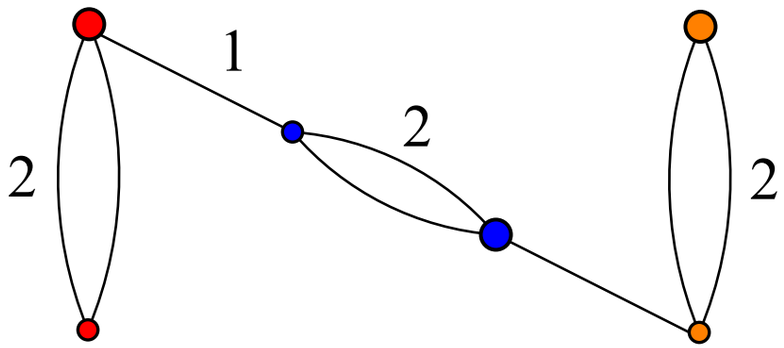

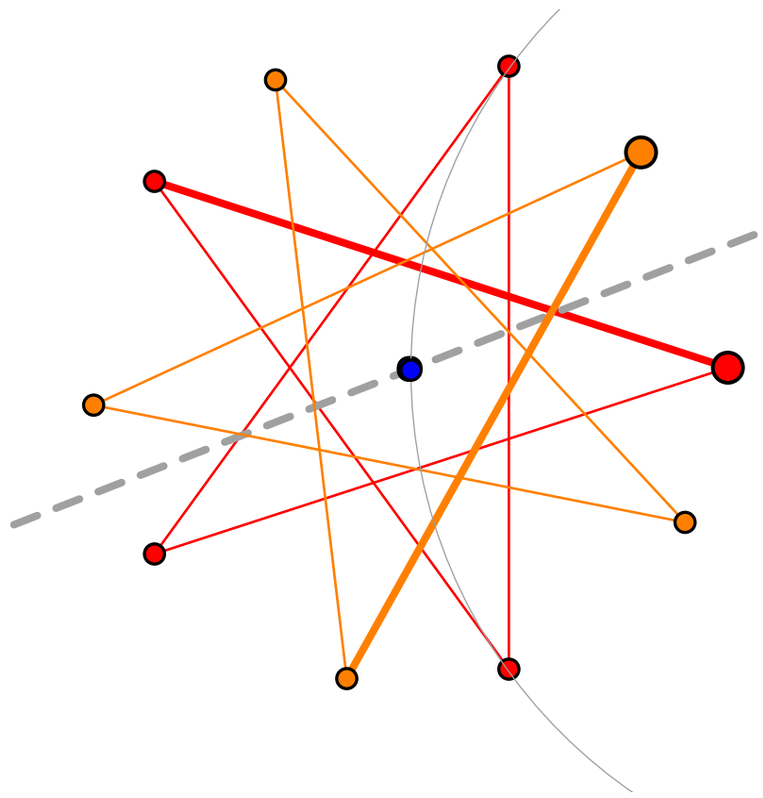

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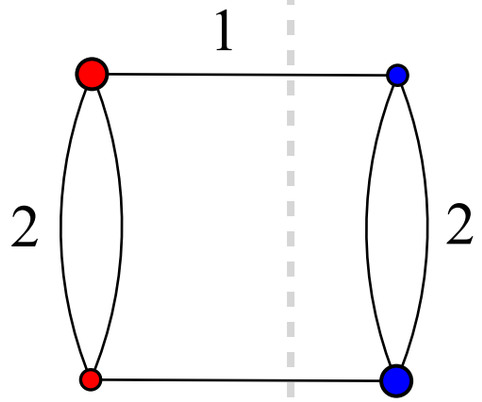

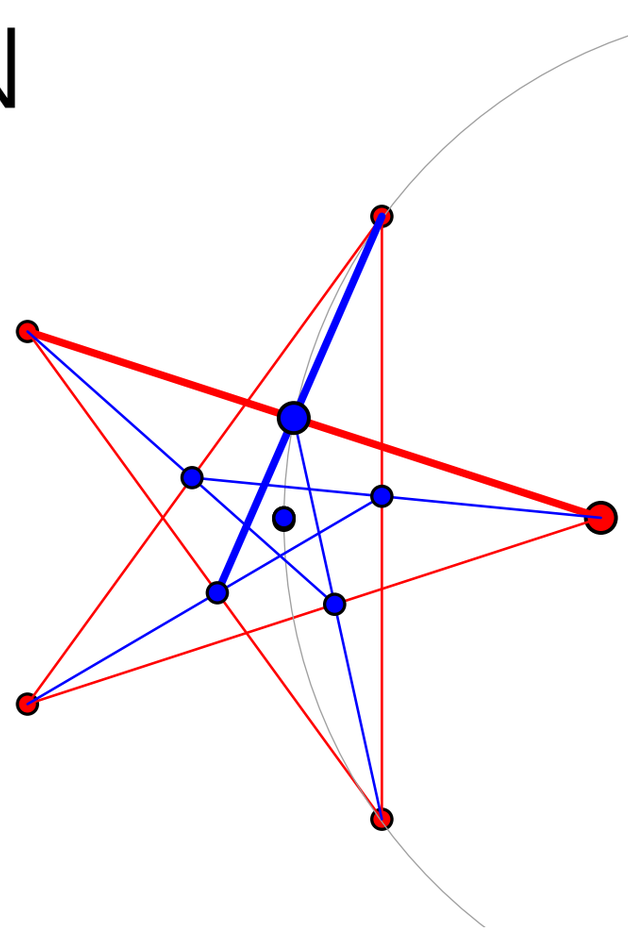

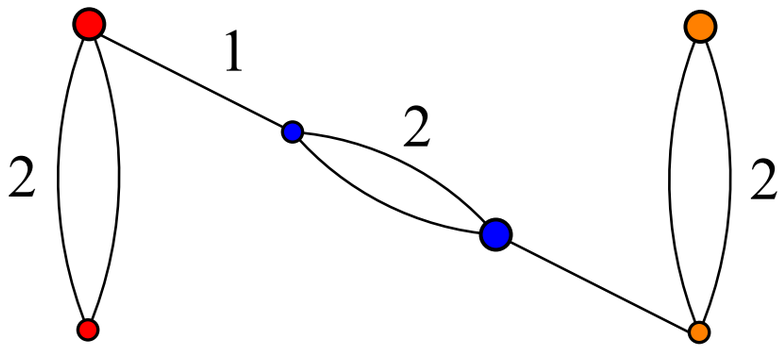

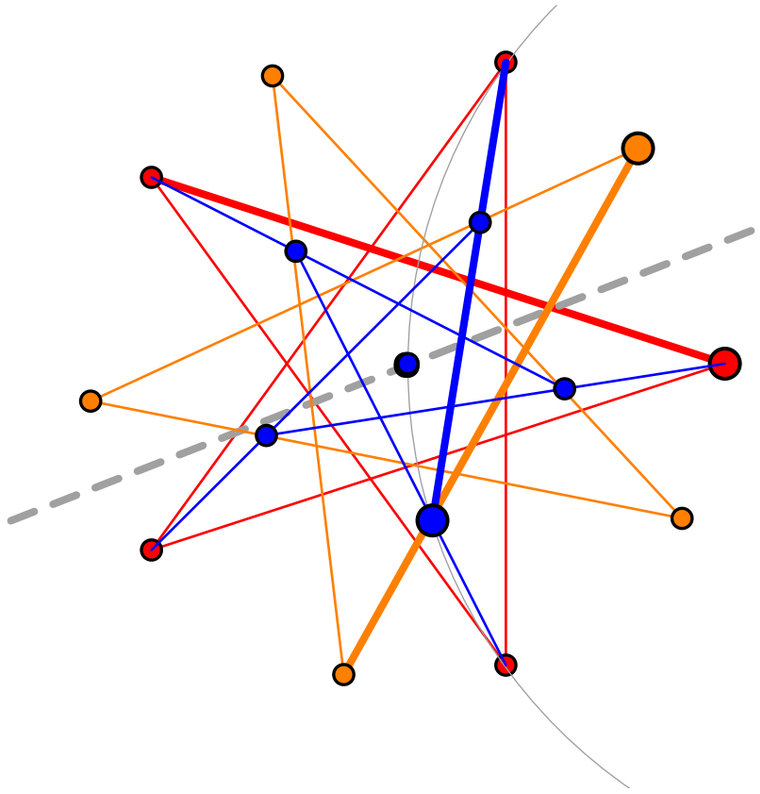

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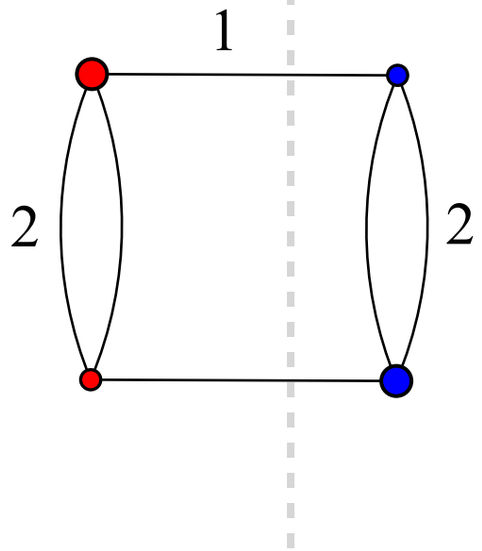

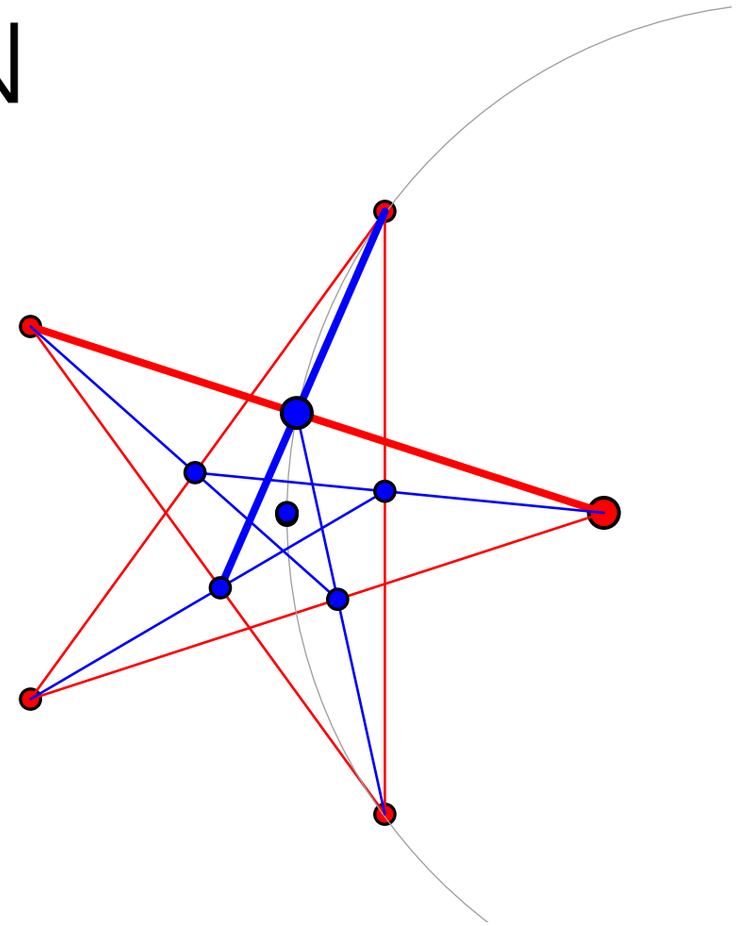

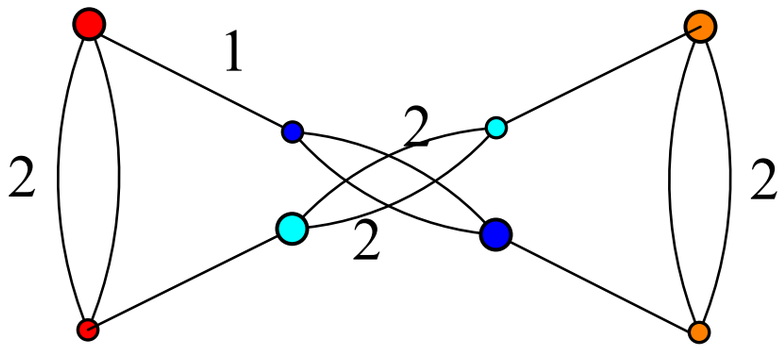

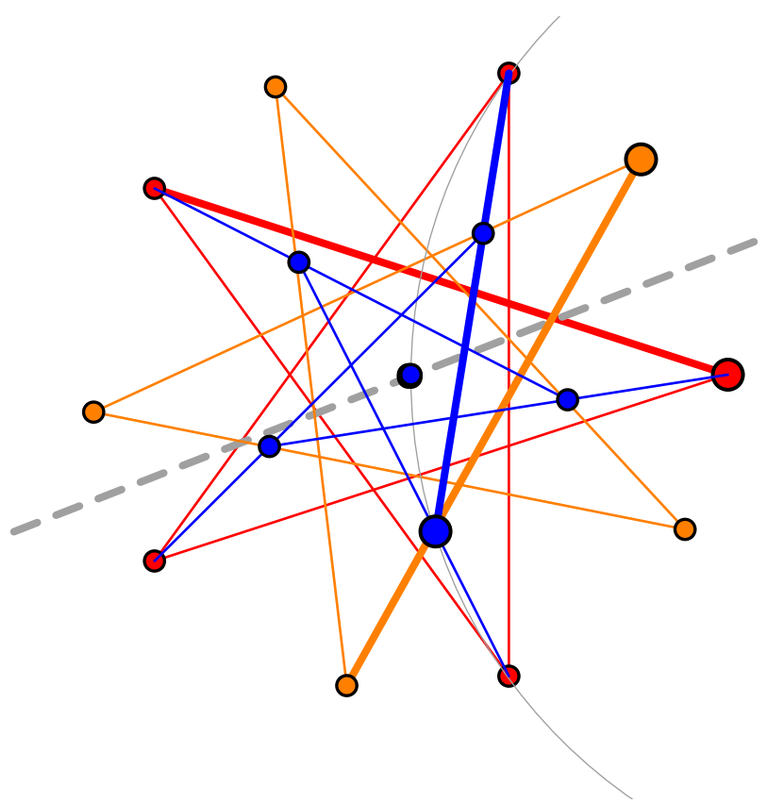

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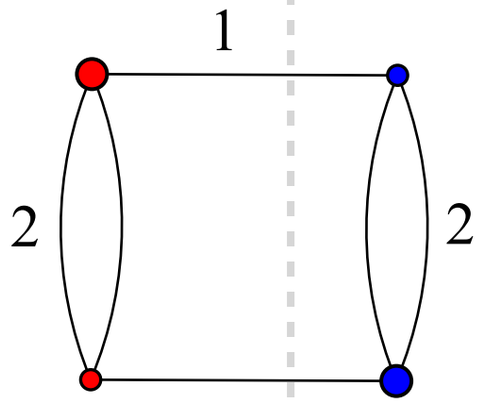

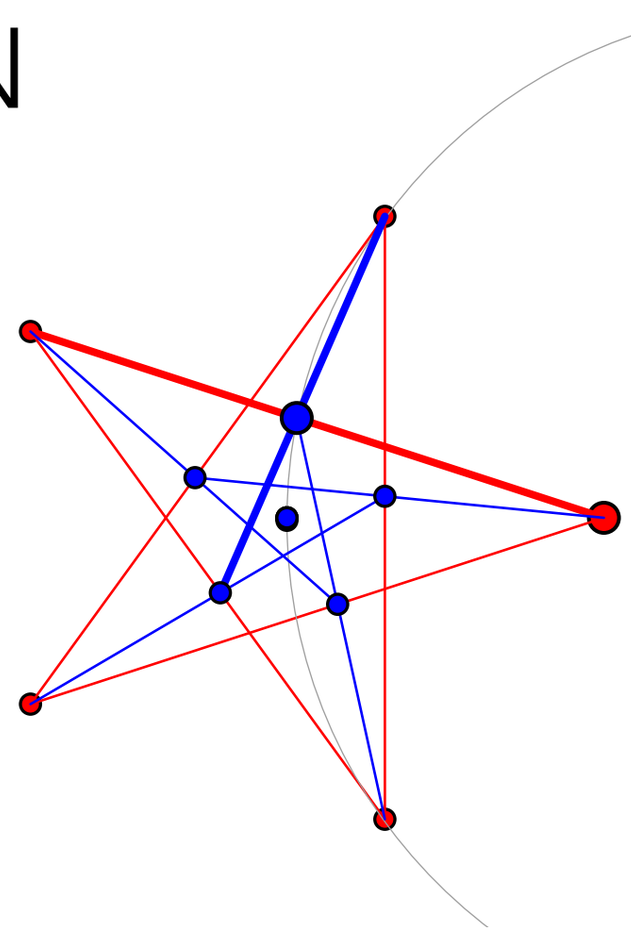

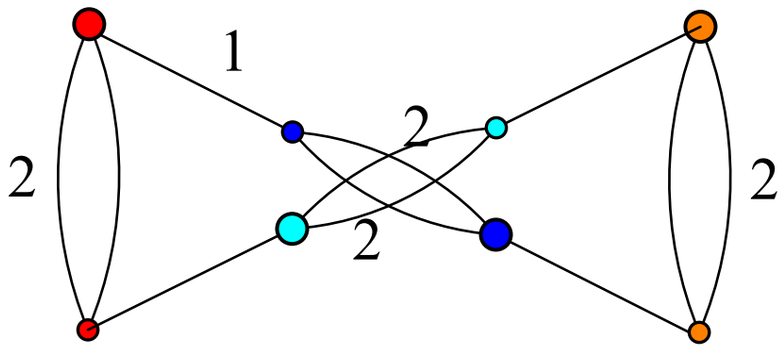

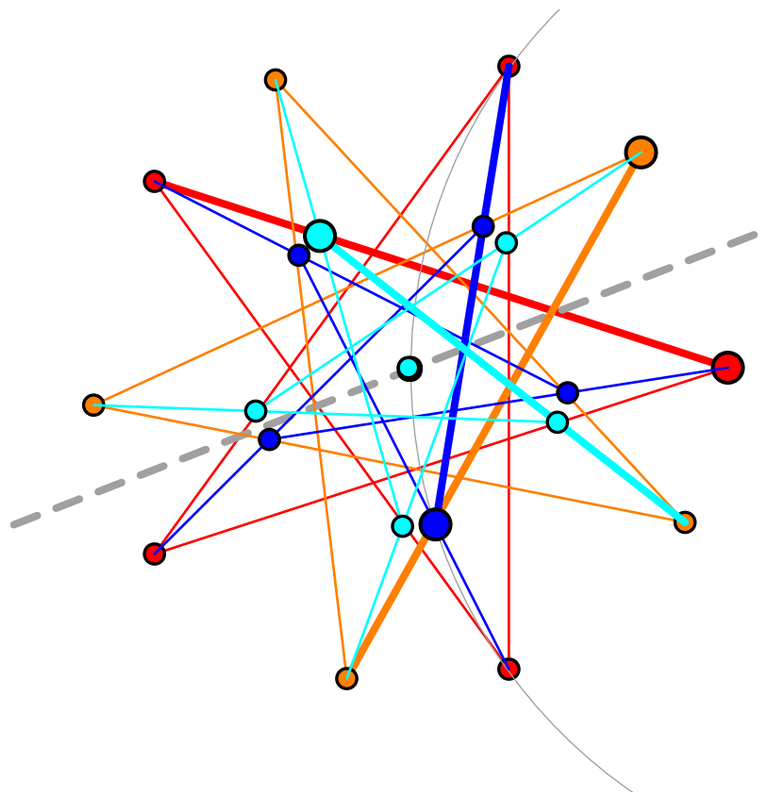

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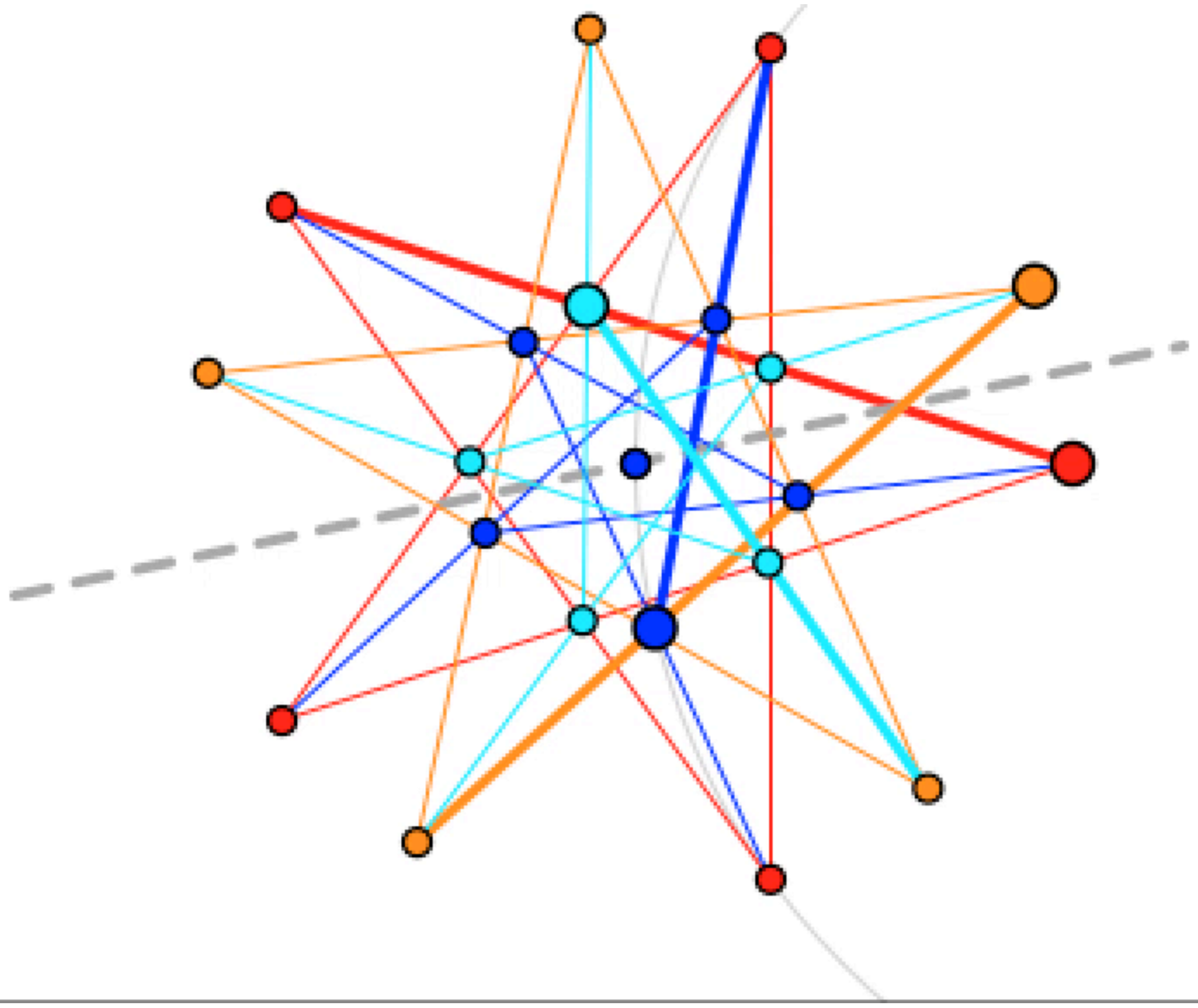

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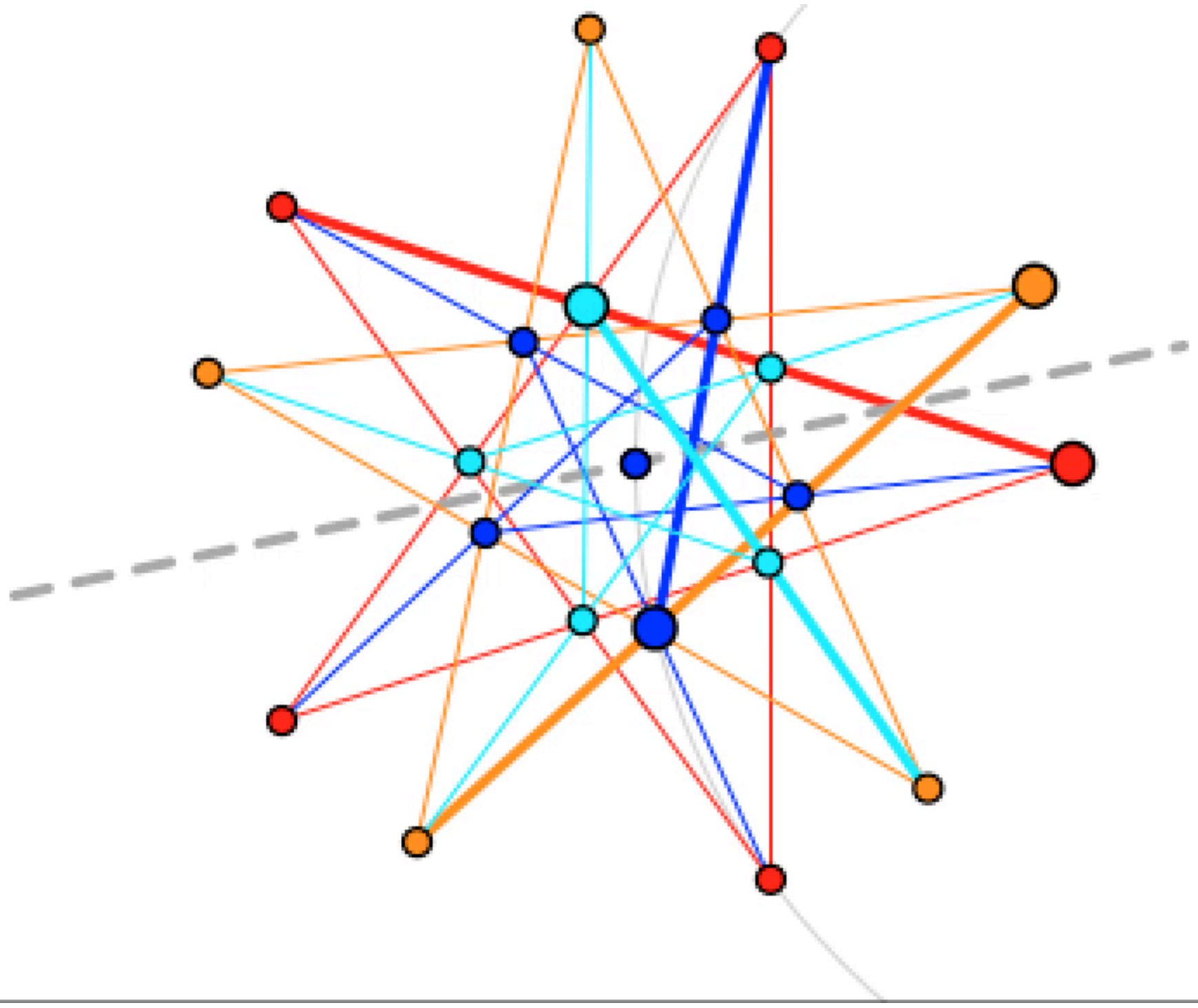
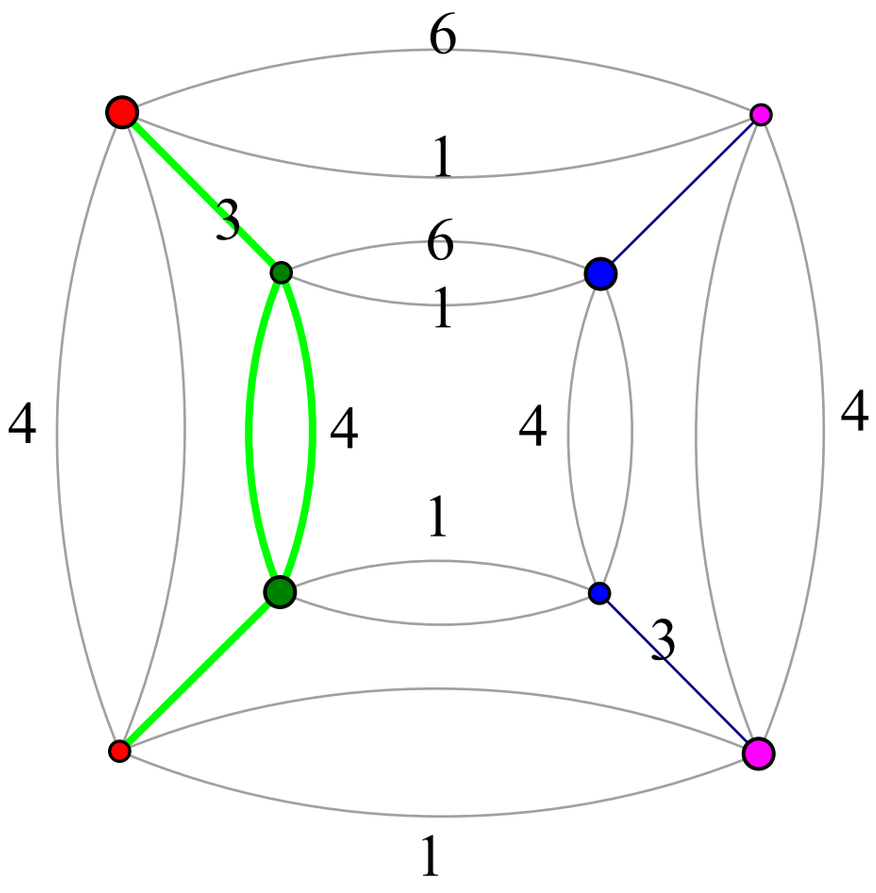
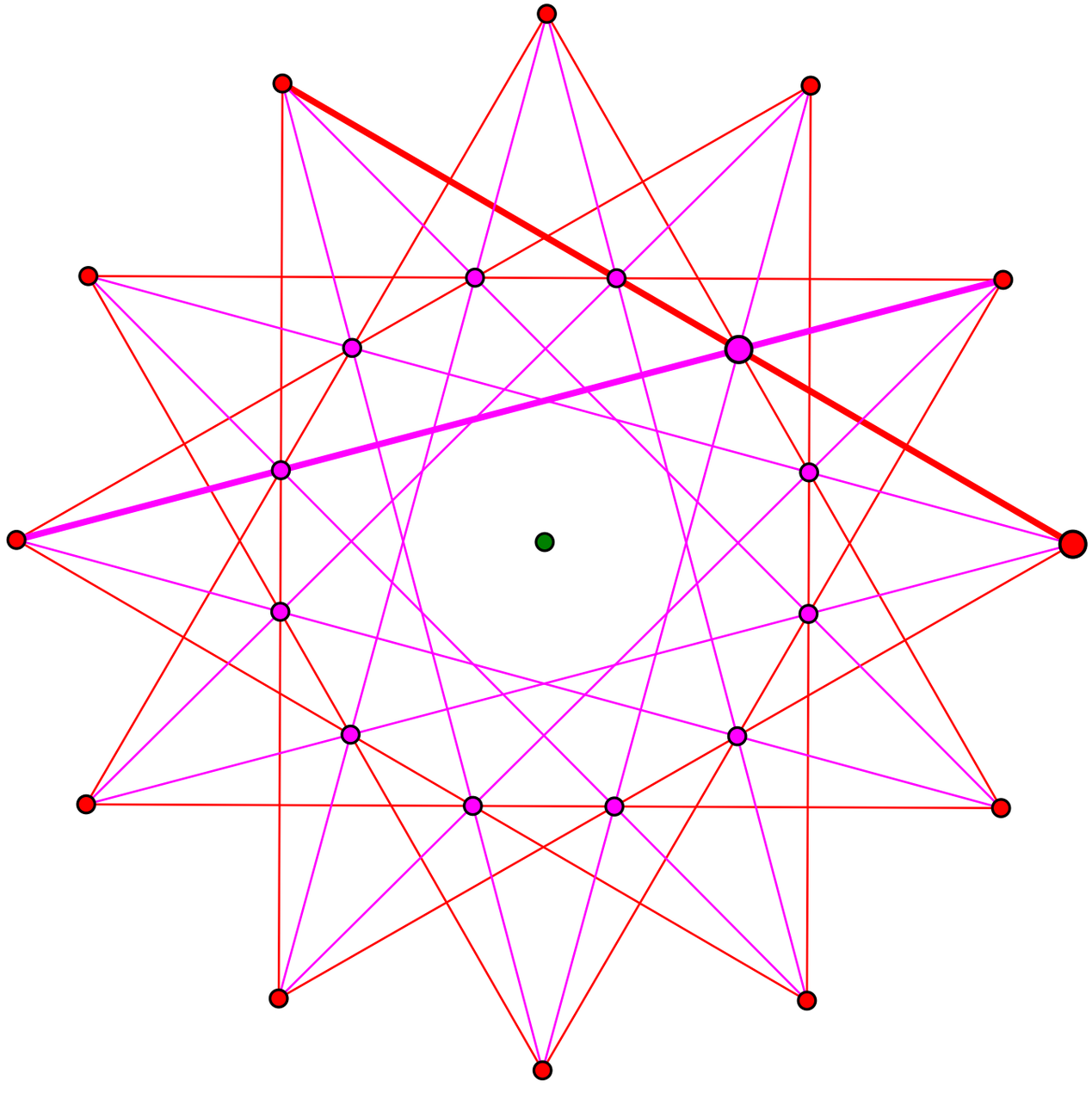


MOVABLE 5-CONFIGURATIONS





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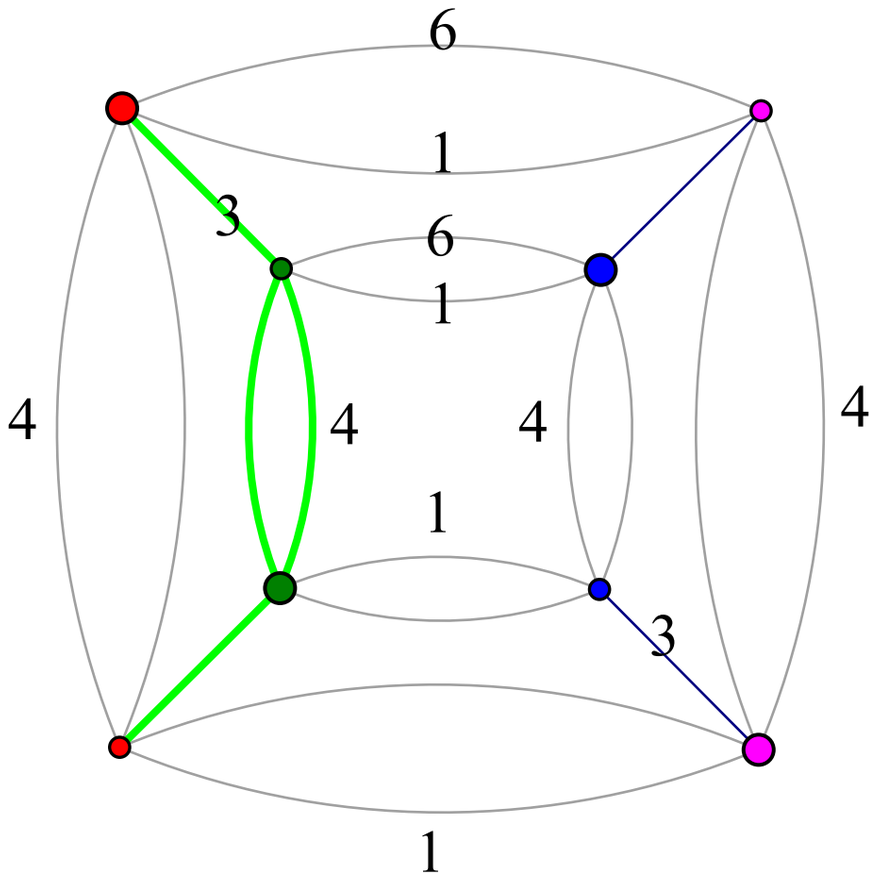
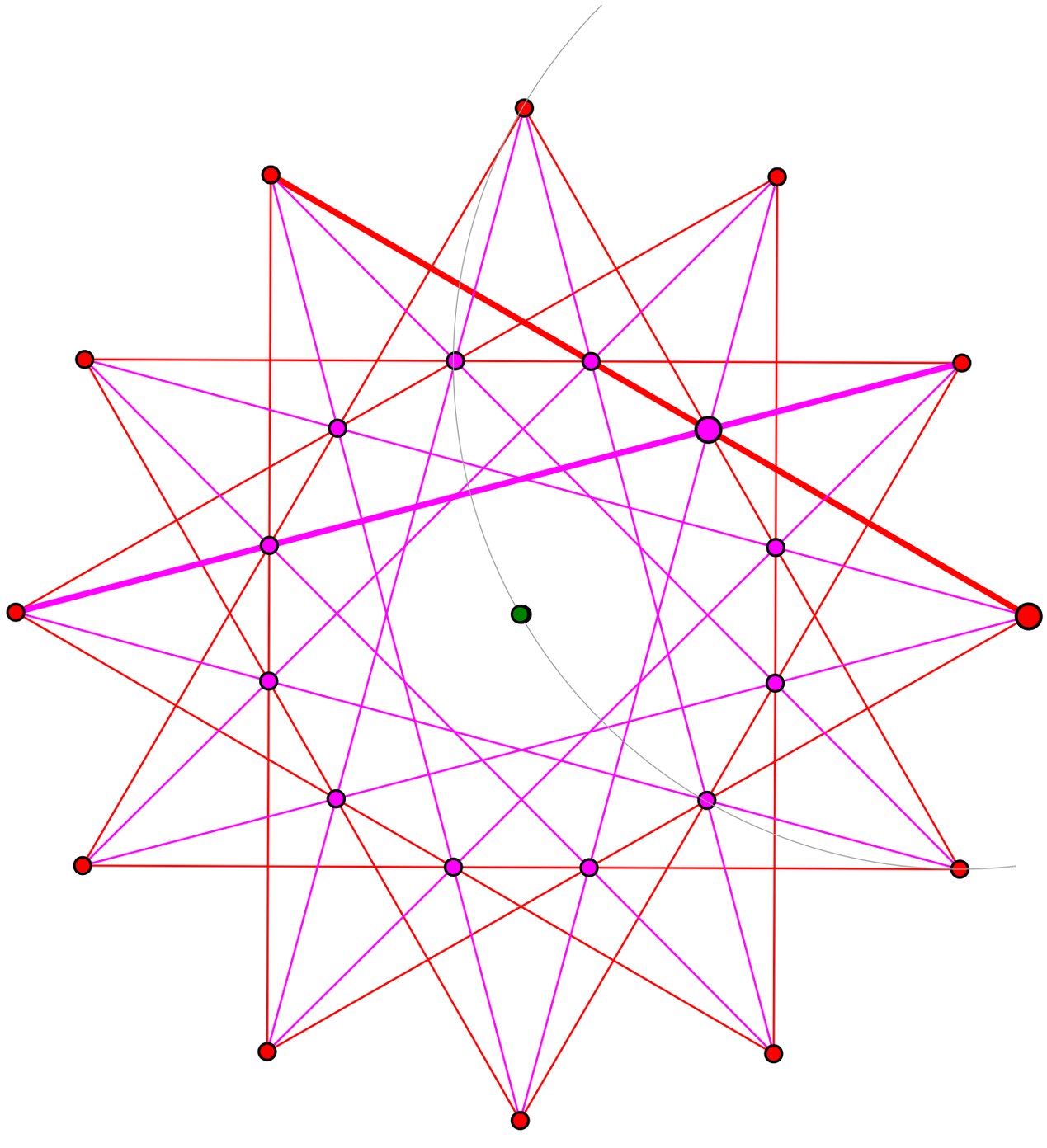
MOVABLE 5-CONFIGURATIONS





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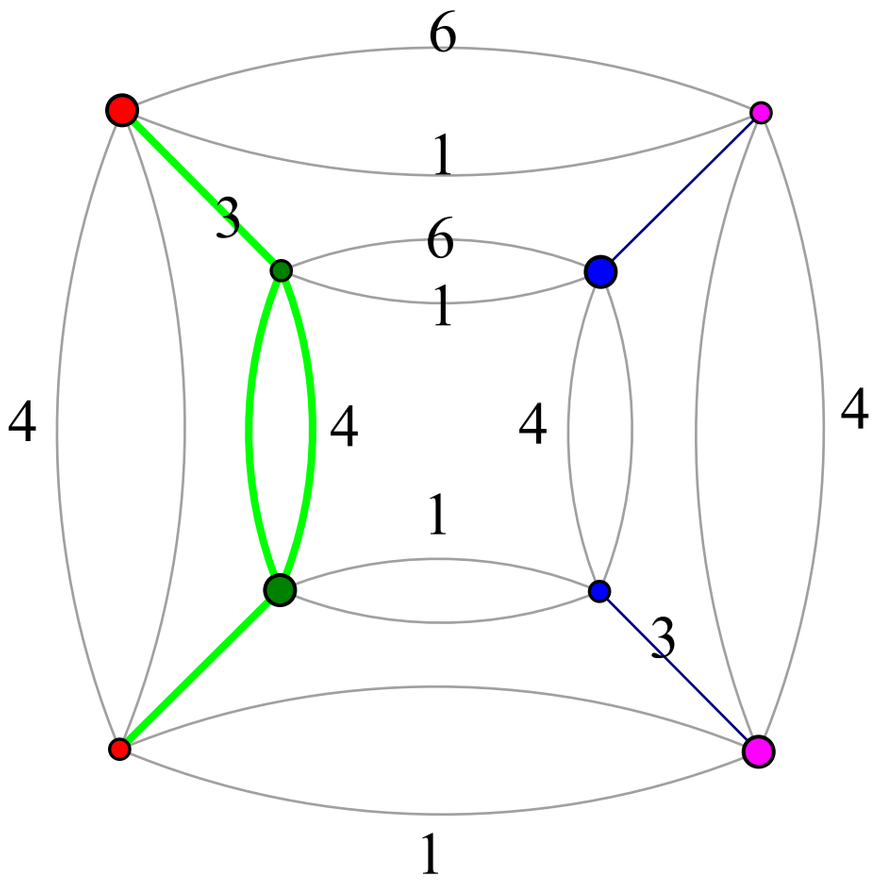

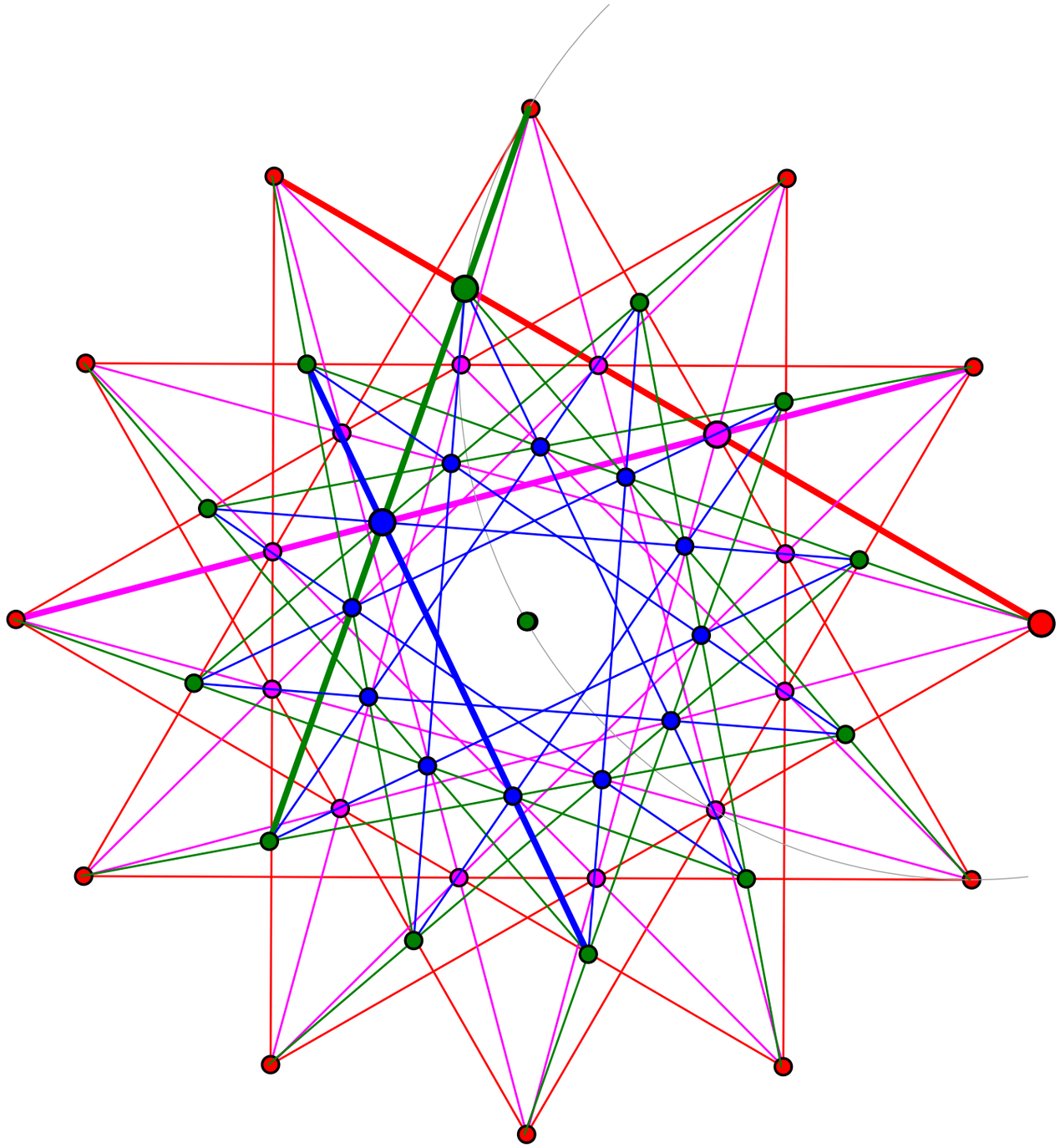

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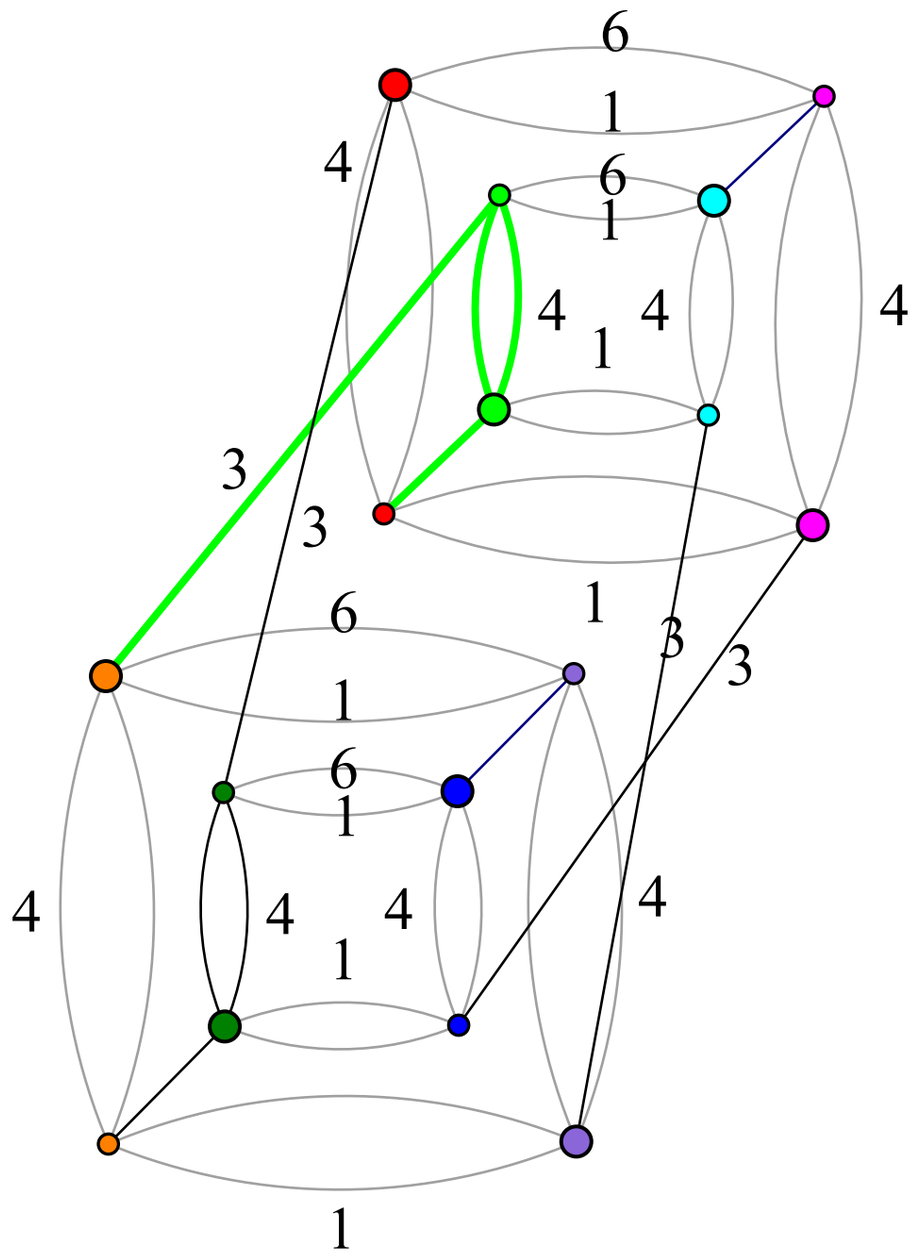

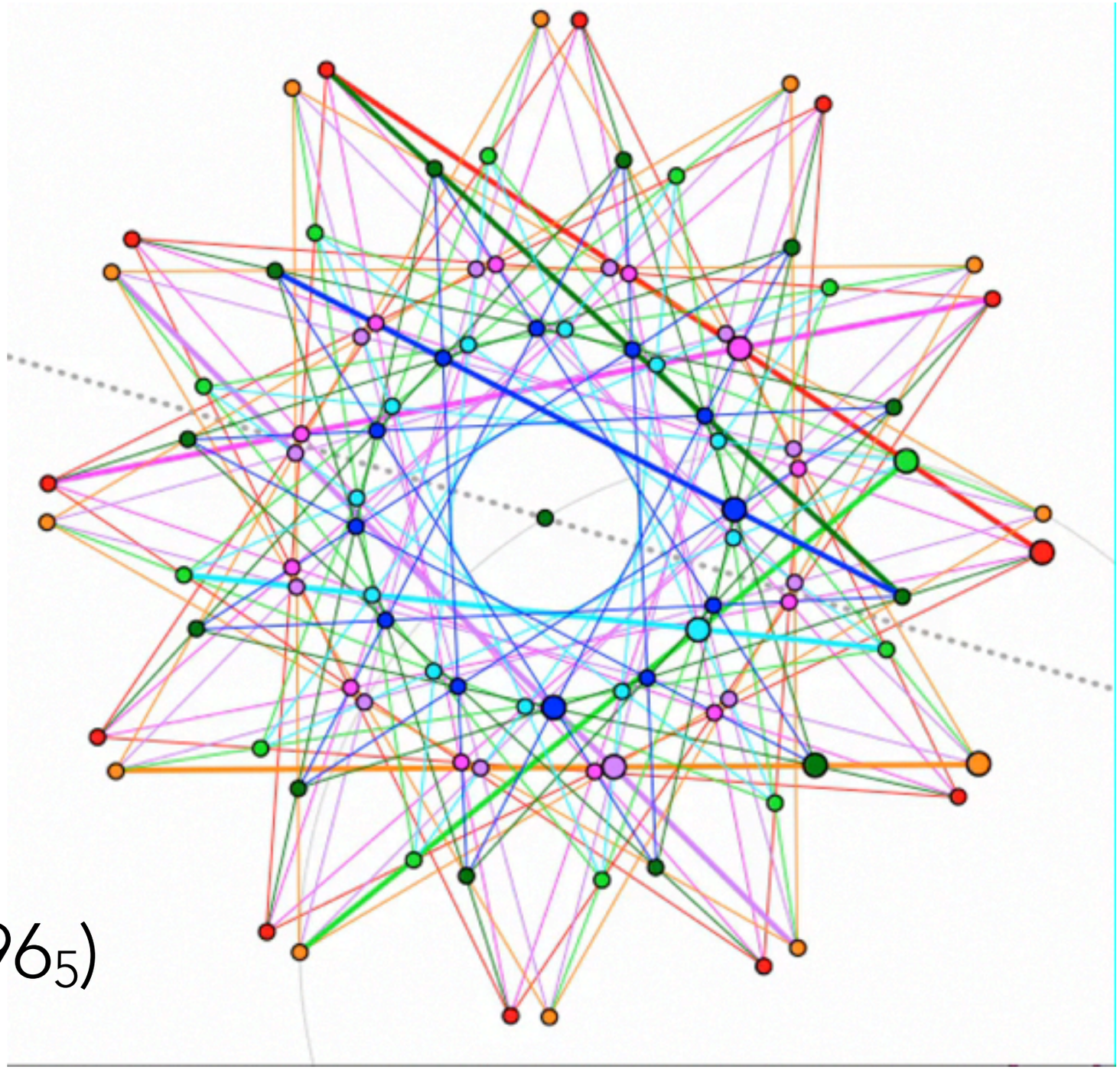

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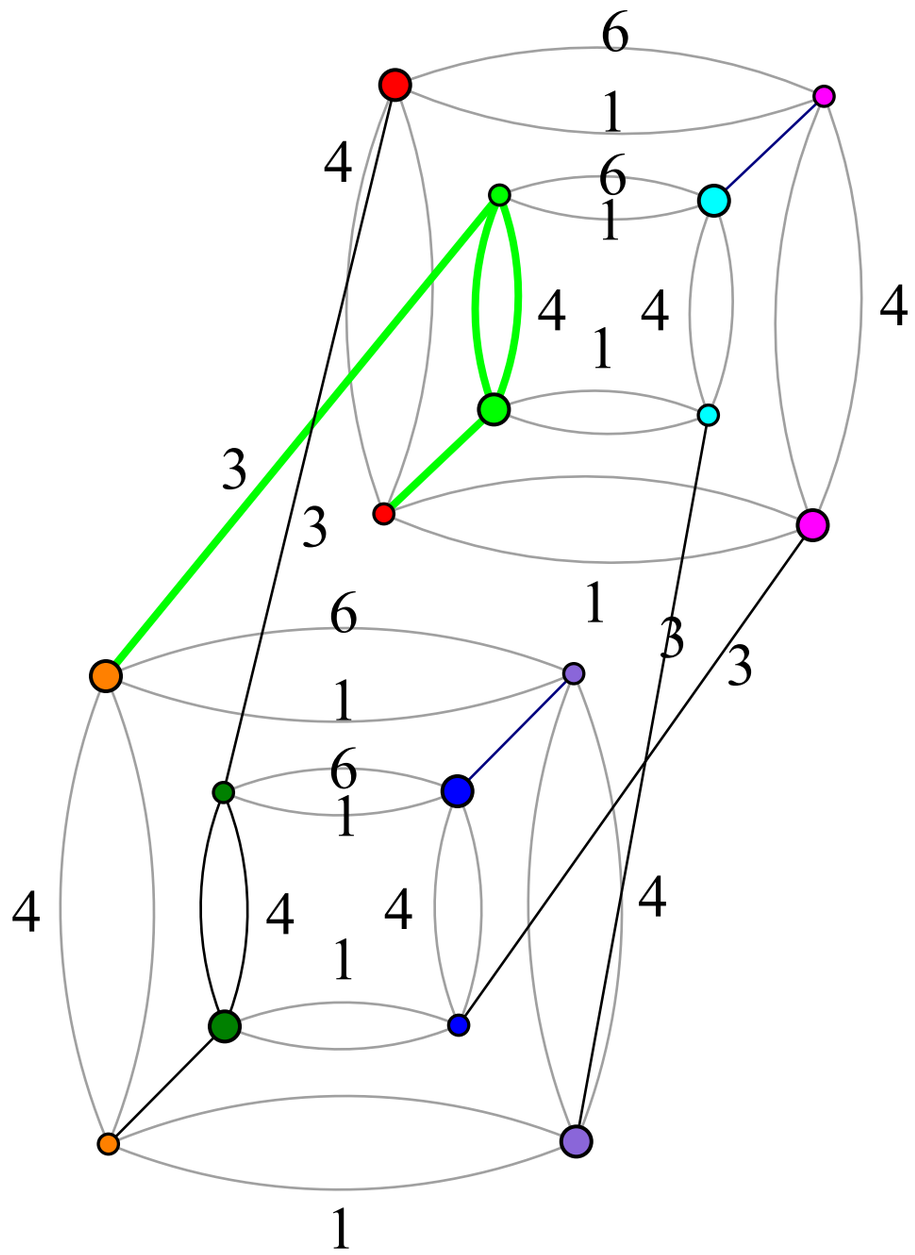
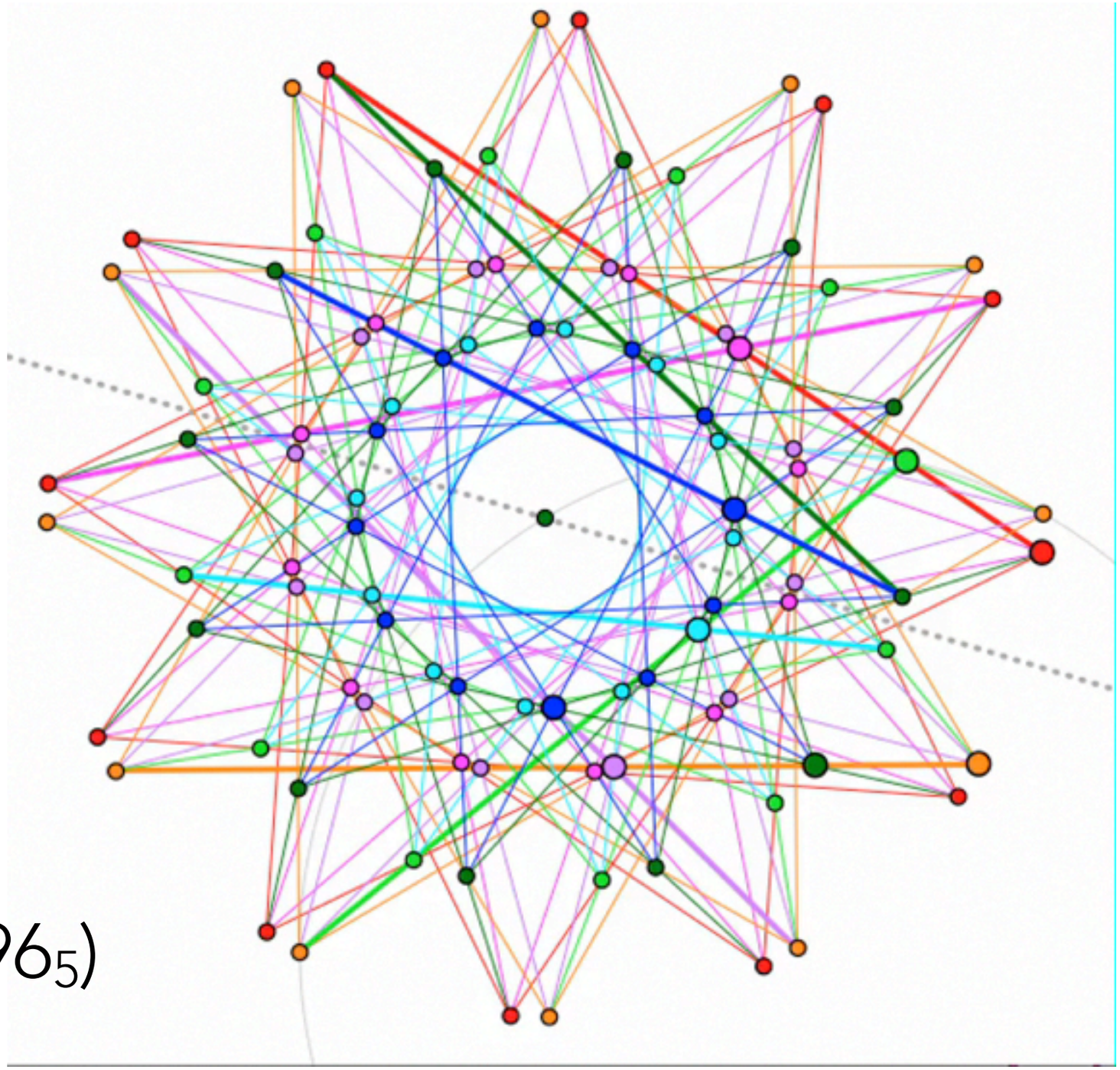
MOVABLE 5-CONFIGURATIONS





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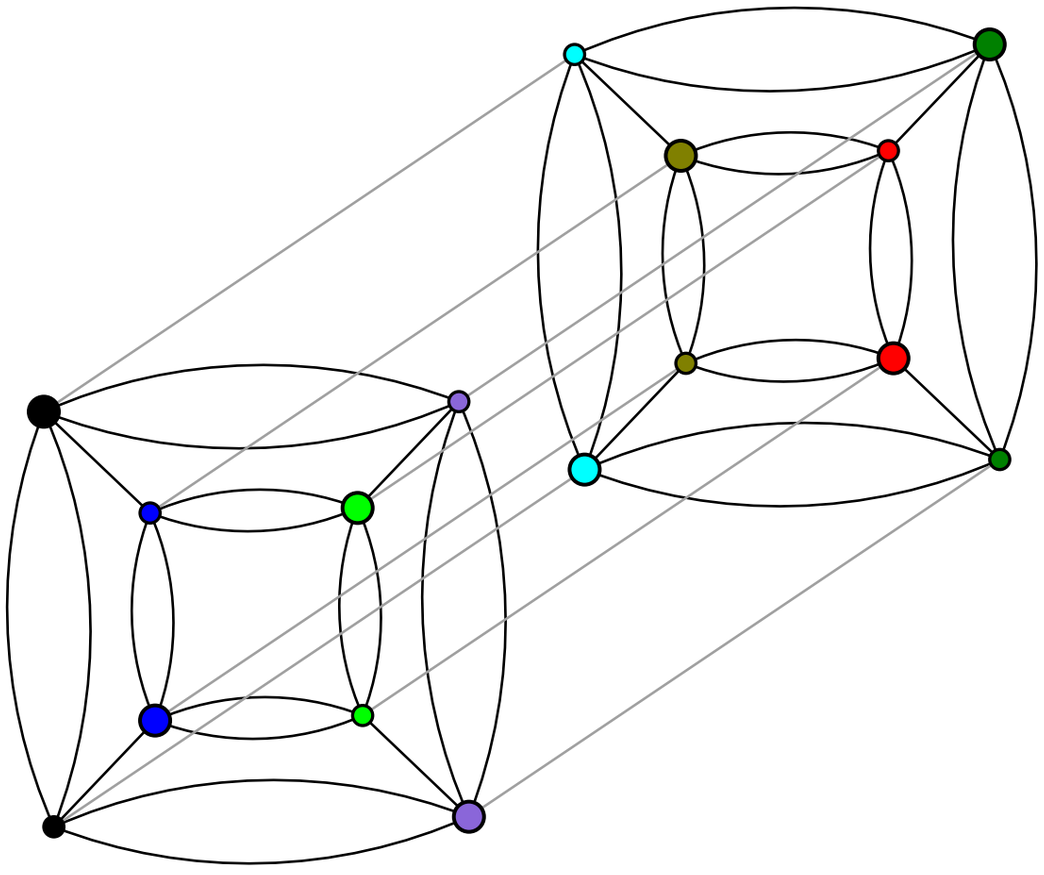
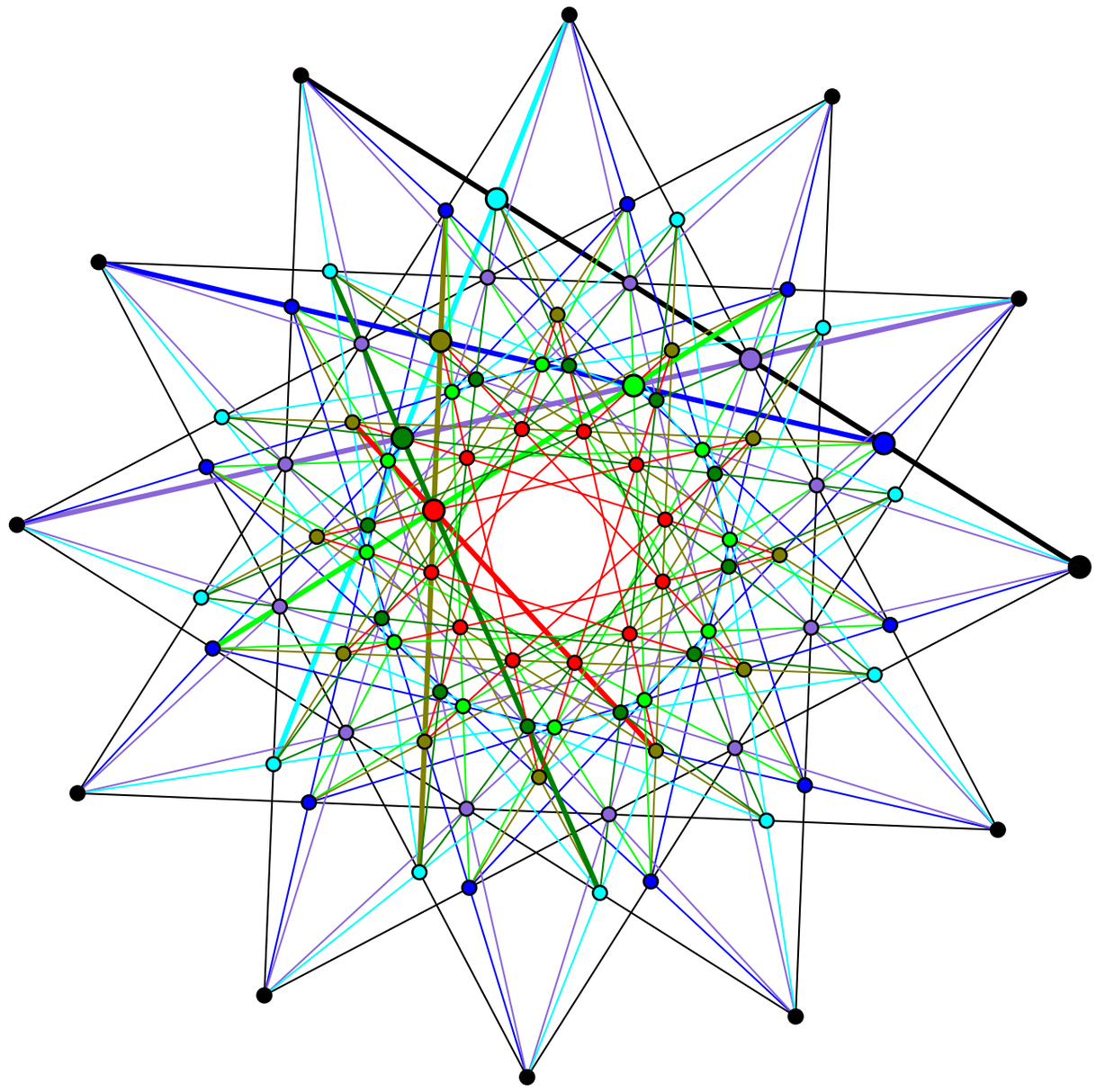
LARGER EXAMPLES?





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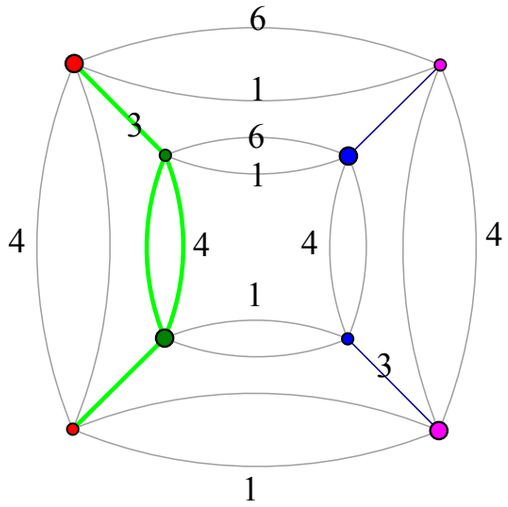
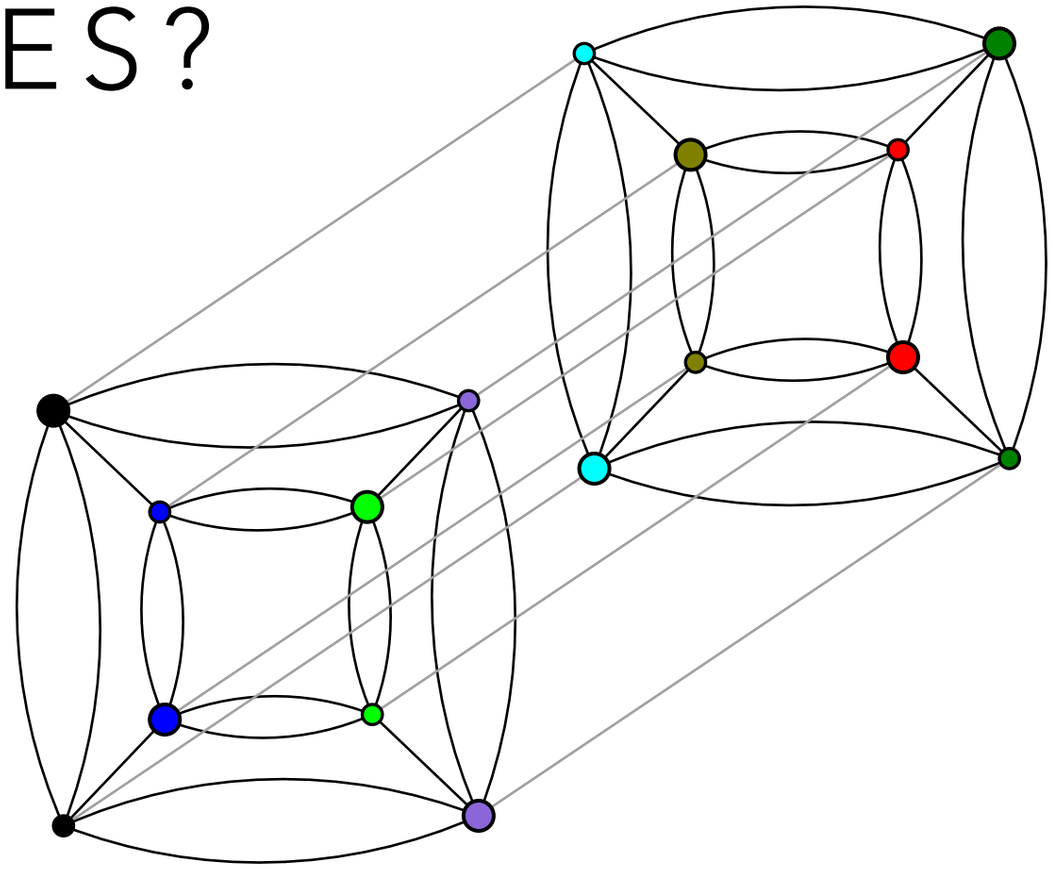
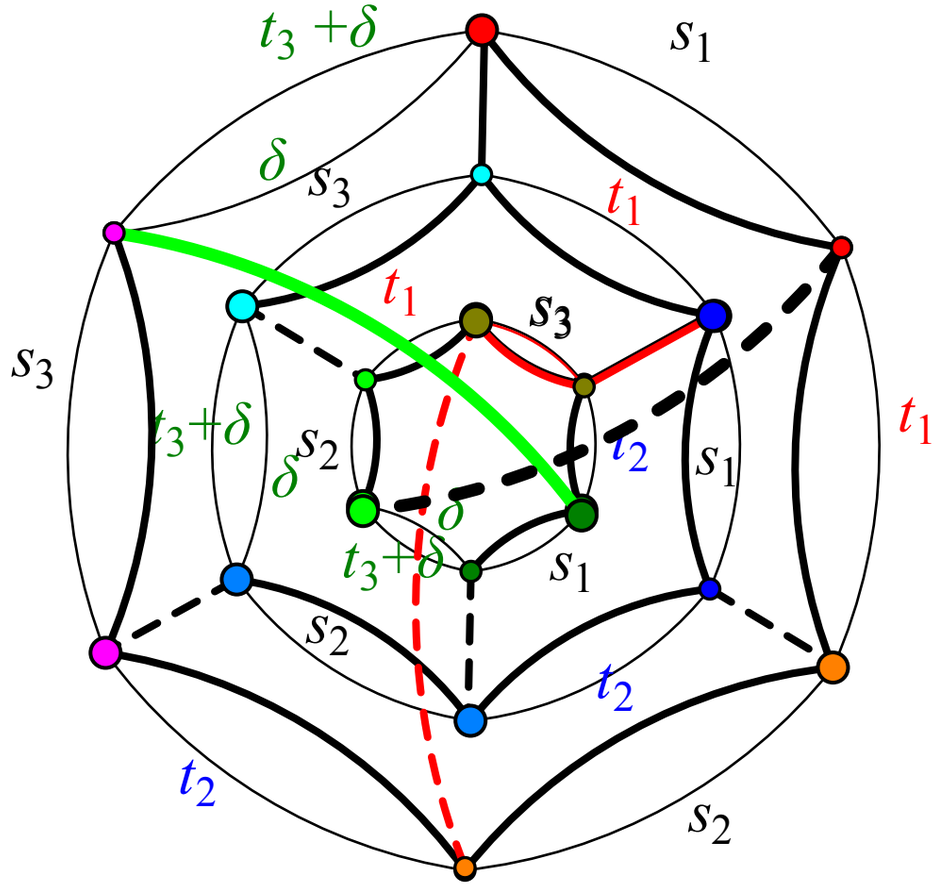
LARGER EXAMPLES?







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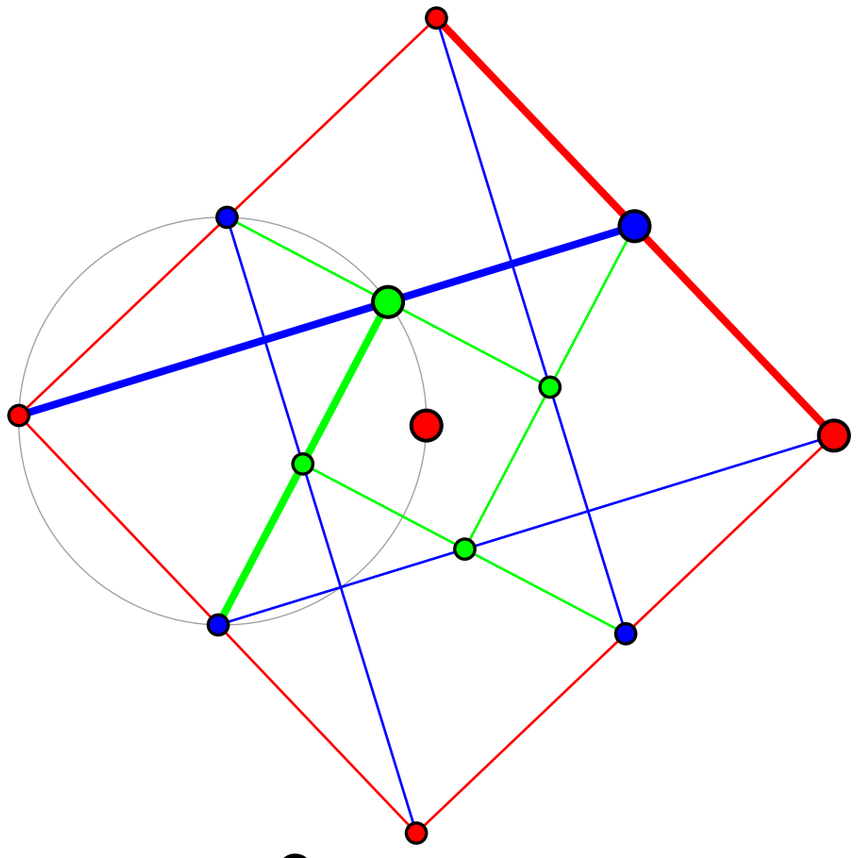
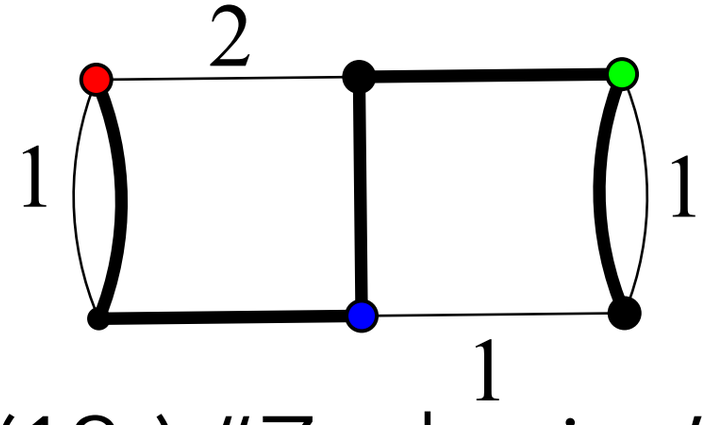
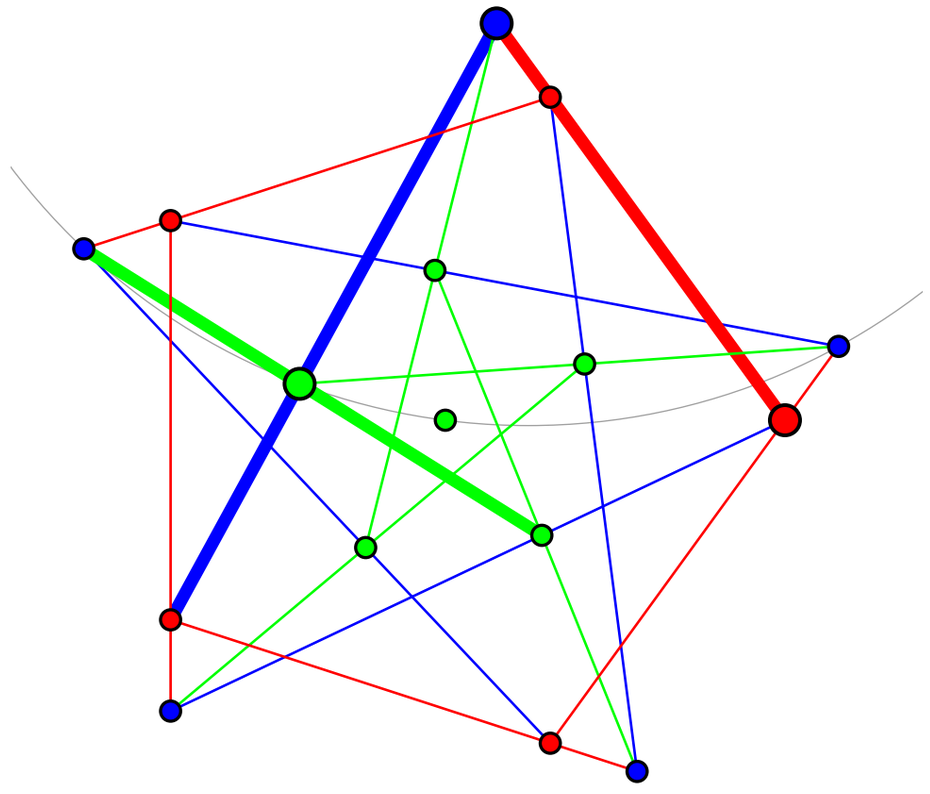
VERY NICE CLASSES?

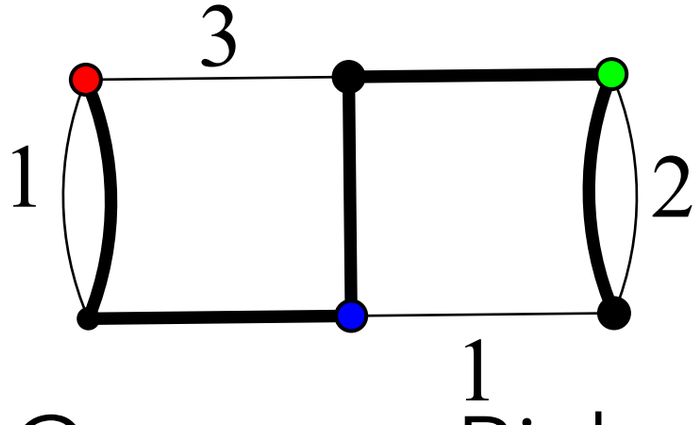
What can we say about the automorphism groups?





(12_3) "Zacharias"





(15_3) Cremona-Richmond

THANK YOU!